Strategic Interactions in Information Decisions with a Finite Set of Players*

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Abstract

This paper considers a tractable class of two-player quadratic games to examine the relation between strategic interactions in actions and in information decisions. We show that information choices become substitutes when actions are sufficiently complementary. For levels of substitutability sufficiently high, information choices become complements for some initial information decisions. When attention is restricted to beauty contest games, our results contrast qualitatively with the case studied by Hellwig and Veldkamp (2009), where the set of players is a continuum. Also, for games different from beauty contests, we show that high levels of external effects favor that information choices be complements for any degree of complementarity in actions.

JEL Classification Numbers: C72, D82, D83

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1. Introduction

The optimal action of a decision maker in a variety of environments—including oligopolistic industries, networks, investment activities, financial markets, and monopolistic competition—depends on her expectation of both an exogenous state and other agents' actions. In these environments, because optimal actions and the state are correlated, information about the latter conveys information about other agents' actions as well. Models incorporating these features have been extensively used to study particular problems in many fields.[1] Most applications of these games have been confined to exogenously given information. In practice, however, the information that a decision maker has about unknown parameters depends to some extent on her decision of how much to learn. This paper studies the relation between strategic interactions in the action choice and the information choice using quadratic preferences. Analyzing this question is central to investigate whether heterogenous beliefs can be endogenously sustained in dynamic situations where players interact repeatedly, provided that there is a new realization of the state in each period.

Consider a group of players and suppose that each of them chooses the informativeness of a private signal about some payoff-relevant state prior to choosing actions. Assume that signals are independent (conditionally on the state) across players. Do information choices become strategic complements/substitutes when actions are strategic complements/substitutes? In a beautiful recent paper, Hellwig and Veldkamp (2009)—henceforth, HV—show that the answer is affirmative when there is a large number of identical small players engaged in a beauty contest game. In contrast, this paper shows that the answer is not always affirmative when the set of players is finite or relatively small. More precisely, Proposition 1 shows that if actions are very complementary, then a player wants to learn less when others want to learn more. This result has interesting implications. When actions are very complementary, heterogeneous beliefs, usually assumed in problems under asymmetric information and in industrial organization models, can be endogenously sustained.

When others become better informed, this has two effects on our incentives to acquire information. On the one hand, the actions of those players who improve their information become more correlated to the state, which makes more valuable our own information about the state when actions are complements. On the other hand, the variance of their actions (conditioned on our signal) may change. Lower variance of others' actions makes our own information about the state less valuable. With a continuum of players,

[1] Incentives of this nature have been considered, among many others, by: (i) Cooper and John (1988) to study coordination failures in macroeconomic models, (ii) Morris and Shin (2002), Hellwig (2005), Cornand and Heinemann (2008), Angeletos and Pavan (2004, 2007), and Colombo and Femminis (2008) to study the welfare effects of public information disclosure, (iii) Morris and Shin (2005) to study the welfare implications of central bank transparency, (iv) Calvó-Armengol and de Martí (2007, 2009) to study efficiency properties of communication networks, (v) Calvó-Armengol, de Martí, and Prat (2009), and Hagenbach and Koessler (2010) to study endogenous information transmission in networks, and (vi) Dewan and Myatt (2008, 2009) to study endogenous communication from party leaders to activists.

the fact that each player is very small compared to the size of the group leads to a negligible change in the variance of the action of a player who improves her information. Consequently, the variance of the average action is not affected and hence we care only about the reduction in the covariance between the state and the average action. Thus, the second effect above plays no role in our incentives to acquire information. However, with a small set of players, better information about the state may reduce the variance of others' actions. Furthermore, when actions are very complementary, the covariance between a player's action and the state is very small regardless her information choice and, therefore, the first effect above vanishes. In this case, the reduction in the variance of the average action overcomes the higher covariance between others' actions and the state, leading to a decrease in our incentives to acquire information. In short, if others' actions are hardly correlated with the state and, in addition, vary less, knowing the state becomes less valuable even when we wish to approximate to others' actions.

In these games, optimal actions depend on arbitrarily higher-order iterated expectations of the state. As in Morris and Shin (2002), the assumption that the set of players is a continuum enables HV to use an average expectation operator to keep track of higher-order expectations. For a finite set of players, this approach is appropriate only when higher-order beliefs are very homogeneous across players. But, if the players begin with heterogeneous beliefs, then the heterogeneity would not necessarily vanish unless one imposes a very restrictive symmetric information structure. Thus, an average expectation operator would be inappropriate to keep track of the required higher-order beliefs with a finite number of players and a flexible information structure.

We consider a finite set of players and follow the approach introduced in the networks literature by Calvó-Armengol and de Martí (2007) of keeping track of higher-order beliefs through a knowledge index. For tractability reasons, we conduct the analysis through a two-player game, though our results continue to hold qualitatively provided that the set of players is finite.

Perhaps more important than extending earlier work, this paper emphasizes that understanding strategic interactions in information depends crucially on whether one considers a large or a small set of players. For a finite number of players, keeping track of higher-order beliefs through a knowledge index leads to conclusions different from those obtained using an average expectation operator. Clearly, the implications of our results are more relevant in environments involving a relatively small number of players, such as oligopolistic competition, organizations or a group of agents engaged in a common task. Our results, however, do not go beyond the insights obtained by HV in markets with a large number of players, as it is typical in models of monopolistic competition.

Our class of preferences also allows us to examine the role played by second order external effects. Proposition 2 shows that, when actions are complements, high levels of the externality favor that information choices be complements even when actions are very complementary. This is intuitive since the externality provides an additional incentive for players to be concerned about the suitability of others' actions with the state.

A growing number of papers have recently drawn attention to quadratic games with endogenous information acquisition. A paper closely related to ours is Myatt and Wallace (2010), which considers a continuum of players endowed with beauty contest preferences. They analyze information decisions when players are allowed to choose both which signals they observe and the precision of such signals. An important difference with our model is in that they allow signals to be correlated across players. This provides a nice framework to study how public information can emerge endogenously from private information decisions. Although their information structure is different from that in HV, the result that complementarity in actions always leads to complementarity in information decisions also applies in their model. Colombo and Femminis (2008) introduce endogenous information acquisition into the beauty contest game with a continuum of players proposed by Morris and Shin (2002) to analyze whether public information can reduce welfare. In their model, higher precision in public information is welfare-improving. Related work can be also found in political science. Dewan and Myatt (2008) analyze communication between leaders and activists in political parties. Party leaders can choose the precision of the signals through which they communicate their opinions while activists can affect the precision of the signals through which they learn leaders' opinions.

All those papers above consider a continuum of players and, therefore, can invoke an average expectation operator to compute higher-order beliefs. However, most related work has not considered a finite set of players. One exception is Calvó-Armengol, de Martí, and Prat (2009). They study costly information transmission among players who are connected through a network. In their model, players have quadratic preferences and are both senders and receivers of information. They use a knowledge index to compute the higher-order beliefs required to characterize optimal actions. The question analyzed in our paper is different from theirs as we consider information acquisition from external sources. Finally, Chatterjee and Harrison (1988) consider a two-player sealed-bid auction where players can acquire private information about the value of the object before submitting their bids. For this game of complementary actions, they show that information decisions can be substitutes, a result which agrees with ours for a different payoff structure. Hence, their assumption of a finite set of players may help explain why they obtain substitutive information decisions when actions are complements.

The rest of the paper is organized as follows. Section 2 introduces the model. We examine players' optimal actions and the nature of their strategic interactions in the information choice in Section 3. All the proofs are relegated to the Appendix.

2. The Model

2.1. *Actions and Payoffs*

We consider two players, $i = 1, 2$, who make decisions in a two-stage game. In the first stage, nature selects a state of the world $\theta \in \mathbb{R}$, which is unobservable for the players.

