Anticipating Future Expected Utility and Coordination Motives for Information Decisions in Networks*

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Abstract

This paper develops a model of costly information acquisition by agents who are connected through a network. For an exogenously given network, each agent decides first on information acquisition about an underlying state from her neighbors and then, after processing the information acquired, takes an action. Each agent is concerned about how others align their actions with the state. We propose a novel equilibrium notion, with a behavioral motivation, in which the agents update their beliefs according to Bayes rule. This equilibrium notion requires that the agents, when deciding about information acquisition, anticipate the *expected* utility that they will have in the future with their information choice. Efficient and equilibrium information acquisition decisions are analyzed, and the compatibility between them is related to a measure that informs about the density of the network.

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1 Introduction

Networks within groups or organizations serve as conduits that carry news, information, and opinions about products, job vacancies and political programs: from our neighbors, friends, and co-workers, we acquire information which helps us to improve our knowledge about uncertain (payoff-relevant) variables. For example, researchers obtain information from their colleagues to understand better certain scientific problems, unemployed agents request information from their friends about job vacancies, and investors newly arrived to a sector acquire information from more experienced investors to obtain accurate estimates of the sector profitability.

Information acquisition is ubiquitous within networked groups and yet little is known about this phenomenon. How do agents interact with respect to their information acquisition decisions when they are connected through a network? How is the compatibility between efficient and equilibrium information acquisition related to the network architecture? To study these questions, this paper provides a game theoretical framework that models the transmission of information as a Bayesian belief revision process.

This model focuses on situations where each agent cares both about choosing an action appropriate to the underlying state and about the suitability of the other agents' actions to the state. Thus, we are interested in situations in which we enjoy a positive externality from the fact that other agents make correct decisions. This externality is clearly relevant in organizations in which its members are engaged in a joint enterprise. In these situations, we benefit from the performance or prestige of the entire organization: we naturally prefer to be members of an organization which performs well and has good reputation. This reputation is often earned because others make decisions which are appropriate to the underlying state. Theory suggests that the productivity and wages of an organization increase with the performance of its members in their tasks. For example, a branch of the literature on organization capital (e.g., Jovanovich, 1979; Prescott and Visscher, 1980; Becker, 1993) emphasizes the importance for the firm of its employees matches to the assigned tasks. In this paper, we label this concern about our colleagues' performance as *team concern*. In addition, in contrast with the approach followed by most of the related literature, we consider that, in principle, agents are not interested in matching their own actions with the actions taken by others. More precisely, we consider situations in which there are no strategic interactions in actions.

Suppose that, before choosing actions, the agents can make some information decisions to improve their knowledge of the state. For the class of preferences described above, our first conjecture could be that there are no strategic reasons for us to care neither about other agents' beliefs of the state nor about their information acquisition decisions. In fact, a standard game theoretical approach to this sort of situations (using perfect Bayes-Nash equilibrium, or any weaker or stronger concept) would conclude that strategic interactions in information decisions are also absent (see, e.g., Hellwig and Veldkamp, 2009).

That conclusion, however, does not provide a satisfactory explanation of some well documented forms of interdependence in information decisions present in groups or organizations such as collective denial or information avoidance. The theory of strategic information acquisition with strategic interactions in actions (e.g., Morris and Shin, 2002; Angeletos and Pavan, 2007; Calvó-Armengol and de Martí, 2007; Hellwig and Veldkamp, 2009; Myatt and Wallace, 2010, among many others) has provided important insights in several fields but does not rationalize the contagion of beliefs observed in a variety of real-world situations in which there are no underlying strategic interactions in actions. This form of interdependence of beliefs is viewed by a large literature of social and organizational psychology as a factor which plays a relevant role in a variety of phenomena such as market bubbles, political crises, investment and financial crashes, and organization failures. For instance, analyzing political episodes such as the Bay of Pigs invasion, the Cuban missile crisis and the escalation of the Vietnam war, Janis (1972) identified a pattern of spread of beliefs unrelated to any interdependence of actions, which he coined with the term "groupthink." Among other features, organizational psychologists identify selection bias in collecting information as a major symptom of groupthink.[1] Also, economic historians (e.g., Mackay, 1980; Kindleberger and Aliber, 2005; Shiller, 2005) account for many examples in which "contagious delusions", "manias", and "financial folly" are important contributing factors of financial crises.[2]

[1] Groupthink and its consequences have been documented, among others, by Cohan (2002), Hersh (2004), Eichenwald (2005), and Isikoff and Corn (2007).

[2] See Bénabou (2012) for an exhaustive survey of the literature on contagion of distorted beliefs and emergence of collective beliefs.

1.1 Motivation of the Solution Concept

This paper is an attempt to set out a model that allows for the presence interdependencies in beliefs without strategic interactions in actions when information is endogenous. In addition, as mentioned earlier, we wish to use the model to analyze information acquisition decisions when the agents are connected through a network.

Information acquisition is typically modeled through a two-stage game where the agents have incomplete information about the state. A simple specification is one in which nature chooses first a state realization (which remains unknown to the agents) and then the agents make information choices (about the state) in the first stage and choose actions in the second stage. Thus, some dynamic process is naturally involved in this sort of interactions. The solution concept typically used for these games is that of perfect Bayes-Nash equilibrium—henceforth, PBE—(or some weaker concept or a refinement of it). For the information acquisition game that we have described, PBE requires that each agent (a) chooses optimally her action in the second stage given the amount of information that she has collected in the first stage and (b) chooses optimally the amount of information that she acquires in the first stage, taking as given the actions that the agents optimally choose in the second stage. In this way, despite the fact that agents move sequentially, the game is solved as if decisions were taken simultaneously. This is the standard approach followed when the agents make, in general, different decisions over several periods.

Nevertheless, the games of information acquisition described above have in common a particular feature that makes them very special compared to other games: the choice made in the first stage does not simply corresponds to a payoff-relevant action but it determines the (posterior) beliefs (and, thus, the expected utility) that the agent has in the second stage. In other words, the preferences that the agents use to make their decisions in the second stage are endogenously chosen in the first stage. Then, how do agents anticipate in the first stage the form that their preferences will have in the second stage? This question is particularly relevant in environments in which we care about how others perform in their tasks. In these situations, our information choice in the first stage determines the precision with which we perceive in the second stage how the others align their actions with the state.

Although applying PBE to the class of games described above entails that we do take into account the future optimal actions that all the agents will choose (using their information decisions), it fails to capture how our information choice affects the way in which we anticipate our (endogenous) perception of our future utility. A realistic description of the problem that an agent faces in this sort of situations should be consistent with the following facts: (a) sequentiality matters since the agent obtains her preferences for the second stage as a result of her decision in the first stage (not to mention that in practice there might be a large temporal lag between both stages); (b) in the second stage, the agent is uncertain both about the state and the information received by others prior to choosing her action (unless she acquires full information); and (c) in the first stage, before making her information choice, the agent is not only uncertain (with her priors) about the state and about what others know, but she also anticipates that she will be uncertain (with her posteriors) in the second stage. In view of (c) above, a plausible theory of how the agent computes her expected utility in the first stage should capture the fact that she considers that in the second stage her preferences will be represented by an expected utility form which makes use of her posteriors.

As mentioned earlier, by using PBE to solve this class of games, one considers that both information and action decisions are taken simultaneously. More important, in order to compute an agent's expected utility in the first stage, one substitutes the optimal actions that all the agents choose in the second stage into the agent's utility *under certainty.* Thus, once optimal actions are determined for each information choice, the game is solved as if the agent anticipates in the first stage that she will face no uncertainty in the second stage (neither about the state nor about the private information finally held by others). In the standard approach there is no realistic story of how the agent evaluates her utility in the first stage by anticipating preferences under certainty for the second stage.

Also, using PBE, one assumes that in the first stage the agent takes as given the information choices of the others. Using this, the agent can obtain the *correct* distribution over the pieces of private information finally held by the others. In a realistic description, the agent should rather anticipate in the first stage that, in the second stage, she will use her posteriors (obtained through her information decision) to predict both the private information received by the others and the proximity of their actions to the state.

