"Versioning" with Advertising in Social Networks*

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Abstract

This article investigates second-degree discrimination through a two-version service that a platform serves over a random network. Consumers choose between purchasing a premium version or a free version of the service, which comes with advertising about some product (unrelated to the service). Under the assumption that advertising rotates clockwise the inverse demand of the product, I explore how the random network, and the market conditions for the advertised product, relate to optimal pricing of the service. Hazard rate functions are crucial for optimal pricing and first-order stochastic dominance of the degree distribution characterizes welfare implications. The model provides foundations for empirical analysis on degree distributions and hazard rate functions underlying complex social networks.

Keywords: Social networks, second-degree discrimination, advertising, rotation of demand, degree distributions, hazard rate

JEL Classification: D83, D85, L1, M3

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1 Introduction

This article develops a model of second-degree discrimination with advertising that is applicable to the provision of two-version services over social networks in the presence of usage externalities. The motivation comes from the observation that most online platforms have the choice of offering two versions of their services: a *premium version*, which consumers purchase at a price, and a *free version*, where consumers pay no price but, in exchange, they receive ads of some other products (usually unrelated to the service). The attached ads have two effects. First, exposure to advertising is costly for the consumers, which establishes the relevant trade-off to choose one version or the other. Secondly, advertising influences free version adopters' opinions about the advertised product and, therefore, the revenue from the sales of such a product. Thus, when the platform receives a compensation based on the product sales, the optimal pricing of the premium version is also affected.

Over the last decade, the expansion of the Internet, mobile devices, and other communication technologies has facilitated a substantial increase of online services that are served over social networks. The networks targeted by such platforms tend to feature positive externalities: the usage of the service is more or less beneficial to a consumer depending on whether or not his neighbors (e.g., other users, family, friends, or co-workers) are using it as well. Given such network externalities, the platform could benefit from implementing some type of price discrimination. One plausible strategy is to offer individualized prices as a function of the consumer's position in the network. On this important issue of perfect price discrimination, the theoretical literature on networks has recently yielded a number of key insights which relate the firm's optimal pricing to consumer centrality (Candogan et al. (2012) and Bloch & Quérou (2013)). Perfect price discrimination, however, is hard to implement in many real-world complex social networks.[1]

Alternatively, as the technology available for providing online services has made it possible for other firms (from a variety of industries, in principle unrelated to that of the

[1] First, it requires the platform to have full information about the entire network, which may be unfeasible in complex (and, sometimes, rapid evolving) networks. Secondly, setting a particular price to each consumer depending on his position in the network may result too costly (or simply unpractical) for the platform even if it is able to identify completely the neighbors of each consumer.

platform) to use it as a channel to advertise their own products, platforms often implement some sort of second-degree discrimination with advertising. A widely established pattern is one where the platform offers its service over some social network and another firm uses the platform's diffusion technology to advertise its own product. The platform receives then a compensation, which is usually based on the amount of ads served or on the profits from the advertising activity. An additional feature of these business models is that the service is designed in a way such that the premium version usually allows consumers to enjoy the network externality to a larger extent than the free version. This type of second-degree quality discrimination, which is extensively used in many real-world social networks,[2] is commonly referred to as "versioning" in the business literature. Whereas the incentives for perfect price discrimination in social networks are now fairly well understood by economists, second-degree discrimination through "versioning" with advertising has received little formal study. Understanding the simple economics of this type of second-degree discrimination is the goal of this article.

To overcome the difficulties of dealing with complex networks, the platform can rely on the empirical regularities that exhibit most social networks.[3] Considering random networks allows for a tractable framework. Based on random networks, this article explores how (a) the degree distribution of the network and (b) the optimal pricing decisions of the advertised product affect (1) the incentives of the platform to pursue "versioning" with advertising, (2) the platform's optimal pricing strategy, (3) the platform's optimal profits, and (4) consumer surplus.

The notion of advertising used in the article builds upon the general framework proposed by Johnson & Myatt (2006) to consider rotations of demand induced by informative

[2]Providing services over social networks using technologies that allow to attach ads is the business model of companies such as Google (general communication and information), Facebook, Twitter, and Snapchat (social interaction), Whatsapp, Skype, and Line (communication) AirBnB (accommodation search), Waze (traffic and route forecasting), The Weather Channel (weather forecasting), Yelp and Foursquare (review and rating), YouTube, Vimeo, and Spotify (entertaining and information), Strava (exercise and health tracking), Box (second-hand trade), or Tinder (dating). All these platforms provide either a free version, a premium version, or pursue "versioning" with both versions. Over the recent years most of these platforms have switched, at least once, from one regime to another. Currently, most of them seem to opt either for "versioning" or for serving only the free version, which seems to indicate that advertising does play an important role in their business models.

[3]See, e.g., Jackson (2008) where empirical regularities of social networks such as *small worlds*, *clustering*, or *assortativity* are discussed at length.

advertising. Following Lewis & Sappington (1994) and Johnson & Myatt (2006), I assume that advertising helps consumers to improve their knowledge of their true underlying preferences for the product. In particular, as in Johnson & Myatt (2006), such informative advertising is assumed to rotate clockwise the inverse demand of the product. Intuitively, advertising makes "more disperse" or "more heterogeneous" the consumers' valuations of the product. While this assumption does not place strong restrictions, it encompasses models of signaling advertising, as explained in Subsection 2 (Example 1), or models where advertising lowers price-elasticity of the demand, as considered by Assumption 3.