Then, each player i makes an information choice $x_i \in [0,1]$ about the state. Information decisions are taken simultaneously. After this, nature sends a private signal $s_i \in \mathbb{R}$ about the state to each player i. In the second stage, each player i observes her signal and chooses an action $a_i \in \mathbb{R}$. Actions are taken simultaneously too.

Payoffs depend on the state θ and on the action profile $a = (a_1, a_2)$. More precisely, the payoff to each player i is given by the (common) quadratic function:

$$u(a_i, a_j, \theta) = (a_i, a_j, \theta) \begin{pmatrix} \gamma_{ii} & \gamma_{ij} & \gamma_{i\theta} \\ \gamma_{ij} & \gamma_{jj} & \gamma_{j\theta} \\ \gamma_{i\theta} & \gamma_{j\theta} & \gamma_{\theta\theta} \end{pmatrix} (a_i, a_j, \theta)^T,$$

where $\gamma_{i\theta} \neq 0$ and $\gamma_{j\theta} \neq 0$. That is, there is a strategic relation between the state and each action. The assumption that u is quadratic guarantees linearity of optimal action strategies. We assume strict concavity at the individual level, $\gamma_{ii} < 0$, as well as at the aggregate level, $\gamma_{ij} + \gamma_{ii} < 0$. Strict concavity at the individual level guarantees that best responses in actions are well-defined. In addition, to ensure that optimal actions are well-defined and to guarantee equilibrium existence, we need to assume $\gamma_{ij} > \gamma_{ii}$. Finally, in order to consider nonnegative (second order) external effects, we assume concavity in the other player's action, $\gamma_{jj} \leq 0$.

Given those assumptions above on preferences, we specify the following pair of parameters: $\lambda := -\gamma_{ij}/\gamma_{ii}$ and $\pi := \gamma_{jj}/\gamma_{ii}$. By combining $\gamma_{ii} < 0$, $\gamma_{ij} + \gamma_{ii} < 0$, and $\gamma_{ij} > \gamma_{ii}$, it follows $\lambda \in (-1, 1)$. Also, $\gamma_{ii} < 0$ together with $\gamma_{jj} \leq 0$ imply $\pi \geq 0$. Parameter λ measures the degree of strategic complementarity ($\lambda > 0$) or substitutability ($\lambda < 0$) in actions. Parameter π is useful to measure the level of the externality generated on a player by the other player's action.

Remark 1. Beauty contest payoffs are a particular case satisfying our assumptions above on preferences. The beauty contest game studied by HV (adapted to our two-player case) is given by the payoff $u(a_i, a_j, \theta) = -(1-\lambda)^{-2}\big[a_i - (1-\lambda)\theta - \lambda a_j\big]^2$. In this case, we have: $\gamma_{ii} = -2/(1-\lambda)^2$, $\gamma_{ij} = 2\lambda/(1-\lambda)^2$, $\gamma_{jj} = -2\lambda^2/(1-\lambda)^2$, $\gamma_{i\theta} = 2/(1-\lambda)$, and $\gamma_{j\theta} = -2\lambda/(1-\lambda)$. As a consequence, strategic interactions in actions and external effects are related through the relation $\pi = \lambda^2$.

2.2. *Information Structure and Information Decisions*

We consider a Gaussian information structure for tractability. In the first stage of the game, nature draws a state realization θ from a normal distribution with mean μ and variance σ_θ^2. This distribution summarizes the (common) priors of the players about the state. In addition, each player i makes an information decision x_i and observes a signal realization s_i. We assume that a player's information decision determines the correlation between the state and the private signal that she receives. Thus, the player makes a decision on her own belief revision process (following Bayes' rule) and ends up with some posteriors about θ, which she can use in the second stage to choose her action. We use $\tilde{z}$ to denote a random variable with realization z.

Definition 1. An *information choice for player* i, $x_i \in [0,1]$, is the square of a value for the correlation coefficient between the random variables $\tilde{\theta}$ and $\tilde{s}_i$.

Thus, we model information decision as a continuous choice. Higher values of x_i indicate higher degrees of informativeness for player i's signal. Let $x = (x_1, x_2) \in [0,1]^2$ be an information profile. We interpret an information choice as a decision on the quality of the channel though which the player receives information about the state. We assume that each player observes the information decision of the other before she chooses her action in the second stage. This assumption is natural if we think that affecting the technology through which we receive information requires some investments and has observable consequences.[2] Nevertheless, we assume that a player i does not observe the signal s_j received by the other player.

In order to capture information acquisition, the signals that players receive must be correlated with the state. We should also expect some degree of correlation between signals. The precise way in which signals relate with the state is the result of some information-generating procedure. We abstract from this information-generating procedure by considering the joint distribution of the state and the signals as a primitive in the model. We assume that the vector of the state and signals follows a multi-normal distribution, $(\tilde{\theta}, \tilde{s}_1, \tilde{s}_2) \sim (\mu\underline{1}, \Sigma)$, where a $\underline{1}$ is a 3-dimensional vector of ones and $\Sigma = (\sigma_{kl})_{k,l=\theta,1,2}$ is a general variance-covariance matrix for the random triple. The only relevant assumption that we impose on this information structure is that signals are independent, conditionally on the state. In other words, we assume that there is no public information component resulting from the underlying information-generating procedure. Formally, we assume that $\sigma_{12} = \sigma_{\theta 1}\sigma_{\theta 2}\sigma_\theta^{-2}$. Notice that this condition is satisfied if and only if $\mathrm{Cov}[\tilde{s}_1, \tilde{s}_2 \,|\, \theta] = 0$. Also, for pure notational convenience, we assume that both signals have the same variance, $\sigma_1^2 = \sigma_2^2 = \sigma_s^2$. Taking into account these assumptions, the variance-covariance matrix Σ, which characterizes our information structure, can be written as:

$$\Sigma = \begin{pmatrix} \sigma_\theta^2 & \sigma_{\theta 1} & \sigma_{\theta 2} \\ \sigma_{\theta 1} & \sigma_s^2 & \sigma_{\theta 1}\sigma_{\theta 2}\sigma_\theta^{-2} \\ \sigma_{\theta 2} & \sigma_{\theta 1}\sigma_{\theta 2}\sigma_\theta^{-2} & \sigma_s^2 \end{pmatrix}. \tag{1}$$

Also, given our specification above of an information choice x_i, the definition of correlation coefficient between and $\tilde{\theta}$ and $\tilde{s}_i$, leads to $\sigma_{\theta i} = \sqrt{x_i}\sigma_\theta\sigma_i$ and $\sigma_{\theta 1}\sigma_{\theta 2}\sigma_\theta^{-2} = \sqrt{x_1 x_2}\sigma_s^2$.

Some basic algebra on normal distributions yields $\mathrm{Var}[\tilde{\theta} \,|\, s_i, x] = (1 - x_i)\sigma_\theta^2$. That is, the posterior variance of the state is strictly decreasing in the information choice $x_i \in [0,1]$ and, therefore, the informativeness of a signal can be naturally ranked according to the induced posterior variance of the state.

[2]Consider, for instance, a university changing its library resources or a firm changing the hardware employed for information acquisition and processing.

Remark 2. In their paper, HV assume that each player can choose a subset from a finite set of signals so that the information choice is discrete. They rank the informativeness of the information choice in terms of the induced posterior variance of the state, conditioned on the signals observed. Our model specifies the information choice in a different way and, in particular, it considers a continuous information choice. Nevertheless, both approaches are similar in the sense that both rank analogously the informativeness of a signal using posterior variances.