A plausible theory of how we acquire information when our future utility is affected by how others collect information should consider both that we anticipate that our future preferences will be under uncertainty, and that we incorporate the effects of our present information decisions over our future expected utility. Our view is that PBE is a solution concept suitable broadly for sequential games under incomplete information but that our approach might be (at least under some circumstances) more finely tailored to the specific problem at hand.

In this paper, we propose a solution concept for the class of information acquisition games described above which is based on an alternative theory/description of how the agents evaluate their utilities at the moment in which they acquire information. This approach has the central concern that the agents, when evaluating their first-stage preferences, (a) anticipate that they will have preferences under uncertainty in the future and (b) include the effects induced by their information decisions over their future expected utilities. The key element of our approach is that, when deciding about information acquisition, an agent cannot predict correctly what other agents will learn with their information choices and, therefore is uncertain about their optimal actions. In the first stage, each agent puts herself in the situation under uncertainty that she will face in the second stage with her information choice. This assumption can be seen as a failure of (or lack of confidence in) contingent thinking.

More specifically, we assume that each agent computes her first-stage expected utility as the nested expected value (according to her priors and taking as given the information acquisition strategies of all agents) of her possible *expected* utilities (according to her posteriors) in the second stage. In this way, the agent anticipates her future expected utility. In contrast, in a PBE, the agent computes her first-stage expected utility as the expected value of her future utilities under certainty, evaluated in the agents' optimal actions. This is the only difference between our approach and the standard one. In addition, exactly as one considers in a PBE, we assume that each agent's second-stage expected utility is computed as the expected value of her utility (according to her posteriors), that posterior beliefs are consistent with strategies according to Bayes rule, and that each agent is sequentially rational in each stage of the game. These conditions define the solution concept, Equilibrium with Anticipation of Expected Utility (EAEU), that we use in this paper to analyze interactions in information decisions.

Following the motivation given above, we then use the EAEU solution concept to explore information acquisition between agents who are connected through an exogenously given network. We use a two-stage game as the one described earlier. In the first stage, each agent chooses the amount of information that she acquires, at a cost, from her neighbors. In the second stage, each agent chooses an action. We assume that the agents are able to receive information only from their direct neighbors in the network.

1.2 Preview of the Results

Because each agent anticipates, with some degree of uncertainty, how others align their actions with the state, there arise linkages between the agents' information acquisition decisions. In our model, the network structure affects the incentives of the agents to acquire information as others improve their own information. In particular, Proposition 2 shows that the incentives of an agent to acquire information from a neighbor decrease with the amount of information that the rest of neighbors of that neighbor collect from her. Thus, substitutability in information acquisition arises between agents indirectly connected (through a common neighbor) in the network. This interdependence of information decisions leads to an emergence of correlated (distorted) posteriors which is consistent with the various phenomena mentioned earlier of contagion of beliefs without strategic interactions in actions. The substitutive nature of information decisions is also consistent with recent theoretical developments on investment decisions in international financial markets. For instance, Nieuwerburgh and Veldkamp (2009) provide an explanation for the home bias puzzle which is based on the presence of substitutive information acquisition decisions between investors.

Under the class of preferences assumed in the present paper, each agent is risk-averse with respect to the discrepancy between each of the other agents' actions and the state. This assumption plays an important role in the mechanism which drives our results. When some agent k acquires more information from a neighbor j of us, k's optimal action becomes closer to the true value of the state. Since we are risk-averse with respect to the distance between the state and the action taken by agent k, we benefit from a decrease on the level of riskiness of our uncertain stream of utilities. On the other hand, by acquiring more information from our neighbor j, we increase the precision with which we perceive both our future utility and how agent k's action approaches the state. Thus, both information decisions enter multiplicatively in our first-stage expected utility and, in particular, they interact in a substitutive way.

The model's welfare analysis provides conditions in terms of a precise measure of the network density, namely, the minimum degree of the network, under which efficient information acquisition can be attained in equilibrium. These implications, stated in Corollary 1, suggest that the compatibility between efficiency and equilibrium is favored when the network allows each agent to acquire information from a sufficiently large number of neighbors.

1.3 Related Literature

An approach closely related to the one pursued in this paper is that recently followed by Bénabou (2012). Motivated by the phenomenon of groupthink, and by other instances of documented contagion of distorted beliefs, he proposes a model with endogenous information in which the agents have anticipatory preferences. As in the present paper, he considers that there are neither strategic interactions in actions nor signals that could lead to herding or social learning. The key mechanism in his model is that, when the agents anticipate their utilities from future prospects, there arise cognitive linkages that relate how they accept (or deny) new private information with the way in which others deal with their own new pieces of information. In his model, "thinking styles thus become strategic substitutes or complements" despite the assumption that there are no strategic interactions in actions. The message conveyed by Bénabou's (2012) model has a clear resemblance with the insights provided by the present paper. Both models share the common feature that there is a connection between the information that the agent receives about the information possessed by others and her "overall" utility. Our model, however, differs from his in that he considers explicitly that the agents derive utility from anticipating their future preferences while we assume that the agents, through their information decisions, affect the precision with which they anticipate their perception of how others improve their knowledge from their information decisions.

Our approach is also related to the literature on cognitive dissonance[3] since, given our assumption which specifies how the agents compute their first-stage expected utilities, an agent's information choice can be viewed as a decision over how she perceives her future utility. In their seminal paper on cognitive dissonance within the field of economics, Akerlof and Dickens (1982)

[3]See, e.g., Akerlof and Dickens (1982), Schelling (1986) Rabin (1994), Bénabou and Tirole (2002, 2004), Compte and Postlewaite (2004), and Di Tella, Galiani, and Schargrodsky (2007).

(p. 307) describe the behavioral features studied in their model as:

> "First, persons not only have preferences over states of the world, but also about their beliefs about the state of the world. Second, persons have some control over their beliefs . . . . Third, it is of practical importance for the application of our theory that beliefs once chosen persist over time."

The approach pursued in this paper captures these three features above as important ingredients of the model. Yet, the present model is different from the models of cognitive dissonance in that we consider that agents anticipate future situations under uncertainty while cognitive dissonance implies that the agents make (intentionally) incorrect assessments about the state. In addition, our model studies the interaction between the beliefs of agents who face a common enterprise, a question which is not in principle addressed by the literature on cognitive dissonance.

Our approach to modeling strategic interactions in information decisions stems from these earlier contributions within the behavioral economics literature. Yet, following the traditional approach, some papers have recently analyzed communication networks using Bayesian belief revision processes to model information transmission. A common feature of these models is that they consider strategic interactions in actions. Calvó-Armengol and de Martí (2007) consider a framework where agents communicate through a given network as a result of a Bayesian belief revision process that takes place in successive rounds. An important difference with the present paper is in that they do not consider endogenous information transmission decisions. Hagenbach and Koessler (2010) consider a model where each agent decides whether or not to reveal her private information to the others before choosing her own action. The choices on information revelation determine endogenously a communication network. The main difference with the present paper is in the fact that, instead of information acquisition, they deal with information revelation decisions. In their framework, an agent's information choice does not affect her own future preferences under uncertainty. Thus, the motivation given earlier for our approach is not compelling for the problem that they analyze. Two other papers that, following the traditional approach, have recently analyzed endogenous information transmission within networks are Calvó-Armengol, de Martí, and Prat (2011), and Galeotti, Ghiglino, and Squintani (2011).

The rest of the paper is organized as follows. The model, and the notions of EAEU and efficiency are introduced in Section 2. Section 3 studies efficient and equilibrium information acquisition decisions, and presents the results that relate the compatibility between equilibrium and efficiency to the network density. Section 4 concludes with a discussion of the results. The proofs of propositions 1 and 2 are grouped together in the Appendix.