Remark 3. A number of papers that consider a continuum of players, including among many others, Morris and Shin (2002), and Angeletos and Pavan (2007), assume an information structure where players receive both private and public signals. Furthermore, they assume that signals are equal to the state plus some idiosyncratic noise (private information) and some common noise (public information). When there is no public information, that information structure for our two-player game can be summarized by assuming:

$$\tilde{\theta} \sim N(\mu, \sigma_\theta^2), \qquad \tilde{s}_i = \tilde{\theta} + \tilde{\varepsilon}_i, \qquad \tilde{\varepsilon}_i \sim N(0, \psi_i^2), \qquad i = 1, 2,$$

where ψ_i^2 is the variance of the noise of player i's signal, and $\tilde{\theta}$ and $\tilde{\varepsilon}_i$ are assumed to be independent, as well as $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. Notice that this information structure is obtained by imposing conditions on the marginal distributions of the state and signals, and by assuming a linear relation between signals and the state. Our information structure follows instead by considering a general multi-normal distribution with the only restriction of independence (conditionally on the state) between signals. Therefore, when there is no public information, our information structure is more general than the one summarized above. In particular, the information structure above is accommodated as a special case by our information structure if we further impose $\sigma_{\theta i} = \sigma_\theta^2$ and $\sigma_s^2 = \sigma_\theta^2 + \psi_i^2$ for each $i = 1, 2$. However, under this information structure, we cannot change $\mathrm{Cov}[\tilde{\theta}, \tilde{s}_i]$ without affecting σ_θ^2. Because $\tilde{\theta}$ and $\tilde{\varepsilon}_i$ are independent, affecting the variance of $\tilde{\varepsilon}_i$ does not change the covariance between the state and signal $\tilde{s}_i$. Thus, under that information structure, x_i cannot be taken as endogenous when the variance of the state is exogenously given.

Regarding the Bayesian updating of beliefs under our information structure, some basic results on normal distributions lead to that the random variables $[\tilde{\theta} \mid s_i, x]$ and $[\tilde{s}_j \mid s_i, x]$ are normally distributed with respective means:

$$\mathbb{E}[\tilde{\theta} \mid s_i, x] = \mu + \sqrt{x_i} \left(\frac{\sigma_\theta}{\sigma_s} \right) (s_i - \mu), \qquad \mathbb{E}[\tilde{s}_j \mid s_i, x] = \mu + \sqrt{x_i x_j}(s_i - \mu). \tag{2}$$

Let us denote the incomplete information game that we have described by $(u, N(\mu\underline{1}, \Sigma))$.

2.3. *Equilibrium*

An action strategy for player i is a function $\alpha_i : \mathbb{R} \times [0,1]^2 \to \mathbb{R}$, where $\alpha_i(s_i, x)$ is the action chosen by player i when the information profile is x and the signal that she receives is s_i. Let $\alpha = (\alpha_1, \alpha_2)$ denote an action strategy profile.

The backward induction process in equilibrium is as follows. Given an information profile x, each player i chooses in the second stage, for each signal realization s_i, an action a_i in order to maximize the expected payoff $\mathbb{E}[u(a_i, a_j, \tilde{\theta}) \mid s_i, x]$. By solving this optimization problem, we obtain player i's best response in actions. Given an action strategy profile α, the expected payoff to player i in the first stage is defined by:

$$V_i(x; \alpha) := \mathbb{E}[\, u(\alpha_i(\tilde{s}_i, x), \alpha_j(\tilde{s}_j, x), \tilde{\theta})\,]. \tag{3}$$

Thus, given α, in the first stage player i chooses x_i so as to maximize $V_i(x_i, x_j; \alpha)$. This gives us player i's best response in information decisions.

We restrict attention to equilibrium in pure strategies.

Definition 2. A strategy profile (x^*, α^*) is a *Perfect Bayes-Nash Equilibrium (PBE)* of $(u, N(\mu\underline{1}, \Sigma))$ if the following conditions are satisfied for each player $i = 1, 2$:

(i) for each $x \in [0,1]^2$ and each $s_i \in \mathbb{R}$,

$$\mathbb{E}[u(\alpha_i^*(s_i, x), \alpha_j^*(\tilde{s}_j, x), \tilde{\theta}) \mid s_i, x] \geq \mathbb{E}[u(a_i, \alpha_j^*(\tilde{s}_j, x), \tilde{\theta}) \mid s_i, x] \qquad \text{for each } a_i \in \mathbb{R};$$

(ii)
$$V_i(x_i^*, x_j^*; \alpha^*) \geq V_i(x_i, x_j^*; \alpha^*) \qquad \text{for each } x_i \in [0,1];$$

(iii) for each $x \in [0,1]^2$ and each $s_i \in \mathbb{R}$, the expectation operator $\mathbb{E}[\cdot | s_i, x]$ satisfies the conditions in (2).

3. Main Results

To answer the question addressed in this paper, we characterize optimal action strategies and use them to study how the pair of parameters (λ, π) affects the sign of the second order derivative $\partial^2 V_i(x; \alpha^*)/\partial x_i \partial x_j$. We emphasize that this paper does not characterize information decisions in equilibrium and does not study either the properties of the set of PBE of the game $(u, N(\mu\underline{1}, \Sigma))$.

3.1. *Optimal Action Strategies*

To ease notation, let us denote by $\mathbb{E}_i[\cdot] = \mathbb{E}[\cdot \mid s_i, x]$ player i's expectation operator conditioned on observing signal realization s_i and on any information profile x. From our assumptions on preferences, it follows that, for each information profile $x \in [0,1]^2$, player i's optimal action strategy is given by:

$$\alpha_i^*(s_i, x) = (1-\lambda)\mathbb{E}_i[\tau_0 + \tau_1 \tilde{\theta}] + \lambda \mathbb{E}_i[\alpha_j(\tilde{s}_j, x)],$$

where $\tau_0, \tau_1 \in \mathbb{R}$ are some constants. By iterating recursively, we obtain

$$\alpha_i^*(s_i, x) = \tau_0 + (1-\lambda)\tau_1 \sum_{k=0}^{\infty} \lambda^k \mathbb{E}_i \mathbb{E}_j \mathbb{E}_i \cdots \mathbb{E}_{p(k)}[\tilde{\theta}],$$

where $\mathbb{E}_i\mathbb{E}_j\mathbb{E}_i \cdots \mathbb{E}_{p(k)}[\tilde{\theta}]$ denotes the $(k+1)$-order iterated expectations of $\tilde{\theta}$. The subindex $p(k)$ equals i if k is either zero or even and equals j if k is odd.

At this point, we need an operator that allows us to keep track of the discounted nested expectations of the players, in order to obtain a closed expression for optimal action strategies. In the case with a continuum of players, each player cares about the average of higher-order iterated beliefs and one can invoke an average expectation operator to approximate such beliefs, an approach introduced by Morris and Shin (2002). However, with a finite number of players who have heterogeneous beliefs, one needs instead to account for the iterated beliefs of each particular player. To do so, we make use of the knowledge index introduced by Calvó-Armengol and de Martí (2007) in their work on communication in networks.

As it is common in the related literature, a player's optimal action is linear in the signal that she observes. The following lemma characterizes optimal action strategies for our game.

Lemma 1. *The unique optimal action strategy profile α^* in any PBE of $(u, N(\mu\underline{1}, \Sigma))$ is given by:*

$$\alpha_i^*(s_i, x) = \tau_0 + \tau_1\mu + m_i\,(s_i - \mu),$$

for each $x \in [0,1]^2$, each $s_i \in \mathbb{R}$, and each player i=1,2, where $\tau_0, \tau_1 \in \mathbb{R}$, and

$$m_i = \tau_1\left(\frac{\sigma_\theta}{\sigma_s}\right)\frac{(1-\lambda)(1+\lambda x_j)\sqrt{x_i}}{(1-\lambda^2 x_i x_j)}.$$

By substituting the optimal action strategy profile α^*, characterized in Lemma 1, into the ex-ante expected payoffs, we obtain a single-stage game where players make information choices and receive their payoffs according to $V_i(x_i, x_j; \alpha^*)$. In this game, each player has infinitely many (pure) information strategies, which could in principle lead to equilibrium existence problems. Nevertheless, since such strategies lie in the compact set $[0,1]$ and the payoff function $V_i(x_i, x_j; \alpha^*)$ is continuous in (x_i, x_j), the results in Fudenberg and Levine (1986) regarding approximate games and approximate equilibrium[3] can be invoked to guarantee the existence of PBE for our game $(u, N(\mu\underline{1}, \Sigma))$.

3.2. *Strategic Interactions in Information Decisions*

We state now our main result regarding a player's incentives to acquire information when the other player increases her own information.

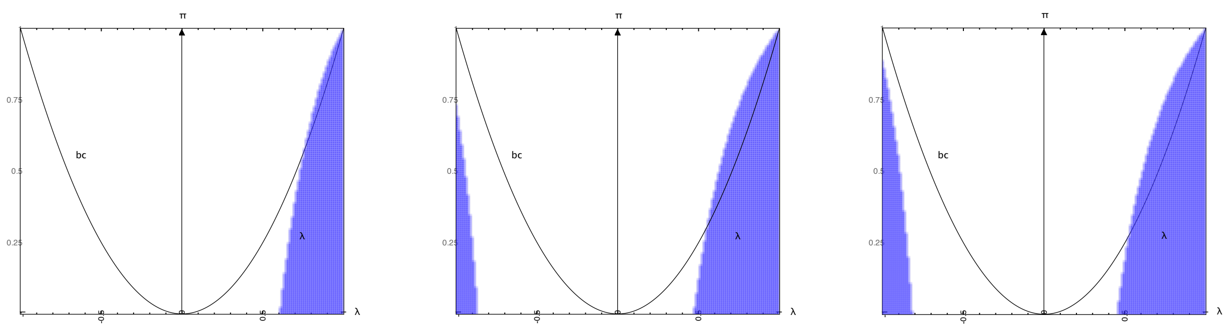
Proposition 1. *Let α^* be the unique optimal action profile for $(u, N(\mu\underline{1}, \Sigma))$ characterized in Lemma 1 and suppose that player j increases locally her information choice x_j. Then,*

(i) if there is no strategic interaction in actions ($\lambda = 0$), then player i has no incentives to change her information choice, $\partial^2 V_i(x;\alpha^*)/\partial x_i \partial x_j = 0$;

[3] Approximate equilibrium is formally defined by Radner (1980).

(ii) there exists some bound $\varepsilon > 0$ such that (a) if actions are moderately complementary ($0 < \lambda < \varepsilon$), then player i has incentives to increase her information choice, $\partial^2 V_i(x; \alpha^*)/\partial x_i \partial x_j > 0$, whereas (b) if actions are moderately substitutive ($-\varepsilon < \lambda < 0$), then player i has incentives to decrease her information choice, $\partial^2 V_i(x; \alpha^*)/\partial x_i \partial x_j < 0$;
(iii) for each level of the external effect $\pi \in [0, 1)$, there exists some bound $\kappa(\pi) \in (\varepsilon, 1)$ such that if actions are sufficiently complementary ($\kappa(\pi) < \lambda < 1$), then player i has incentives to decrease her information choice, $\partial^2 V_i(x; \alpha^*)/\partial x_i \partial x_j < 0$;
(iv) for each level of the external effect $\pi \in [0, 1)$, there exists some bound $\rho(\pi) \in (-1, 0)$ such that if actions are sufficiently substitutes ($-1 < \lambda < \rho(\pi)$), then player i has incentives to increase her information choice for some initial values of the information profile x, $\partial^2 V_i(x; \alpha^*)/\partial x_i \partial x_j > 0$ for x_i small enough and x_j large enough;
(v) for the case of beauty contest payoffs ($\pi = \lambda^2$), there exists some bound $\beta \in (0, 1)$ such that if actions are sufficiently complementary ($\beta < \lambda < 1$), then player i has incentives to decrease her information choice, $\partial^2 V_i(x; \alpha^*)/\partial x_i \partial x_j < 0$.

Figure 1 below illustrates the results stated in Proposition 1.



(a) $x_1 = x_2 = 0.5$ (b) $x_1 = 0.1$, $x_2 = 0.9$ (c) $x_1 = 0.05$, $x_2 = 0.95$

FIGURE 1

For (λ, π) in the shadowed region, the information choice does not have the same strategic coordination motives as the action choice. The line bc is $\pi = \lambda^2$ and corresponds to the beauty contest game.

Thus, strategic interactions in the information choice have the same motives as those in actions when the degree of coordination is moderate, as stated in Proposition 1 (i) and (ii). This agrees with the main result by HV for a game with a continuum of players. However, when actions are very complementary, information decisions turn out to be substitutes, as stated in Proposition 1 (iii) and (v) above. This result contrasts qualitatively with the main result in HV.

If the state were known to player i, then her optimal action would be

$$a_i^* = (1 - \lambda)(\tau_0 + \tau_1 \theta) + \lambda a_j,$$

which summarizes her desire to match both the underlying state and the other player's action. In our incomplete information game, player i observes the information profile

x and her private signal s_i, but her optimal action is uncertain. Then, the posterior variance $\text{Var}[\tilde{a}_i^* \,|\, s_i, x]$ gives us a measure of how valuable is the information about the state to player i. More variance of the player's optimal action is associated with a more uncertain objective. More uncertainty reflects that information is more valuable to the player. We have

$$\text{Var}[\tilde{a}_i^*|s_i, x] = (1-\lambda)^2\tau_1^2\text{Var}[\tilde{\theta}|s_i, x] + \lambda^2\text{Var}[\tilde{a}_j|s_i, x] + 2(1-\lambda)\lambda\tau_1\text{Cov}[\tilde{\theta}, \tilde{a}_j|s_i, x]. \quad (4)$$

Thus, when actions are complements ($0 < \lambda < 1$), more covariance between the state and the other player's action increases the variance of our optimal action and, therefore, makes information more valuable for us. One can imagine that as the other player becomes better informed, her beliefs are closer to the true value of the state so that her action is more correlated with the state. In that case, her action reflects with better precision the true value of the state. This is indeed the reason for the result in HV's model that a player wants to increase her information when others do so. In our model, we obtain this effect too. As the other player increases (locally) her information, we have

$$\frac{\partial\text{Cov}[\tilde{\theta}, \tilde{a}_j|s_i, x]}{\partial x_j} = \frac{(1-\lambda)\tau_1\sigma_\theta^2(1-x_i)(1+\lambda x_i)}{(1-\lambda^2 x_i x_j)^2} > 0.$$

Note, however, that when λ approaches one this effect becomes very small. In other words, when actions are very complementary, any player's optimal action approaches the optimal action that she chooses when she acquires no information at all. Thus, regardless players' information choices, this effect is very weak when complementarity in actions is large.

On the other hand, the variance of the other player's action can also be affected by her information decision. As the other player becomes better informed, one can expect that the variance of her action decreases because she is more certain about the actual value of the state. This effect makes less valuable our own information about the state. This effect is absent in HV's model because each player is very small with respect to the size of the group. In that case, the variance of a player's action is not affected by her information decision, neither is affected the variance of the average action. It can be verified that

$$\frac{\partial\text{Var}[\tilde{a}_j|s_i, x]}{\partial x_j} = \frac{(1-\lambda)^2\tau_1^2\sigma_\theta^2(1+\lambda x_i)^2}{(1-\lambda^2 x_i x_j)^3}\left[1-(2-\lambda^2)x_i x_j\right],$$

which takes negative values for $0 < \lambda < 1$, for some values of x. Then, we observe from the expression in (4) that, when λ is high enough, the reduction in the variance of the other player action's can compensate for the (low) increase in the covariance between her action and the state, leading to a decrease in the value of our information.