2 The Model

2.1 Network Notation

We follow the notation developed by Jackson and Wolinsky (1996). There is a finite set of agents $N := \{1, \ldots, n\}$, with $n \geq 2$. The shorthand notation ij denotes the subset of N, of size two, containing agents i and j, which is referred to as the *link* ij. A *communication network* g is a collection of links where $ij \in g$ means that i and j are directly linked and able to acquire information from each other under network g. We assume that the architecture of the network itself is exogenously given and common knowledge. Let G denote the set of all possible networks on N. For a network $g \in G$, the set of agent i's *neighbors* is $N_i(g) := \{j \in N : ij \in g\}$ and the number of her neighbors is $n_i(g) := |N_i(g)|$. We consider that $ii \in g$ for each $i \in N$ and each $g \in G$ so that $i \in N_i(g)$ by construction. Finally, let $\delta(g) := \min_{i \in N} \; n_i(g)$ denote the minimum degree of network g, a measure which informs us about the density of network g.

2.2 Information Structure and Preferences

Given a network $g \in G$, agents are engaged in a game that is played in two consecutive stages, numbered 1 and 2. In stage 1, each agent i decides the amount of information that she acquires from each agent in her neighborhood $N_i(g)$. We consider that each agent acquires full information from herself. Information decisions are simultaneous. In stage 2, each agent chooses an action using the information that she has acquired from her neighbors in stage 1. Actions are simultaneous as well.

We consider an information structure with complementarities. The initial private information held by each agent i is described by her *type* $\theta_i \in \Theta_i := \mathbb{R}$. A *state of the world* is a vector $\theta := (\theta_i)_{i \in N} \in \Theta := \times_{i \in N} \Theta_i = \mathbb{R}^n$ so that agent i's type is the respective coordinate θ_i of the actual state θ.[4] All aspects of this game, except θ, are common knowledge. Thus, the assumed information structure exhibits complementarities in the sense that any two distinct agents improve their knowledge of the underlying state by sharing their pieces of private information. In particular, the true state is always obtained by combining the pieces of private information of all the agents.[5]

Although the proposed information structure relates generally to situations with informational complementarities, the main motivation of this structure comes from situations where the agents face (independently) a common decision problem with several "aspects" so that solving the problem requires to solve the various aspects. Each agent can be seen as an "expert" in one aspect so that the knowledge about how to solve the problem is improved by information sharing. The goal of this information structure is to capture environments where the agents face a joint enterprise and there are complementarities in the initial information that they possess.

An *action* for agent i is an n-coordinate vector $a_i := (a_{ik})_{k\in N} \in A_i := \mathbb{R}^n$. Notice that the action space available to each agent coincides with the state space. The idea here is to think of an action as a collection of all the independent steps that an agent must take in order to solve her decision problem (i.e., one step for each aspect of the problem). Thus, the k-th coordinate a_{ik} of agent i's action vector summarizes the action taken by agent i with respect to the k-th aspect of the decision problem. Let $a := (a_i)_{i\in N} \in A := \times_{i\in N} A_i = \mathbb{R}^{n^2}$ denote an *action profile.*

We consider a class of preferences under which there are no strategic interactions over actions. Each agent wishes to match her own action with the true state and, in addition, is concerned about the extent to which the other agents align their actions with the state. We use the term *team concern* to designate this second motive. We wish to capture situations in which the organization's profits (either monetary or in terms of prestige) increase with the performance of its members in their tasks and in which each member is accordingly rewarded in terms of reputation or monetary payments.[6]

[4] The proposed set of states is similar to those used in models of multidimensional cheap talk. See, e.g., Chakraborty and Harbaugh (2007), and Levy and Razin (2007).

[5] Jiménez-Martínez (2006) proposes an analogous information structure to study a two-agent information sharing problem.

[6] Think, for example, of a set of investors choosing their investment strategies in a new sector, where the profitability of the sector increases with the number of investors who make good investment decisions. In this case, each investor naturally cares about the extent to which other investors align their actions with the state.

The utility to agent i is described by a function $u_i : A \times \Theta \to \mathbb{R}$, specified as

$$u_i(a, \theta) = -(1-r)\|\theta - a_i\|^2 - \frac{r}{n-1}\sum_{j\neq i}\|\theta - a_j\|^2, \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm. The parameter $r \in [0, 1]$ above measures agent i's concern about the alignment of the others' actions with the state. The first term in equation (1) is the quadratic loss in the (Euclidean) distance between agent i's own action and the state. The second term is the team concern, i.e, the payoff loss derived from the discrepancy between the other agents' actions and the state. Since an agent's utility is strictly decreasing with respect to the distance between her action and the state, she has incentives to acquire information in order to pick actions better suited to the state. Note also that the specified preferences represent common interests for the agents.

Although the class of preferences described above is very specific, it can be viewed as a second-order approximation of a more general class of convex preferences. While the assumptions imposed on preferences make the analysis tractable, they enable us to work with all the relevant ingredients in an environment without strategic interactions over actions and with (positive) external effects.

2.3 The Information Transmission Process

Each agent can receive information from another agent through a message realization, whose distribution is conditional on the sender's type. A message received by an agent i from another agent j is denoted by $m_{ij} \in \mathbb{R}$. We use $m_i := (m_{ij})_{j\in N} \in \mathbb{R}^n$ to denote a vector of messages received by agent i, and $m := (m_i)_{i\in N} \in M := \mathbb{R}^{n^2}$ to denote a *message profile.* A natural way to interpret a message $m_{ij} = \theta_j$, privately listened by agent i, is as a statement that agent j's type takes the value θ_j.

Note that, in principle, we allow each agent to receive messages from any agent in the group (including herself). However, the network structure g restricts the information that the agents ultimately receive from others. In particular, an agent i will obtain no information whatsoever from the message m_{ij} received by any agent $j \notin N_i(g)$, i.e., who is not in her neighborhood. Also, an agent i will obtain full information from her own message m_{ii} (this information, however, is redundant since she already knows her type). Below we describe in detail how these features are modeled.

Following the pertinent literature, we consider a Gaussian information structure for tractability. At the beginning of stage 1, nature draws a state realization θ from a multi-normal distribution with mean vector $\mu\underline{1}$ and variance-covariance matrix $\sigma^2 I$, where $\mu \in \mathbb{R}$, $\sigma^2 > 0$, $\underline{1}$ denotes the n-dimensional vector of ones, and I denotes the $n \times n$ identity matrix. This distribution summarizes the (common) priors of the agents about the state. Notice that we are assuming that each type θ_i is normally distributed with mean μ and variance σ^2 and that, furthermore, the agents' types are independent.

After the state realization is drawn, each agent i learns her type and chooses, for each of agent $j \in N$, the precision of the message through which she receives information from j. This choice regarding another agent's message can be naturally interpreted as a decision about the quality of the message service through which we receive information from her. In practice, this choice is often costly. Then, according to this choice, the agent receives privately a message realization from each agent in the group (including herself). All messages are sent simultaneously.

We assume that, for each $i, j \in N$, $m_{ij} = \theta_j + \varepsilon_{ij}$, where ε_{ij} is an idiosyncratic noise in the information acquisition process of agent i from agent j. This additive structure for the information transmission process can be also expressed as $m_i = \theta + \varepsilon_i$, where $\varepsilon_i := (\varepsilon_{ij})_{j \in N}$. For each agent $i \in N$, we assume that each noise term ε_{ij} is independent of the type θ_j, as well as of each ε_{ik}, $k \neq j$. In addition, we assume that each pair of random vectors ε_i and ε_k, $i \neq k$, are independent.

We further assume that each noise term ε_{ij} is normally distributed with zero mean and variance $\psi_{ij}^2 \geq 0$. Notice that the informativeness of the message m_{ij} can then be made endogenous by allowing agent i to choose the variance ψ_{ij}^2 at a cost. This modeling choice to endogenize information acquisition decisions is standard in the related literature.[7] Intuitively, lower variances of the associated noise corresponds to more informative messages. Formally, we define an *information acquisition choice for agent i with respect to agent j* as a value for the parameter $\omega_{ij} := \sigma^2/(\sigma^2 + \psi_{ij}^2) \in [0, 1]$, which reflects the precision of the message strategy with which agent i acquires information from agent j. The precision ω_{ij} chosen by an agent i with respect to a neighbor $j \in N_i(g)$ determines, via Bayesian updating, her posteriors about θ_j, which she uses in stage 2 to choose her action. If $j \notin N_i(g)$, then the network structure g does not allow

[7] See, e.g., Angeletos and Pavan, (2007), Dewan and Myatt (2008), and Myatt and Wallace (2010).

agent i to receive information from agent j. We model this by setting $\omega_{ij} := 0$ for $j \notin N_i(g)$. Also, we set $\omega_{ii} := 1$ for each $i \in N$, which implies that each agent receives full information from herself (though this information is obviously redundant since she already knows her type). Given these conventions, let $\omega_i := (\omega_{ij})_{j\in N} \in [0,1]^n$ denote an *information acquisition strategy* for agent i and let $\omega := (\omega_i)_{i\in N} \in \Omega := [0,1]^{n^2}$ denote an *information acquisition profile.*

The *cost of information acquisition* for each agent $i \in N$ with respect to each agent $j \in N$ is described by a function $c(\omega_{ij})$, which is assumed to satisfy $c(0) = 0$, and to be strictly increasing and (weakly) convex in $\omega_{ij} \in [0,1]$.

Each agent i is able to observe the information decision ω_{-i} of the others before she chooses her action in the second stage. This assumption is natural if we think that affecting the technology through which we receive information requires some investments and has observable consequences. Nonetheless, an agent i cannot observe the particular message m_{kj} received by any other agent $k \neq i$ from any of her neighbors $j \in N_k(g)$. In other words, an agent can observe the message service used by others to receive information but not the particular messages that they receive.

Note that, since types are independent, an agent can update her beliefs over the state of the world by doing separately the corresponding update over each of her neighbors' types. Let $p(\theta)$ denote the density function that describes any agent's priors about the state and let $q(\theta, m_{-i}|m_i, \omega_i)$ denote the density function which describes agent i's posteriors about the state and the messages received by the other agents. Since types are independent and identically distributed, we can write $p(\theta) = \prod_{i\in N} h(\theta_i)$, where $h(\theta_i)$ denotes the (marginal) density for any type θ_i, $i \in N$.

Regarding the Bayesian updating of beliefs, some basic results on normal distributions imply that the random variable $\theta_j|m_{ij}, \omega_{ij}$ is normally distributed with mean and variance

$$E[\theta_j|m_{ij}, \omega_{ij}] = \omega_{ij} m_{ij} + (1-\omega_{ij})\mu, \qquad \mathrm{Var}[\theta_j|m_{ij}, \omega_{ij}] = \sigma^2(1-\omega_{ij}). \tag{2}$$

From the expression in (2) above, we observe that the posterior variance $\mathrm{Var}[\theta_j|m_{ij}, \omega_{ij}]$ is strictly decreasing in the precision ω_{ij} of agent i's information decision with respect to agent j. Hence, the informativeness of an information choice about an agent's type can be completely ranked according to the induced posterior variance of that agent's type.

Other interesting inferences that an agent makes under our information structure are as follows. Consider a pair of distinct agents $i, k \in N$ who can acquire information from a common neighbor, $j \in N_i(g) \cap N_k(g)$. It follows that the random variable $m_{kj}|m_{ij}, \omega_{ij}$ is normally distributed with mean and variance

$$E[m_{kj}|m_{ij}, \omega_{ij}] = \omega_{ij} m_{ij} + (1 - \omega_{ij})\mu, \qquad \text{Var}[m_{kj}|m_{ij}, \omega_{ij}] = \sigma^2(1 - \omega_{ij}\omega_{kj})/\omega_{kj}. \tag{3}$$

Also, the conditional covariance between the type θ_j and the message received by agent k from agent j is given by

$$\text{Cov}[\theta_j, m_{kj}|m_{ij}, \omega_{ij}] = \sigma^2(1 - \omega_{ij}). \tag{4}$$

Finally, note that the assumed information structure also implies that, for any pair of distinct agents $i, k \in N$ who can acquire information from a common neighbor j, we have $\text{Cov}[m_{ij}, m_{kj}|\theta_j, \omega] = 0$. Thus, any two messages m_{ij} and m_{kj} received by different agents from a common neighbor are (conditionally) independent and there is no public information component which could give rise to spread of beliefs between indirectly connected agents.

2.4 Equilibrium (EAEU) and Efficient Information Acquisition

An *action strategy for agent i with respect to aspect j* is a function $\alpha_{ij} : M \times [0, 1] \to \mathbb{R}$ that associates her choice of action $a_{ij} = \alpha_{ij}(m_{ij}, \omega_{ij})$ (over coordinate j of the action space) to the message m_{ij} that she receives from agent j and to her information acquisition choice ω_{ij}. Since types are independent and all messages are sent simultaneously, an agent's choice of action over a particular coordinate depends *only* on the message that she receives from the expert in that aspect of the problem, as specified. An *action strategy for agent i* is then a function $\alpha_i : \mathbb{R}^n \times [0, 1]^n \to A_i$ defined by $\alpha_i := (\alpha_{ij})_{j \in N}$. An action strategy α_i associates agent i's action a_i to the vector of messages m_i that she receives and to her information acquisition strategy ω_i. Let $\alpha := (\alpha_i)_{i \in N}$ denote an *action strategy profile.* Note that an action strategy profile $\alpha(m, \omega)$ specifies an action profile for a message profile m and an information acquisition profile ω.

The *stage 2–expected utility* of agent i, for a given information acquisition profile ω and a given vector of messages m_i received by agent i, is

$$\begin{aligned} U_{i,2}(\alpha; \omega, m_i) &:= E\big[u_i\big(\alpha_i(m_i, \omega_i), (\alpha_j(m_j, \omega_j))_{j \neq i}, \theta\big) \bigm| m_i, \omega\big] \\ &= \int_\Theta \int_{M_{-i}} q(\theta, m_{-i}|m_i, \omega_i) u_i\big(\alpha_i(m_i, \omega_i), (\alpha_j(m_j, \omega_j))_{j \neq i}, \theta\big) dm_{-i} d\theta. \end{aligned}$$

Of course, this is the expected utility that one considers for stage 2 using the traditional approach as well.

To illustrate how agents compute their expected utilities in the first stage under our approach, let us consider first the traditional approach. Note that, under our assumptions on the information transmission process, the random vector $m|\theta,\omega$ follows a multi-normal distribution. Let $f(m|\theta,\omega)$ denote the corresponding density, which describes the probability with which the message profile m is sent conditioned on the state θ and on the information acquisition profile ω. Also, since messages are sent independently, we can write $f(m|\theta,\omega)=\prod_{i\in N}\prod_{j\in N}g(m_{ij}|\theta_j,\omega_{ij})$, where $g(m_{ij}|\theta_j,\omega_{ij})$ denotes the (marginal) conditional density for message m_{ij}.

In the first stage, for a given information acquisition profile ω, each agent uses the density $f(m|\theta,\omega)$ to compute the expectation of her utility evaluated in the agents' action strategies. In addition, using her priors, the agent computes her expectation of that expected value. Then, given an action strategy profile α and an information acquisition profile ω, the term $\int_M f(m|\theta,\omega)u_i\big(\alpha(m,\omega),\theta\big)dm$ can be naturally interpreted as the anticipation of agent i's future stream of utilities $\big\{u_i\big(\alpha(m,\omega),\theta\big)\,:\,m\in M\big\}$ (that is, under certainty since no expected utility is considered) for a given $\theta\in\Theta$. Given this, note that in a PBE (or a weaker or stronger solution concept), agent i anticipates in the first stage her future stream of utilities by computing

$$E\Big[E\big[u_i\big(\alpha(m,\omega),\theta\big)\,\big|\,\theta,\omega\big]\Big]=\int_\Theta p(\theta)\Big[\int_M f(m|\theta,\omega)u_i\big(\alpha(m,\omega),\theta\big)dm\Big]d\theta.$$

Therefore, the agent anticipates the set of her possible ex-post utilities $\big\{u_i\big(\alpha(m,\omega),\theta\big)\,:\,m\in M\big\}$. The agent anticipates as well that she will consider such utilities under certainty. PBE entails the idea that the agents make simultaneously their decisions for both stages of the game. This logic is appropriate to study most sequential games. However, some time separation seems a fundamental aspect of a game in which the agents decide first on their future posteriors and then use those posteriors to make another decision. Our view is that there are limits, due maybe to cognitive restrictions, in the agents' abilities to compute simultaneously both the influence of everyone's information decision on optimal actions and how this ultimately affects their own utilities.