Notice, however, that those arguments above make use of the precision $1/\text{Var}[\tilde{a}_i^*|s_i, x]$ as indicative of the value of information while Proposition 1 uses $V_i(x; \alpha^*)$ to measure the value of information. This is why the external effect captured by parameter π plays no role in the intuitions given above.

Remark 4. Our restriction to the two-player case is for tractability. With a larger number of players, one must resort to computational numerical methods to obtain the required inverse of matrix $[I-\lambda\Omega]$. This inverse determines the slope of a player's optimal action in her private signal. Nevertheless, the implication that the variance of the action of a player who improves her information decreases (for some information profiles) continues to hold with an arbitrary finite number of players. Suppose instead that our game is played by a finite set of players $N = \{1, \dots, n\}$ and take two players $i, j \in N$, $i \neq j$. The assumed information structure implies $\text{Var}[\tilde{s}_j|s_i, x] = (1 - x_i x_j)\sigma_s^2$. Since $\text{Var}[\tilde{a}_j|s_i, x] = m_j^2 \text{Var}[\tilde{s}_j|s_i, x]$, it can be checked that

$$\frac{\partial \text{Var}[\tilde{a}_j|s_i, x]}{\partial x_j} = m_j \sigma_s^2 \Big[2 \frac{\partial m_j}{\partial x_j}(1 - x_i x_j) - x_i m_j \Big],$$

which takes negative values for each $\lambda \in (0, 1)$, for some values of x. Then, from the expression in (4), we obtain $\partial \text{Var}[\tilde{a}_i^*|s_i, x]/\partial x_j < 0$ by choosing $\lambda > 0$ sufficiently close to one. Thus, the result that information decisions are substitutes when actions are very complementary continues to hold provided that the number of players is finite. However, because higher-order average expectations approximate the average of higher-order expectations when the number of players tends to infinity, our results asymptotically converge to the result by HV.

Remark 5. Multiple equilibria may arise in a game with endogenous information decisions even when equilibrium is unique for the corresponding game without information choice. When information choices are complements, this problem is indeed present in HV's model because it assumes a discrete information choice. In our model, information acquisition is a continuous choice in a compact set, which ensures uniqueness of PBE for those values of (λ, π) under which the value of additional information is either strictly increasing or decreasing in the other player's information. To see this, suppose that player i increases her initial information choice x_i by a certain amount $\Delta x_i > 0$. Then,

$$\eta(x_j) := V_i(x_i + \Delta x_i, x_j; \alpha^*) - V_i(x_i, x_j; \alpha^*)$$

gives us the value of the additional information Δx_i to player i. Because there is no cost of information acquisition, two information choices $x_j, x_j' \in [0, 1]$, $x_j \neq x_j'$, by player j can be part of a PBE only if $\eta(x_j) = 0$ and $\eta(x_j') = 0$. However, both equalities cannot hold simultaneously when $\partial V_i(x; \alpha^*)/\partial x_i$ is either strictly increasing or strictly decreasing in x_j. As a result, for those values of (λ, π), identified in Proposition 1, under which $\partial^2 V_i(x; \alpha^*)/\partial x_i \partial x_j$ is either positive or negative, there is a unique $x_j^* \in [0, 1]$ which satisfies the equilibrium requirement $\eta(x_j^*) = 0$.

External effects have also an important role in the analysis of the question studied in this paper. When actions are complements, the shape of the shadowed regions in Figure 1 suggests that the threshold level of complementarity above which information decisions become substitutes increases with the second order externality. In addition, those regions seem to indicate that, for a level of the externality high enough ($\pi \geq 1$), information decisions are complements for any degree of complementarity in actions. These results indeed follow in our model, as stated in the next proposition.

Proposition 2. *Suppose that actions are complements ($0 < \lambda < 1$), then the critical threshold $\kappa(\pi)$ identified in Proposition 1 (iii) (above which information decisions become substitutes) is continuous and nondecreasing in π. Moreover, for a level of the external effect sufficiently high, information decisions are complements regardless the level of complementarity in actions, $\kappa(1) = 1$.*

Thus, when the external effect is sufficiently high, the message delivered by the main result in HV's model is restored. Notice, however, that this implication follows in our model for quadratic payoffs different from the beauty contest case. The second order externality serves as an additional incentive for players to coordinate their actions. They become more concerned about the suitability of others' actions with the state, which makes them value more learning about the state when others do so. Consider, for instance, a group of firms competing in a Bertrand oligopoly where the market demand depends on some unknown parameter. In this case, the second order externality is high when the offered products are very homogenous. Then, each firm has a huge incentive to learn more about the parameter when others are well informed in order to choose a price appropriate to the market conditions but slightly below the prices set by its competitors.

5. Concluding Remarks

This paper investigated the relation between strategic interactions in information decisions and interactions in actions for a tractable class of games with complementary or substitutive actions, externalities, and a fairly general information structure.

Our analysis highlighted the differences in the nature of interactions when the set of players is finite with respect to the case with a continuum of players. From a methodological viewpoint, keeping track of players' higher-order beliefs through a knowledge index leads to conclusions different to those obtained using an average expectation operator. The main reason behind this discrepancy is in the implication that, with a small number of players, the variance of others' actions may decrease as they become better informed, making less valuable our own information about the state.

As the number of players increase, our results (for the beauty contest game) converge to the main result obtained by HV and, therefore, information decisions tend to inherit the same strategic motives as actions. Our results are relevant in environments with a relatively small number of players, such as industrial competition settings, but lose importance for the case of markets with many traders. For such large markets, usually studied in general equilibrium and macro models, the insights obtained by HV are very useful for the analysis of strategic interactions in information decisions.

The role played by external effects in our model points out the importance of considering payoffs that allow for a relation between strategic interactions and external effects more flexible than that imposed by beauty contests.

This paper has not analyzed the welfare consequences of strategic interactions in information decisions. A full characterization of the set of PBE for our game is required to conduct such analysis. Angeletos and Pavan (2007) have successfully examined the comparative statics of welfare with respect to the information structure in a setting with a continuum of players and without endogenous information choice. With endogenous information choice and a finite set of players, a comparative statics exercise along those lines seems an interesting direction for future research.

Appendix

Proof of Lemma 1. Using the posterior expectations in (2), we obtain

$$\alpha_i^*(s_i, x) = \tau_0 + (1-\lambda)\tau_1 \sum_{k=0}^{\infty} \lambda^k \left[\mu + \sqrt{x_{p(k)}(x_1 x_2)^k} \left(\frac{\sigma_\theta}{\sigma_s} \right) (s_i - \mu) \right].$$

Now, consider the pair of matrices

$$\phi := \left(\frac{\sigma_\theta}{\sigma_s} \right) (\sqrt{x_1}, \sqrt{x_2})_{1\times 2}, \qquad \Omega := \begin{pmatrix} 0 & \sqrt{x_1 x_2} \\ \sqrt{x_1 x_2} & 0 \end{pmatrix}_{2\times 2}.$$

Then,

$$\begin{aligned} \alpha_i^*(s_i, x) &= \tau_0 + \tau_1\mu + (1-\lambda)\tau_1\, \phi \cdot \sum_{k=0}^{\infty} \lambda^k \Omega^k \cdot e_i (s_i - \mu) \\ &= \tau_0 + \tau_1\mu + (1-\lambda)\tau_1\, \phi \cdot [I - \lambda\Omega]^{-1} \cdot e_i\, (s_i - \mu), \end{aligned}$$

where I denotes the 2×2 identity matrix and e_i is the ith vector of the canonical basis of $\mathbb{R}^2$. The infinite sum $\sum_{k=0}^{\infty} \lambda^k \Omega^k = [I - \lambda\Omega]^{-1}$ above is well-defined since we are assuming $\lambda \in (-1, 1)$. More precisely, Debreu and Herstein (1953) show that the convergence of $\sum_{k=0}^{\infty} \lambda^k \Omega^k$ is guaranteed if $|\lambda|$ is strictly less than the inverse of the largest eigenvalue of Ω. It can be easily verified that this largest eigenvalue equals $\sqrt{x_1 x_2}$ so that its inverse is strictly larger than one.