In addition, note that application of PBE to our two-stage game implies that an agent i uses in the first stage the density $f(m|\theta,\omega)$ to anticipate her future utilities. Then, at the moment in which she decides about information acquisition, she takes as given the densities $g(m_{kj}|\theta_j, \omega_{kj})$ chosen by other agents k (with respect to any other agent $j \neq i$) and, accordingly, anticipates with certainty not only her own future utility but also the message realizations m_{kj} privately received by any other agent k. Thus, following the traditional approach, one computes an agent's expected utility in the first stage as if the agent not only observes the message service used by others but as if she also learns the correct distribution over the private messages that others receive. A plausible theory about how the agent computes her expected utility in the first stage should consider that she rather anticipates her posteriors about the messages received by other agents.

Our approach is based on the assumption that the agents are not able to make simultaneously their decisions for both stages and that they anticipate in the first stage their stream of future utilities under uncertainty (or interim utilities). Specifically, we assume that, given an action strategy profile α and an information acquisition profile ω, each agent $i \in N$ anticipates her stream of utilities $\{U_{i,2}(\alpha; \omega, m_i) \, : m_i \in \mathbb{R}^n\}$, which are computed using the posteriors that her information decision generates. Thus, each agent i anticipates in the first stage her future stream of utilities by computing

$$E\Big[E\big[U_{i,2}(\alpha;\omega,m_i)\,\big|\,\theta,\omega\big]\Big] \\ = \int_{\Theta} p(\theta)\left[\int_M f(m|\theta,\omega)\Big(\int_{\Theta}\int_{M_{-i}} q(\theta, m_{-i}|m_i,\omega_i) u_i\big(\alpha(m,\omega),\theta\big) dm_{-i} d\theta\Big) dm\right] d\theta.$$

Consequently, we assume that agent i's *stage 1–expected utility*, $U_{i,1}$, is specified as

$$U_{i,1}(\omega) := \int_{\Theta} p(\theta)\left[\int_M f(m|\theta,\omega) U_{i,2}(\alpha;\omega,m_i) dm\right] d\theta - \sum_{j \in N_i(g)} c(\omega_{ij}). \tag{5}$$

Under the formulation above, we are assuming that the agent anticipates her stage 2–expected utility $U_{i,2}(\alpha; \omega, m_i)$ instead of her utility under certainty $u_i(\alpha(m, \omega), \theta)$. This is the only difference between our approach and the traditional one.

Finally, note that our approach can be alternatively interpreted as considering that the agent is divided into two selves so that the stage 1–self anticipates the self that she will be in stage 2 with her information decision. On the other hand, under the traditional approach, one assumes that the agent can make use in stage 1 of the others' information decisions in order to obtain inferences about their private pieces of information (i.e., the messages that they will receive). However, the agent will in general not be able to make such inferences in stage 2. In the traditional approach, prior to making her information decision, the agent can predict (correctly) the private messages that anyone will obtain in stage 2 and can compute the contingent optimal actions of all agents. Furthermore, the traditional approach considers that all these computations are made simultaneously in a single period. Our assumption on how the agent computes her stage 1–expected utility is based on our view that there may be cognitive limits to the way in which the agent can make such calculations.

We restrict attention to equilibrium in pure strategies.

Definition 1 (EAEU). Given a network $g \in G$, an *Equilibrium with Anticipation of Expected Utility* (EAEU) is an information acquisition profile ω^* and an action strategy profile α^* such that the following conditions are satisfied for each agent $i \in N$:

(i) *sequential rationality in stage 2;* for each $\omega \in \Omega$ and each $m_i \in \mathbb{R}^n$,

$$U_{i,2}(\alpha^*; \omega, m_i) \geq U_{i,2}(\alpha_i, \alpha^*_{-i}; \omega, m_i) \quad \text{for each} \quad \alpha_i.$$

(ii) *sequential rationality in stage 1;* for each α,

$$U_{i,1}(\omega^*) \geq U_{i,1}(\omega_i, \omega^*_{-i}) \quad \text{for each} \quad \omega_i \in [0,1]^2.$$

(iii) *belief consistency;* for each $\omega \in \Omega$ and each $m_i \in \mathbb{R}^n$, (a) each random variable $\theta_j | m_{ij}, \omega_{ij}$ is normally distributed with mean and variance given by (2), and (b) each random variable $m_{kj} | m_{ij}, \omega_{ij}$, for $k \in N_j(g) \setminus \{i\}$, is normally distributed with mean and variance given by (3).

The welfare measure that we propose to gauge the efficiency properties of equilibrium profiles makes use of the sum of the expected utilities of all the agents in stage 1. We require that the agents choose optimally their action strategies in stage 2 and then compare information acquisition profiles. Hence, we allow the planner to change the information initially acquired by the agents, who will then pay the corresponding *new* cost of information acquisition and use such information to optimally choose their actions. Specifically, for each message profile $m \in M$ and each information acquisition profile $\omega \in \Omega$, the planer considers in stage 2 a welfare function

W_2, defined by

$$W_2(\theta; m, \omega) := \sum_{i \in N} u_i(\alpha^*(m, \omega), \theta),$$

where each α_i^*, $i \in N$, satisfies the (stage 2) sequential rationality requirement stated in condition (i), using the posteriors described in condition (iii), of Definition 1 above. An important difference in our model between an agent's behavior and the planner's choice should be noted. Although the planner is able to change the agents' posteriors, the planner's choice does not affect his own perception of the agents' stream of future utilities. Therefore, we assume that the planner computes his expected utility in stage 1 by anticipating future utilities under *certainty.* This is in contrast with our key assumption about how an agent anticipates her own future possible utilities. This modeling assumption is based on the idea that the planner's choice in stage 1 affects the agents' future preferences but not his own. Thus, our proposal is that the planner considers a *stage 1–welfare function* W_1, specified as

$$\begin{aligned} W_1(\omega) &:= E\Big[E\big[W_2(\theta; \omega, m) \,\big|\, \theta, \omega\big]\Big] - \sum_{i \in N} \sum_{j \in N_i(g)} c(\omega_{ij}) \\ &= \int_{\Theta} p(\theta) \left[\int_M f(m|\theta, \omega) \sum_{i \in N} u_i\big(\alpha^*(m, \omega), \theta\big) dm \right] d\theta - \sum_{i \in N} \sum_{j \in N_i(g)} c(\omega_{ij}), \end{aligned} \tag{6}$$

where each α_i^*, $i \in N$, satisfies the sequential rationality requirement stated in condition (i), using the posteriors described in condition (iii), of Definition 1.

Definition 2. Given a network $g \in G$, an information acquisition profile $\widehat{\omega}$ is efficient if $W_1(\widehat{\omega}) \geq W_1(\omega)$ for each $\omega \in \Omega$.

3 Main Results

This section studies efficient and equilibrium (using EAEU) information acquisition, and relates the compatibility between them to the network density.

An important observation that follows from our utility specification (equation (1)) is that the welfare function evaluated in stage 2, W_2, takes the form

$$W_2(\theta; m, \omega) = -\sum_{i \in N} \sum_{j \neq i} \big(\theta_j - \alpha_{ij}^*(m_{ij}, \omega_{ij})\big)^2.$$

Hence, the planner seeks to keep the action of each agent close to the underlying state and ignores the team concern of each agent. This is due to the fact that agents are ex-ante identical so that the influence of each agent's action on any other agent's utility is homogenous across agents.

We restrict attention to interior information decisions. A clarification is then in order since, by construction, $\omega_{ij} = 0$ if $j \notin N_i(g)$ and $\omega_{ii} = 1$. In the sequel, by interior information decision profile, we mean a profile in which each agent $i \in N$ chooses an interior information decision $\omega_{ij} \in (0, 1)$ with respect to each neighbor other than herself, i.e, for each $j \in N_i(g) \setminus \{i\}$. The following two propositions analyze, respectively, efficient and equilibrium profiles in which all information decisions are interior.

Proposition 1. *For each network $g \in G$, an interior efficient information acquisition profile $\widehat{\omega}$ is characterized by the condition $c'(\widehat{\omega}_{ij}) = \sigma^2$ for each $i \in N$ and each $j \in N_i(g)$.*

The result in Proposition 1 above simply expresses the intuitive insight that efficiency is characterized by the condition that, for each link ij in the network, the marginal cost of information acquisition must equal the marginal benefit of information acquisition.

The next proposition characterizes an agent's best response in information decisions. In our model, the optimal decision of an agent i with respect to the information that she acquires from a neighbor $j \in N_i(g)$ depends on the information decisions of the other neighbors of agent j. Thus, under the assumption that the agents anticipate their future stream of expected utilities, there arise linkages that relate the information decisions of agents who are indirectly connected through common neighbors.

Proposition 2. *Consider a network $g \in G$ and an interior information acquisition profile ω^* corresponding to some EAEU. Then, the optimal information decision of an agent i with respect to a neighbor $j \in N_i(g)$ is characterized by the condition*

$$c'(\omega^*_{ij}) = 2(1-r)\sigma^2 + r\sigma^2/(n-1) \sum_{k \in N_j(g) \setminus \{i\}} [1 + \omega^*_{kj}(\omega^*_{kj} - 2)].$$

Consider two agents i and k who can acquire information from a common neighbor j. Since the term $\omega_{kj}(\omega_{kj} - 2)$ decreases with $\omega_{kj} \in (0, 1)$, it follows from the result in Proposition 2 above, and from the assumption that $c(\omega_{ij})$ is convex in ω_{ij}, that higher precision of agent k about the information that she receives from agent j leads to a decrease in agent i's incentives to improve the information that she receives from agent j. Thus, by restricting attention to interior information choices, we show that information decisions are strategic substitutes in our context.

Given our assumptions on preferences, the expected utility of agent i in stage 2 incorporates her concern about how agent k learns about agent j's type. In stage 2, agent i observes agent k's information decision ω_{kj} with respect to agent j. Recall that agent i does not observe the message realization m_{kj} that agent k receives. However, using ω_{kj} together with her own information choice about agent j's type, agent i can make inferences about m_{kj}. In stage 2, agent i cares about the (conditional) covariance between the message received by agent k from agent j and agent j's type and about the (conditional) variance of that message m_{kj}.

On the one hand, higher covariance between θ_j and m_{kj} implies that the knowledge about θ_j reflects with better precision the value of m_{kj}. This implication lowers the value of agent i's information about θ_j. Nevertheless, in our information structure, agent k's information decision about θ_j does not affect the covariance $\text{Cov}[\theta_j, m_{kj}|m_{ij}, \omega_{ij}]$. In fact, this covariance is only affected by agent i's information choice ω_{ij}. Recall from (4) that $\text{Cov}[\theta_j, m_{kj}|m_{ij}, \omega_{ij}] = \sigma^2(1 - \omega_{ij})$.

On the other hand, higher variance $\text{Var}[m_{kj}|m_{ij}, \omega_{ij}]$ implies that the message m_{kj} is more uncertain for agent i and, therefore, increases the value of agent i's information about θ_j. When agent k improves her information about agent j's type, this naturally makes m_{kj} to fluctuate less around θ_j and leads to lower (conditional) variance of the message m_{kj}. Recall from (3) that $\text{Var}[m_{kj}|m_{ij}, \omega_{ij}] = \sigma^2(1 - \omega_{ij}\omega_{kj})/\omega_{kj}$.

Under the EAEU solution concept, in which the agents anticipate their future expected utilities, the implications above are anticipated by agent i in the first stage and, therefore, an increase in the precision of the information that agent k acquires from agent j makes less valuable agent i's information acquisition from agent j. Since information acquisition is costly, this leads to a decrease in the precision of the information that agent i acquires from agent j. In this way, substitutability of information decisions propagates along the network between indirectly connected agents who have common neighbors.

Inspection of the condition given in Proposition 2, which characterizes an agent's best response in information decisions, directly reveals that equilibrium is not unique. Nevertheless, despite the multiplicity of equilibria, we are able to relate the compatibility between the efficient and the equilibrium information decisions to the minimum degree of the network, provided that attention is restricted to interior information choices.

3.1 Equilibrium, Efficiency, and the Network Density

Suppose first that there is no team concern, i.e., $r = 0$, so that each agent cares only about solving her own task. Then, for each agent i and each neighbor $j \in N_i(g)$, by combining the results in propositions 1 and 2, and by using the assumption that $c(x_{ij})$ is convex, we obtain that $\omega^*_{ij} > \widehat{\omega}_{ij}$ for each pair of (interior) equilibrium and efficient information acquisition profiles, respectively, ω^* and $\widehat{\omega}$. Thus, agents over-invest in information acquisition with respect to the efficient information choice. The reason for this inefficient behavior is intuitive under the approach followed in this paper. The agents anticipate future situations in which, in general (i.e., when they acquire some information), they will be better informed about the state than in the present. As a consequence, they anticipate future lower variances of the true state and are willing to invest more in information acquisition in the present. Thus, our model delivers the behavioral implication that the agents are overoptimistic about the effects of their information acquisition decisions. Note that this inefficiency is present in our model regardless of the network structure.

We turn now to study how the network structure relates to the compatibility between efficient and equilibrium interior information decisions. Note first that by combining the results in propositions 1 and 2, it follows that an (interior) equilibrium information profile ω^* is efficient only if the equality

$$\frac{r}{n-1} \sum_{k \in N_j(g) \setminus \{i\}} [1 + \omega^*_{kj}(\omega^*_{kj} - 2)] = 2r - 1$$

is satisfied for each agent $i \in N$ and each neighbor $j \in N_i(g)$. Now, it can be verified that, for levels of the team concern given by $r \in [0, 1/2]$, the left-hand side of the expression above is always strictly positive while the right-hand side is either negative or zero so that any interior equilibrium profile is inefficient. In addition, for $r = 1$, the left-hand side of the expression above is always less than one so that any interior equilibrium profile is inefficient as well. Thus, the compatibility between equilibrium and efficient (interior) information decisions necessarily requires some concern about our own performance ($r < 1$) and relatively high levels of the team concern ($r > 1/2$). Then, for levels of the team concern described by $r \in (1/2, 1)$, the following corollary follows by combining the results in Proposition 1 and Proposition 2.

Corollary 1. *Suppose that $r \in (1/2, 1)$ and let $\widehat{\omega}$ be an interior efficient information acquisition profile. Then, there exists a function $\phi : (1/2, 1) \to (1, n)$, which is strictly increasing in r, such that if the density of the underlying network g is sufficiently high so that $\delta(g) \geq \phi(r)$, then $\widehat{\omega}$ can be achieved as an equilibrium (EAEU) information profile. Conversely, if $\delta(g) < \phi(r)$, then the efficient information profile $\widehat{\omega}$ cannot be achieved as an equilibrium (EAEU) information profile.*

The result in Corollary 1 is obtained by specifying, for each $i \in N$ and each $j \in N_i(g)$, the function

$$F_{i,j}(\omega) := \frac{r}{n-1} \sum_{k \in N_j(g) \setminus \{i\}} [1 + \omega_{kj}(\omega_{kj} - 2)],$$

which happens to be continuous in each $\omega_{kj} \in (0, 1)$, for each $k \in N_j(g) \setminus \{i\}$. We note that

$$F_{i,j}(\omega) \in \left(0, \frac{r(n_j(g) - 1)}{n-1}\right)$$

for each interior ω. To achieve an interior information profile $\widehat{\omega}$ as equilibrium, we need then to choose a profile ω^* so as to obtain the equality $F_{i,j}(\omega^*) = 2r - 1$ for each $i \in N$ and each $j \in N_i(g)$. Given the continuity properties of each function $F_{i,j}$, it follows that ω^* can be chosen as required if the following sufficient condition holds for each $j \in N$:

$$\frac{r\big(n_j(g) - 1\big)}{n-1} \geq 2r - 1 \Leftrightarrow n_j(g) \geq 1 + \frac{(2r-1)(n-1)}{r}.$$

Now, all these conditions are satisfied if $\delta(g) \geq 1 + (2r-1)(n-1)/r$. Then, we can specify $\phi(r) := 1 + (2r-1)(n-1)/r$, a strictly increasing function whose image lies in the interval $(1, n)$. In addition, if $\delta(g) < 1 + (2r-1)(n-1)/r$, then we know that

$$\frac{r\big(n_j(g) - 1\big)}{n-1} < 2r - 1$$

for some $j \in N_i(g)$ and some $i \in N$. Then, necessarily,

$$\frac{r}{n-1} \sum_{k \in N_j(g) \setminus \{i\}} [1 + \omega_{kj}(\omega_{kj} - 2)] < 2r - 1,$$

an inequality which, given that $c(\omega_{ij})$ is convex, implies $\omega_{ij} < \widehat{\omega}_{ij}$. Thus, in this case, agent i invests in information acquisition from her neighbor j less than in the efficient information profile.