Let $m_i := (1-\lambda)\tau_1 \phi \cdot [I - \lambda\Omega]^{-1} \cdot e_i$. Since $[I - \lambda\Omega]$ is a 2×2 matrix, it is easy to compute analytically its inverse to obtain

$$m_i = \tau_1 \left(\frac{\sigma_\theta}{\sigma_s} \right) \frac{(1-\lambda)(1+\lambda x_j)\sqrt{x_i}}{(1-\lambda^2 x_i x_j)}.$$

To show that the action strategy profile α^* is unique, we follow the approach used by Calvó-Armengol and de Martí (2009) to prove uniqueness of Nash equilibrium for a one-stage, beauty contest game with an exogenously given information choice (Theorem 1). First, define the following two-player team payoff function:

$$\Psi(a_i, a_j, \theta) = -(1-\lambda)\left[(a_i-\theta)^2 + (a_j-\theta)^2\right] - \lambda(a_i - a_j)^2.$$

Consider a given information profile $x \in [0,1]^2$. Since the payoff function u is common for both players, the cross-derivatives in actions are the same for them (γ_{ij}). Then, since u is quadratic, it follows from Lemma 6 in Ui (2009) that the payoff u admits Ψ as a potential. In other words, when x is fixed, a player's decision problem in actions for the game $(u, N(\mu\underline{1}, \Sigma))$ can be solved using the payoff function Ψ.[4] Second, Theorem 4 in Radner (1962) can be used to show that optimal actions are unique in the team game with payoffs Ψ if the matrix $Q := (\partial^2\Psi(a,\theta)/\partial a_i \partial a_j)$ is negative definite. We obtain

$$Q = 2\begin{pmatrix} -1 & \lambda \\ \lambda & -1 \end{pmatrix},$$

with eigenvalues $\rho_{1,2} = -1 \pm \sqrt{1-(1-\lambda^2)} < 0$ for each $\lambda \in (-1,1)$. Thus, since the matrix Q above is definite negative, optimal actions are unique for the given x. $\square$

Derivation of a closed expression for the value of information: For the information profile $x = (1,1)$ information is complete and the exact value of θ is known by the players. In that case, both players have the same optimal action, which depends only on the state. Further, because u is quadratic, such optimal action is linear in θ. Let $\tau(\theta) := \tau_0 + \tau_1\theta$, where $\tau_0, \tau_1 \in \mathbb{R}$, be any player's optimal action under complete information. Consider now the incomplete information case and fix an information profile $x \in [0,1]^2$. A first-order Taylor expansion of $u(\alpha_i^*(s_i,x), \alpha_j^*(s_j,x), \theta)$ around $(\tau(\theta), \tau(\theta), \theta)$ yields

$$\begin{aligned} u(\alpha_i^*(s_i,x),&\alpha_j^*(s_j,x),\theta) = u(\tau(\theta),\tau(\theta),\theta) + (\gamma_{ii}/2)[\alpha_i^*(s_i,x) - \tau(\theta)]^2 \\ &+ (\gamma_{jj}/2)[\alpha_j^*(s_j,x)-\tau(\theta)]^2 + \gamma_{ij}[\alpha_i^*(s_i,x)-\tau(\theta)][\alpha_j^*(s_j,x)-\tau(\theta)], \end{aligned}$$

where we have used $\partial u(\tau(\theta),\tau(\theta),\theta)/\partial a_i = \partial u(\tau(\theta),\tau(\theta),\theta)/\partial a_j = 0$. Then, using the expression for α^* obtained in Lemma 1 and the definition of V_i given in (3), we have

$$\begin{aligned} V_i =& \mathbb{E}[u(\tau(\tilde\theta),\tau(\tilde\theta),\tilde\theta)] + (\tau_1^2/2)[\gamma_{ii}+\gamma_{jj}+2\gamma_{ij}]\mathbb{E}[(\tilde\theta-\mu)^2] \\ &+ ([\gamma_{ii}m_i^2 + \gamma_{jj}m_j^2]/2)\mathbb{E}[(\tilde s_i-\mu)^2] + \gamma_{ij}m_i m_j \mathbb{E}[(\tilde s_i-\mu)(\tilde s_j-\mu)] \\ &- \tau_1[\gamma_{ii}+\gamma_{ij}]m_i\mathbb{E}[(\tilde s_i-\mu)(\tilde\theta-\mu)] - \tau_1[\gamma_{jj}+\gamma_{ij}]m_j\mathbb{E}[(\tilde s_j-\mu)(\tilde\theta-\mu)]. \end{aligned}$$

Applying the expectation operators and using the information structure summarized by the variance-covariance matrix Σ in (1), we obtain

$$\begin{aligned} V_i = \mathbb{E}[u(\tau(\tilde\theta),\tau(\tilde\theta),\tilde\theta)] + \frac{\tau_1^2\sigma_\theta^2}{2}\Big[\gamma_{ii}+\gamma_{jj}+2\gamma_{ij}\Big] \\ + \frac{1}{2}\Big[\gamma_{ii}m_i^2\sigma_s^2 + \gamma_{jj}m_j^2\sigma_s^2 + 2\gamma_{ij}m_i m_j\sigma_{i\theta}\sigma_{j\theta}\sigma_\theta^{-2} - 2\tau_1(\gamma_{ii}+\gamma_{ij})m_i\sigma_{i\theta} - 2\tau_1(\gamma_{jj}+\gamma_{ij})m_j\sigma_{j\theta}\Big]. \end{aligned}$$

[4]Potential games are formally defined by Moderer and Shapley (1996) and Bayesian potential games are formally defined by Heumen, Peleg, Tijs, and Borm (1996). See Ui (2009) for a general existence result of Bayesian potential games for quadratic games with symmetric cross-derivatives of payoffs.