Hence, the compatibility between efficient and equilibrium information decisions requires that each agent has a number of neighbors large enough. In this sense, sufficiently dense networks favor that efficient information decisions can be achieved in equilibrium. Note also that, since $\phi(r)$ is strictly increasing in r, the condition identified in Corollary 1 above implies that, for an efficient information profile to be achieved as an equilibrium, higher levels of the team concern require higher density for the underlying network. In particular, note that for $r \to 1$, the required network structure asymptotically approaches the complete network where each agent can acquire information from any other agent in the group.

Finally, inspection of the function $F_{i,j}$ defined above reveals that the attainment of the required equality $F_{i,j}(\omega) = 2r - 1$ is favored when agent j has a sufficiently large number of neighbors (as stated in Corollary 1) but also when each neighbor $k \neq i$ of j acquires relatively low amounts of information from her. This is so because each term $\omega_{kj}(\omega_{kj} - 2)$ is strictly decreasing in $\omega_{kj} \in (0, 1)$. This implication is related to our earlier observation that, in equilibrium, the agents over-invest in information acquisition when there is no team concern. Note that the mechanism which drives the result stated in Corollary 1 explicitly presents the number of neighbors that agent j has as a restriction to achieve the required equality $F_{i,j}(\omega) = 2r - 1$. This can be interpreted as the social or physical restriction imposed (locally over agent j) by the network structure. Yet, if this (exogenous) restriction is overcome so that $n_j(g) \geq \phi(r)$, then reaching efficient information decisions is favored when the neighbors $k \neq i$ of j do not over-invest and, instead, acquire low amounts of information from agent j.

Hence, our model delivers also the following, perhaps paradoxical, behavioral implication. On the one hand, the compatibility between efficient and equilibrium information decisions is favored when the network structure allows each agent to acquire information from a sufficiently large set of neighbors. On the other hand, each agent gets closer to her efficient information decision as she acquires lower amounts of information from her neighbors. Thus, although the attainment of efficient information decisions requires that we have a number of neighbors large enough, we tend to behave more efficiently when we do not over-invest or, in other words, when we do not take full advantage of our links in the network.

4 Concluding Comments

The environment investigated in this paper is one with no conflict of interests over actions and with a positive externality from the suitability of others' actions with the state. The purpose of this paper was two-fold. First, the paper proposed a novel mechanism that generated interdependent information decisions and beliefs without strategic interactions in actions. Private messages were assumed to be independent and any source of public information was ruled out. This is important since most models of herding or interdependent (incorrect) beliefs make use of relations between private signals or of some sort of public information. A notable exception is the approach pursued by Bénabou (2012). Second, the paper studied both the efficient and the equilibrium behavior with respect to information acquisition, and related the compatibility between them to a measure of the network density.

We obtained the result that, when there is no team concern, the agents are overoptimistic about the effects of their information decisions. This implication seems relevant in some settings, identified by the large psychology literature on inference, in which people over-interpret the information they receive from their private signals.

As argued in the Introduction, a natural motivation for our assumption that the agents anticipate their future expected utilities comes from the fact that, in some environments, people are not totally accurate in predicting others' information processing capabilities. Bohren (2010) provides an explanation for (incorrect) herding which is based on the assumption that the agents have some exogenous bias in their perception of others' information processing capabilities. The mechanism proposed in our approach can be viewed as an attempt to endogenize, through information decisions, our perception of how others learn about the state and, accordingly, use their knowledge to choose their actions.

The approach pursued in this paper can be extended to analyze other situations, possibly with a different class of preferences, but in which an agent makes a strategic decision that affects her future perception of her own utility. Nevertheless, a large body of the literature on information transmission in networks (e.g., Hagenbach and Koessler, 2010; Calvó-Armengol, de Martí, and Prat, 2011; Galeotti, Ghiglino, and Squintani, 2011) focuses on situations in which the agents decide instead about information revelation. In these games, an agent makes a strategic decision that affects the future utility of *other* agents and, therefore, the motivation given in this paper for the EAEU solution concept is obviously not compelling. The standard approach and the use of PBE are clearly appropriate to analyze sequential situations in which the agents strategically reveal their private information.

Finally, this paper assumed that information cannot be transmitted through agents indirectly linked in a network. Of course, it would be interesting to investigate the information acquisition problem when such a network effect is allowed for.

Appendix

This appendix contains the proofs of propositions 1 and 2. Note that the utility specification in (1) and the assumed information structure imply that an action strategy α_i^* which satisfies the sequential rationality requirement (i), under the posteriors specified in (iii), in the definition of EAEU (Definition 1) is characterized by

$$\alpha_{ij}^*(m_{ij}, \omega_{ij}) = E[\theta_j | m_{ij}, \omega_{ij}] = \omega_{ij} m_{ij} + (1 - \omega_{ij})\mu \qquad \text{for each} \;\; j \in N.$$

With this observation at hand, we proceed to the proofs.

Proof of Proposition 1. Consider a network $g \in G$. The welfare function evaluated in stage 2, W_2, takes the form

$$W_2(\theta; m, \omega) = -\sum_{i \in N} \sum_{j \neq i} \big((\theta_j - \mu) - \omega_{ij}(m_{ij} - \mu) \big)^2.$$

Then, by using the expression of the welfare function in stage 1 given in (6), we obtain

$$\begin{aligned} W_1(\omega) = &- \int_\Theta p(\theta) \Big[\int_M f(m|\theta, \omega) \sum_{i \in N} \sum_{j \neq i} \big((\theta_j - \mu)^2 + \omega_{ij}^2 (m_{ij} - \mu)^2 \\ &- 2\omega_{ij}(\theta_j - \mu)(m_{ij} - \mu) \big) dm \Big] d\theta - \sum_{i \in N} \sum_{j \in N_i(g)} c(\omega_{ij}) \\ = &- \sum_{i \in N} \sum_{j \in N_i(g)} \Big[\sigma^2 + \omega_{ij}^2 (\sigma^2 + \psi_{ij}^2) - 2\omega_{ij}\sigma^2 + c(\omega_{ij}) \Big] \\ = &\sum_{i \in N} \sum_{j \in N_i(g)} \Big[\sigma^2(\omega_{ij} - 1) - c(\omega_{ij}) \Big]. \end{aligned}$$

Note that the function $W_1(\omega)$ is concave in each ω_{ij} for $j \in N_i(g)$. Then, an interior solution to the planner's problem is characterized by

$$\partial W_1(\widehat{\omega}) / \partial \omega_{ij} = \sigma^2 - c'(\widehat{\omega}_{ij}) = 0 \;\Leftrightarrow\; c'(\widehat{\omega}_{ij}) = \sigma^2$$

for each $i \in N$ and each $j \in N_i(g)$, as stated. ∎

Proof of Proposition 2. Consider a network $g \in G$ and an agent $i \in N$. For a given information profile ω and a given message vector m_i, agent i's expected utility in stage 2, $U_{i,2}$, is given by

$$\begin{aligned} U_{i,2}(\alpha;\omega,m_i) = &-(1-r)\sum_{k\neq i} E\left[\left((\theta_k-\mu)-\omega_{ik}(m_{ik}-\mu)\right)^2 \,\middle|\, m_i,\omega\right] \\ &-\frac{r}{n-1}\sum_{k\neq i}\sum_{j\neq k} E\left[\left((\theta_j-\mu)-\omega_{kj}(m_{kj}-\mu)\right)^2 \,\middle|\, m_i,\omega\right]. \end{aligned}$$

By computing the conditional expectations in the expression above, we obtain

$$\begin{aligned} U_{i,2}(\alpha;\omega,m_i) = &-(1-r)\sum_{k\neq i}\left[\sigma^2(1-\omega_{ik})+\omega_{ik}^2(m_{ik}-\mu)^2-2\omega_{ik}^2(m_{ik}-\mu)^2\right] \\ &-\frac{r}{n-1}\sum_{k\neq i}\sum_{j\neq k,i}\left[\sigma^2(1-\omega_{ij})+\omega_{kj}^2\frac{\sigma^2(1-\omega_{ij}\omega_{kj})}{\omega_{kj}}-2\sigma^2\omega_{kj}(1-\omega_{ij})\right] \\ &-\frac{r}{n-1}\sum_{k\neq i}\sigma^2\frac{1-\omega_{ki}}{\omega_{ki}}. \end{aligned}$$

Therefore, by plugging the expression above for $U_{i,2}(\alpha;\omega,m_i)$ into the definition of agent i's expected utility in stage 1 given in (5), and by using the results about agent i's posteriors stated in (2) , (3), and (4), we obtain

$$\begin{aligned} U_{i,1}(\omega) = &-(1-r)\sigma^2\sum_{k\neq i}(1-2\omega_{ik}) \\ &-\frac{r}{n-1}\sigma^2\sum_{k\neq i}\sum_{j\neq k,i}\left[1-\omega_{ij}-\omega_{kj}+\omega_{ij}\omega_{kj}(2-\omega_{kj})\right] \\ &-\frac{r}{n-1}\sigma^2\sum_{k\neq i}\frac{1-\omega_{ki}}{\omega_{ki}}-\sum_{j\in N_i(g)}c(\omega_{ij}). \end{aligned}$$

Take a given agent $j \in N_i(g)$. It can be verified that the function $U_{i,1}(\omega)$ is convex in $\omega_{ij} \in (0,1)$. Then, the following condition characterizes agent i's optimal choice of her information decision with respect to her neighbor j (when each other agent chooses her optimal information decision as well):

$$\begin{aligned} \partial U_{i,1}(\omega^*)/\partial\omega_{ij} = &2(1-r)\sigma^2+\frac{r}{n-1}\sigma^2\sum_{k\in N_j(g)\setminus\{i\}}\left[1-\omega_{kj}^*(2-\omega_{kj}^*)\right]-c'(\omega_{ij}^*)=0 \\ &\Leftrightarrow\; c'(\omega_{ij}^*)=2(1-r)\sigma^2+\frac{r}{n-1}\sigma^2\sum_{k\in N_j(g)\setminus\{i\}}\left[1+\omega_{kj}^*(\omega_{kj}^*-2)\right], \end{aligned}$$

as stated. ∎

References

[1] Akerlof, G., and W. Dickens (1982): "The Economic Consequences of Cognitive Dissonance," *American Economic Review*, 72, 307-19.

[2] Angeletos, G.-M., and A. Pavan (2007): "Efficient Use of Information and Social Value of Information," *Econometrica*, 75, 4, 1103-1142.

[3] Becker, G. (1993): *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education*, Chicago Press.

[4] Bénabou, R. (2012): "Groupthink: Collective Delusions in Organizations and Markets," mimeo, forthcoming in *Review of Economic Studies.*

[5] Bénabou, R., and J. Tirole (2002): "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics*, 117, 871-915.

[6] ——— (2004): "Willpower and Personal Rules," *Journal of Political Economy*, 112, 884-887.

[7] Bohren, J. A. (2010): "Information-Processing Bias in Social Learning," mimeo.

[8] Calvó-Armengol, A., and J. de Martí (2007): "Communication Networks: Knowledge and Decisions," *American Economic Review*, 97, 2, 86-91.

[9] Calvó-Armengol, A., de Martí, J., and A. Prat (2011): "Communication and Influence," mimeo.

[10] Chakraborty, A., and R. Harbaugh (2007): "Comparative Cheap Talk," *Journal of Economic Theory*, 132, 70-94.

[11] Cohan, J. (2002): " 'I Didn't Know' and 'I was Only Doing My Job': Has Corporate Governance Careened Out of Control? A Case Study of Enron's Information Myopia," *Journal of Business Ethics,* 40, 275-299.

[12] Compte, O., and A. Postlewaite (2004): "Confidence-Enhanced Performance," *American Economic Review*, 94(5), 1536-1557.

[13] Dewan T., and D. P. Myatt (2008): "The Qualities of Leadership: Direction, Communication, and Obfuscation," *American Political Science Review*, 102, 3, 351-368.

[14] Di Tella, R., Galiani, S., and E. Schargrodsky (2007): "The Formation of Beliefs: Evidence from the Allocation of Land Titles to Squatters," *Quarterly Journal of Economics*, 122(1), 209-241.

[15] Galeotti, A., Ghiglino, C., and F. Squintani (2011): "Strategic Information Transmission in Networks," mimeo.

[16] Eichennwald, K. (2005): *Conspiracy of Fools: A True Story.* New York, Broadwaybooks.

[17] Hagenbach, J., and F. Koessler (2010): "Strategic Communication Networks," *Review of Economic Studies*, 77, 1072-1099.

[18] Hellwig, C., and L. Veldkamp (2009): "Knowing what Others Know: Coordination Motives in Information Acquisition," *Review of Economic Studies*, 76, 1, 223-251.

[19] Hersh, S. (2004): *Chain of Command.* New York, HarperCollins Publishers.

[20] Isikoff, M., and D. Corn (2007): *Hubris.* New York, Three Rivers Press.

[21] Jackson, M. O., and A. Wolinsky (1996): "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, 71, 44-74.

[22] Janis, I. (1972): *Victims of Groupthink: Psychological Studies of Policy Decisions and Fiascoes,* Boston, Houghton Miffin.

[23] Jiménez-Martínez, A. (2006): "A Model of Interim Information Sharing Under Incomplete Information," *International Journal of Game Theory*, 34, 425-442.

[24] Jovanovic, B. (1979): "Job Matching and the Theory of Turnover," *Journal of Political Economy*, 87, 5, 972-990.

[25] Kindleberger, C., and R. Aliber (2005): *Manias, Panics, and Crashes: A History of Financial Crises.* John Wiley and Sons.

[26] Levy, G., and R. Razin (2007): "On the Limits of Communication in Multidimensional Cheap Talk: A Comment," *Econometrica*, 75, 3, 885-893.

[27] Mackay, C.. (1980): *Extraordinary Popular Delusions and the Madness of Crowds.* New York: Three Rivers Press.

[28] Morris, S., and H. S. Shin (2002): "Social Value of Public Information," *American Economic Review*, 92, 5, 1521-1534.

[29] Myatt, D. P., and C. Wallace (2010): "Endogenous Information Acquisition in Coordination Games," mimeo, forthcoming in *Review of Economic Studies.*

[30] Nieuwerburgh, S. V., and L. Veldkamp (2009): "Information Immobility and the Home Bias Puzzle," *The Journal of Finance*, 64, 3, 1187-1215.

[31] Prescott, E. C., and M. Visscher (1980): "Organization Capital," *Journal of Political Economy*, 88, 3, 446-461.

[32] Rabin, M. (1994): "Cognitive Dissonance and Social Change," *Journal of Economic Behavior and Organization*, 23, 177-194.

[33] Schelling, T. (1986): "The Mind as a Consuming Organ," in D. Bell, Raiffa H. and A. Tversky, eds., *Decision Making: Descriptive, Normative, and Prescriptive Interactions.* Cambridge University Press.

[34] Shiller, R. (2005): *Irrational Exuberance.* Second Edition. Princeton, Princeton University Press.