Because $(-1/\gamma_{ii})$ is a positive constant, the sign of $\partial^2 V_i/\partial x_i \partial x_j$ coincides with the sign of $(-1/\gamma_{ii})\partial^2 V_i/\partial x_i \partial x_j$. Using this, we find convenient to propose the normalization $\widehat{V}_i := (-1/\gamma_{ii})V_i$ and proceed in terms of $\widehat{V}_i$ instead of V_i. For each player $i = 1, 2$, let us define the function $q_i : [0,1]^2 \to \mathbb{R}$, specified by

$$q_i(x) := \frac{1+\lambda x_j}{1-\lambda^2 x_i x_j}.$$

Then, with the normalization indicated above and by developing the terms in m_i and m_j, we obtain

$$\begin{aligned}\widehat{V}_i = &(-1/\gamma_{ii})\mathbb{E}[u(\tau(\tilde{\theta}),\tau(\tilde{\theta}),\tilde{\theta})] + \frac{\tau_1^2\sigma_\theta^2}{2}\Big[2\lambda - \pi - 1\Big] \\ &+ \frac{(1-\lambda)^2\tau_1^2\sigma_\theta^2}{2}\Big[-q_i^2 x_i - \pi q_j^2 x_j + 2\lambda q_i q_j x_i x_j + 2q_i x_i - 2\Big(\frac{\lambda-\pi}{1-\lambda}\Big)q_j x_j\Big].\end{aligned}$$

We now compute the second order derivative $\partial^2 \widehat{V}_i/\partial x_i \partial x_j$ from the expression above. For the required algebra, it is useful to take into account

$$\frac{\partial q_i}{\partial x_i} = \frac{\lambda^2 x_j q_i}{1-\lambda^2 x_i x_j}, \qquad \frac{\partial q_i}{\partial x_j} = \frac{\lambda q_j}{1-\lambda^2 x_i x_j}, \qquad \frac{\partial^2 q_i}{\partial x_i \partial x_j} = \frac{\lambda^3 x_j q_j + \lambda^2 q_i}{(1-\lambda^2 x_i x_j)^2}.$$

Using that, it can be verified that

$$\begin{aligned}\frac{\partial^2 \widehat{V}_i}{\partial x_i \partial x_j} =& \lambda\left[\frac{(1-\lambda)\tau_1\sigma_\theta}{1-\lambda^2 x_i x_j}\right]^2 \Bigg[(2-\pi)\lambda^2 x_i x_j q_i q_j \\ &+ \lambda\Big[x_i q_i - \Big(\frac{\lambda-\pi}{1-\lambda}\Big)x_j q_j + (1-\pi)x_j q_j^2\Big] + q_j - \Big(\frac{\lambda-\pi}{1-\lambda}\Big)q_i - \pi q_i q_j\Bigg].\end{aligned} \tag{5}$$

For the particular case of a beauty contest, we impose $\pi = \lambda^2$ to obtain

$$\begin{aligned}\left.\frac{\partial^2 \widehat{V}_i}{\partial x_i \partial x_j}\right|_{\lambda=\pi^2} =& \lambda\left[\frac{(1-\lambda)\tau_1\sigma_\theta}{1-\lambda^2 x_i x_j}\right]^2 \Bigg[-x_i x_j q_i q_j \lambda^4 - x_j q_j^2 \lambda^3 \\ &+ (2x_i x_j q_i q_j - x_j q_j - q_i q_j)\lambda^2 + (x_i q_i + x_j q_j^2 - q_i)\lambda + q_j\Bigg].\end{aligned} \tag{6}$$

With the expressions for the ex-ante expected payoff given by (5) and (6) at hand, we proceed to the proofs of the propositions.

Proof of Proposition 1. Given an information profile $x \in (0,1)^2$, let $h : [-1,1] \times \mathbb{R}_+ \to \mathbb{R}$ be the function specified as

$$\begin{aligned}h(\lambda,\pi) :=& (1-\lambda)\left[x_i x_j q_i q_j (2-\pi)\lambda^2 + x_i q_i \lambda + (1-\pi)x_j q_j^2 \lambda + q_j - \pi q_i q_j\right] \\ &- (\lambda-\pi)\left[x_j q_j \lambda + q_i\right].\end{aligned}$$

Then, for each $\lambda \in (-1,1)$, we have

$$\frac{\partial^2 \widehat{V}_i}{\partial x_i \partial x_j} = \left[\frac{(1-\lambda)\tau_1\sigma_\theta}{1-\lambda^2 x_i x_j}\right]^2 \left[\frac{\lambda}{1-\lambda}\right] h(\lambda,\pi).$$

Note that $\partial^2 \widehat{V}_i / \partial x_i \partial x_j$ and h have the same sign when actions are complements ($\lambda \in (0,1)$) and different sign when actions are substitutes ($\lambda \in (-1,0)$).

Also, given $x \in (0,1)^2$, let $g : [-1,1] \to \mathbb{R}$ be the function specified as

$$g(\lambda) := -x_i x_j q_i q_j \lambda^4 - x_j q_j^2 \lambda^3 + (2 x_i x_j q_i q_j - x_j q_j - q_i q_j)\lambda^2 + (x_i q_i + x_j q_j^2 - q_i)\lambda + q_j.$$

Then, we have

$$\left.\frac{\partial^2 \widehat{V}_i}{\partial x_i \partial x_j}\right|_{\lambda=\pi^2} = \lambda \left[\frac{(1-\lambda)\tau_1 \sigma_\theta}{1-\lambda^2 x_i x_j}\right]^2 g(\lambda).$$

Note that the sign of $\partial^2 \widehat{V}_i / \partial x_i \partial x_j|_{\lambda=\pi^2}$ coincides with the sign of g when actions are complements ($\lambda \in (0,1)$).

(i) The result follows directly from the specification of the function $\partial^2 \widehat{V}_i / \partial x_i \partial x_j$ in (5).

(ii) For a given $x \in [0,1]^2$, we have

$$h(0,\pi) = q_j - \pi q_i q_j + \pi q_i = 1.$$

The result follows since $h(0,\pi) > 0$, $\partial^2 \widehat{V}_i / \partial x_i \partial x_j = 0$ for $\lambda = 0$, $\partial^2 \widehat{V}_i / \partial x_i \partial x_j$ is a continuous function in $\lambda \in (-1,1)$, and (a) $\lambda/(1-\lambda) > 0$ if $\lambda \in (0,1)$ while (b) $\lambda/(1-\lambda) < 0$ if $\lambda \in (-1,0)$.

(iii) Take a given $x \in (0,1)^2$ and a given $\pi \in [0,1)$. It can be checked that $h(0,\pi) = 1$ and

$$h(1,\pi) = (\pi - 1)[x_j q_j + q_i] = (\pi - 1)\left[\frac{x_j(1+x_i) + (1+x_j)}{1 - x_i x_j}\right] < 0.$$

Since $h(0,\pi) > 0$, $h(1,\pi) < 0$, and $h(\cdot,\pi)$ is a continuous function in λ, there is some $\kappa(\pi) \in (0,1)$, bounded away from 1, such that $h(\kappa(\pi),\pi) = 0$ and $h(\lambda,\pi) < 0$ for each $\lambda \in (\kappa(\pi),1)$. The result follows since $\lambda/(1-\lambda) > 0$ for each $\lambda \in (0,1)$.

(iv) Take a given $\pi \in [0,1)$, and consider $x_1 = 0$ and $x_2 = 1$. Then,

$$h(-1,\pi) = 2\left[-(1-\pi)q_j^2 + q_j - \pi q_i\right] - (-1-\pi)\left[-q_j + q_i\right] = \pi - 1 < 0.$$

Since $h(0,\pi) > 0$, $h(-1,\pi) < 0$ for $x_i = 0$ and $x_j = 1$, and $h(\cdot,\pi)$ is a continuous function in λ, in x_i, and in x_j, then there are some $\epsilon(\pi), \delta(\pi) > 0$ and some $\kappa(\pi) \in (-1,0)$, bounded away from -1, such that $h(\lambda,\pi) < 0$ for each $\lambda \in (-1,\kappa(\pi))$, each $0 \le x_i < \epsilon(\pi)$, and each $(1-\delta(\pi)) < x_j \le 1$. The result follows since $\lambda/(1-\lambda) < 0$ for each $\lambda \in (-1,0)$.

(v) For a given $x \in (0,1)^2$, we have

$$g(1) = (x_i x_j - 1)q_i q_j + (x_i - 1)q_i - (x_j - 1)q_j = \frac{(x_i - 1) - (x_i + 3)x_j}{1 - x_i x_j} < 0.$$

Since g is a continuous function in $\lambda \in (-1,1)$, and $g(0) > 0$ while $g(1) < 0$, there is some $\beta \in (0,1)$, bounded away from 1, such that $g(\beta) = 0$ and $g(\lambda) < 0$ for each $\lambda \in (\beta,1)$. The result follows since we are considering $\lambda > 0$. □

Proof of Proposition 2. For $\lambda \in (0,1)$, let $\psi(\lambda)$ be the value of π such that $h(\lambda, \psi(\lambda)) = 0$. From the result in Proposition 1 (iii), we know that there exists some $\kappa(0) \in (0,1)$ such that $\partial^2 \widehat{V}_i / \partial x_i \partial x_j = 0$ for each $x \in (0,1)^2$. This happens if and only if $h(\kappa(0), 0) = 0$. Therefore, $\psi(\kappa(0)) = 0$ with $\kappa(0) < 1$. Also, it can be easily checked that $h(1,1) = 0$ so that $\lim_{\lambda \to 1^-} \psi(\lambda) = 1$. Suppose that the mapping $\psi : (0,1) \to \mathbb{R}_+$ is a function and not a correspondence. Then, since $h(\lambda, \pi)$ is continuous for each $(\lambda, \pi) \in (0,1) \times \mathbb{R}_+$, it must be the case that $\psi(\lambda)$ is continuous and nondecreasing in $\lambda \in (0,1)$ and, therefore, the critical threshold $\kappa(\pi)$ identified in Proposition 1 (iii) is continuous and nondecreasing in π. Therefore, we only need to verify that ψ is indeed a function and not a correspondence.

To show that ψ is a function, take $\pi, \pi' \in \mathbb{R}_+$ such that $\pi \neq \pi'$ and suppose that, for some given $\lambda \in (0,1)$, we have $\psi(\pi) = \psi(\pi')$. This happens if and only if $h(\lambda, \pi) = h(\lambda, \pi') = 0$. Then, on the one hand, $h(\lambda, \pi) = h(\lambda, \pi')$ implies

$$(\pi - \pi') \big[(1-\lambda)[x_i x_j q_i q_j \lambda^2 + x_j q_j^2 \lambda + q_i q_j] - [x_j q_j \lambda + q_i] \big] = 0.$$

Because $\pi \neq \pi'$, it must be the case that

$$[x_i x_j q_i q_j \lambda^2 + x_j q_j^2 \lambda + q_i q_j] = \frac{1}{1-\lambda} [x_j q_j \lambda + q_i].$$

On the other hand, $h(\lambda, \pi) = 0$ implies

$$\frac{1}{\lambda - \pi} \big[x_i x_j q_i q_j (2-\pi) \lambda^2 + x_i q_i \lambda + (1-\pi) x_j q_j^2 \lambda + q_j - \pi q_i q_j \big] = \frac{1}{1-\lambda} [x_j q_j \lambda + q_i]$$

Therefore, by combining the two equalities above, it follows that $\lambda \in (0,1)$ must necessarily satisfy the equation

$$x_i x_j q_i q_j (2-\lambda) \lambda^2 + x_j q_j^2 (1-\lambda) \lambda + x_i q_i \lambda + q_j - \lambda q_i q_j = 0.$$

Now, using the definition of q_i and q_j, we know that if the equation above is satisfied, then we must necessarily have

$$\begin{aligned} (1+&\lambda x_j)(1+\lambda x_i)\lambda \big[(2-\lambda)\lambda x_i x_j - 1 \big] \\ &+ x_j (1+\lambda x_i)^2 (1-\lambda)\lambda + [x_i(1+\lambda x_j)\lambda + (1+\lambda x_i)](1-\lambda^2 x_i x_j) = 0. \end{aligned} \tag{7}$$

But equation (7) above cannot hold for $\lambda \in (0,1)$ because

$$(1+\lambda x_j)(1+\lambda x_i)\lambda \big[(2-\lambda)\lambda x_i x_j - 1 \big] > -\lambda,$$

$$x_j (1+\lambda x_i)^2 (1-\lambda)\lambda + [x_i(1+\lambda x_j)\lambda + (1+\lambda x_i)](1-\lambda^2 x_i x_j) > 1,$$

and $1 - \lambda > 0$. From this contradiction, it follows that ψ is a function.

The last claim of the proposition holds since $\lim_{\lambda \to 1^-} \psi(\lambda) = 1$ implies $\kappa(1) = 1$. □

References

Angeletos, G.-M., and A. Pavan (2004): "Transparency of Information and Coordination in Economies with Investment Complementarities," *American Economic Review*, 94, 2, 91-98.

Angeletos, G.-M., and A. Pavan (2007): "Efficient Use of Information and Social Value of Information," *Econometrica*, 75, 4, 1103-1142.

Calvó-Armengol, A., and J. de Martí (2007): "Communication Networks: Knowledge and Decisions," *American Economic Review*, 97, 2, 86-91.

Calvó-Armengol, A., and J. de Martí (2009): "Information Gathering in Organizations: Equilibrium, Welfare, and Optimal Network Structure ," *Journal of the European Economic Association*, 7, 1, 116-161.

Calvó-Armengol, A., de Martí, J. and A. Prat (2009): "Endogenous Communication in Complex Organizations," mimeo.

Chatterjee, K., and T. P. Harrison (1988): "The Value of Information in Competitive Bidding," *European Journal of Operational Research*, 322-333.

Colombo, L., and G. Femminis (2008): "The Social Value of Public Information with Costly Information Acquisition," *Economics Letters*, 100, 196-199.

Cooper, R., and A. John (1988): "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics*, 103, 441-463.

Cornand, C., and F. Heinemann (2008): "Optimal Degree of Public Information Dissemination," *The Economic Journal*, 118, 528, 718-742.

Debreu G., and I. N. Herstein (1953): "Nonnegative Square Matrices," *Econometrica*, 21, 597-607.

Dewan T., and D. P. Myatt (2008): "The Qualities of Leadership: Direction, Communication, and Obfuscation," *American Political Science Review*, 102, 3, 351-368.

Dewan T., and D. P. Myatt (2009): "On the Rethorical Strategies of Leaders: Speaking Clearly, Standing Back, and Stepping Down," mimeo.

Fudenberg, D., and D. Levine (1986): "Limit Games and Limit Equilibria," *Journal of Economic Theory*, 38, 261-279.

Hagenbach, J., and F. Koessler (2010): "Strategic Communication Networks," *Review of Economic Studies*, 77, 1072-1099.

Hellwig, C. (2005): "Heterogeneous Information and the Welfare Effects of Public Information Disclosures," UCLA Working Paper.

Hellwig, C., and L. Veldkamp (2009): "Knowing what Others Know: Coordination Motives in Information Acquisition," *Review of Economic Studies*, 76, 1, 223-251.

Moderer, D., and L. S. Shapley (1996): "Potential Games," *Games and Economic*

Behavior, 14, 124-143.

Morris, S., and H. S. Shin (2002): "Social Value of Public Information," *American Economic Review*, 92, 5, 1521-1534.

Morris, S., and H. S. Shin (2005): "Central Bank Transparency and the Signal Value of Prices," *Brooking Papers of Economic Activity*, 2, 1-66.

Myatt, D. P., and C. Wallace (2010): "Endogenous Information Acquisition in Coordination Games," mimeo.

Radner, R. (1962): "Team Decision Problems," *Annals of Mathematical Statistics*, 33, 857-881.

Radner, R. (1980): "Collusive Behavior in Oligopolies with Long but Finite Lives," *Journal of Economic Theory*, 22, 136-156.

Ui, T. (2009): "Bayesian potentials and information structures: Team decision problems revisited," *International Journal of Economic Theory*, 5, 271-291.

van Heumen, R., Peleg, B., Tijs, S., and P. Borm (1996): "Axiomatic Characterizations and Solutions for Bayesian Games," *Theory and Decision*, 40, 103-130.