Unlike the classical benchmark of second-degree discrimination proposed by Maskin & Riley (1984), wherein the two types of consumers "self-select" themselves based only on their exogenous valuations of some commodity, the current model considers that consumers self-select (to adopt one or another version of the service) depending on the features of the network and the difference in network externalities across both versions. Then, given such self-selection, advertising affects endogenously the valuations of the product for those consumers that choose the free version. No price discrimination takes place on the advertised product.[4] By choosing the price of the service, the platform affects both the revenue from the sales of the premium version and the proportion of free version adopters. As free version adopters (and only them) are influenced by advertising, this in turn affects the revenue from the advertised product sales. In short, the key tradeoff that the platform faces is whether to rely on the profits from the sales of the premium version of the service or from its advertising activity.

For relatively low optimal prices of the advertised product, Proposition 1 shows that the platform prefers to offer only the premium version rather than pursuing "versioning." For interior optimal prices where the platform optimally serves both versions of the service, the optimal pricing strategy depends crucially on the hazard rate function of the degree distribution (Lemma 1). Application of the classical *first-order stochastic dominance* notion to the degree distribution of the random network characterizes completely the model's main welfare implications. First-order stochastic dominance over the degree distribution follows if and only if the network that dominates yields higher optimal prof-

[4] In the current setting, network externalities affect only the usage of the service and not the consumption of the advertised product.

its to the platform (Proposition 2) and lower values of consumer surplus (Proposition 5). Other welfare insights follow by comparing consumer surplus in situations where the platform optimally pursues "versioning" to others where it optimally serves only the premium or the free version. Consumer surplus is always lower when the platform pursues "versioning," compared to the cases where it serves only the premium version (Proposition 6). Proposition 3 studies the relation between the optimal pricing strategies for the service and the advertised product. For increasing hazard rates of the degree distribution, the optimal prices of the service and of the advertised product always move in the same direction whereas decreasing hazard rates with sufficiently high slope (in absolute value) are required for optimal prices to move in opposite directions. Under the stronger assumption that advertising reduces the price-elasticity of the advertised product (Assumption 3), Corollary 1 provides a clear-cut condition on the hazard rate function of the degree distribution under which the optimal prices of the service and the product move in the same direction.

The empirical literature on social networks suggests that most large but relatively sparse social networks adjust to the pattern of *scale-free networks*, which are governed by *power law* degree distributions and thus feature decreasing hazard rates.[5] In addition, other recent empirical findings suggest that relatively dense and well connected networks tend to diverge from the scale-free/power law pattern and adjust rather to the *exponential* degree distribution pattern, which features constant hazard rate functions.[6] The application of the model to the influential power law degree distribution relates key optimal pricing features in a neat way to the parameter of the distribution. As for the application to the exponential degree distribution, the optimal price of the service always increases in the difference of optimal profits (with and without advertising) from the product sales.

[5]In their seminal study of the World Wide Network, Barabási & Albert (1999) concluded that the degree distribution of the nodes on the Internet adjusts to a power law distribution. More recently, Clauset et al. (2009) found empirical evidence that both the nodes on the Internet (at the level of autonomous systems) and the number of links in websites adjust to a power law distribution. Also, Stephen & Toubia (2009) argue that the emerging social commerce network through the Internet follows a typical power law distribution. In perhaps the largest structural analysis conducted up to date, Ugander et al. (2011) conclude that the Facebook social network features decreasing hazard rates as well, though not necessarily fitting a power law distribution. Also, decreasing hazard rates are always generated by two ubiquitous models of the theoretical literature on random networks: the *preferencial attachment model* proposed by Barabási & Albert (1999) and the *network-based search model* suggested by Jackson & Rogers (2007).

[6]See, e.g., Rosas-Calals et al. (2007) and Ghoshal & Barabási (2011).

Interestingly, the shapes of the hazard rate functions of both degree distributions lead to that the optimal prices of the service and of the price always move in the same direction, so that a rise in the price of the service always makes consumers worse off. Finally, by applying the condition provided in Proposition 4 to both degree distributions, it follows that if the platform finds optimal to set relatively high prices of the service, this may lead it to finally serve only the free version of the service.

The model's results provide empirical predictions of how optimal decisions on "versioning" with advertising, and its implications on welfare, depend on the market features of the advertised product, as well as on the shape of the degree distribution and its associated hazard rate function. Lastly, given the generality of the inverse demand functions and of the degree distributions considered, this article does not explore conditions that guarantee that optimal prices are interior or unique. The optimal prices explored in the main applications, though, are interior and (under very mild conditions) unique.

Related Literature

This article is based on the setting proposed, in a companion article, by Gonzalez-Guerra & Jimenez-Martinez (2016) to explore second-degree discrimination in social networks, wherein advertising is modeled as a signaling device. The main difference between the two articles is that the current setting considers a general class of inverse demand functions for the advertised product and a broader set of plausible effects on demand due to advertising. While the assumptions of Gonzalez-Guerra & Jimenez-Martinez (2016) of uniform valuation distribution for the product and of a purely signaling effect from advertising allow for tractable expressions for optimal pricing (which can be readily used in applications), the current model is able to capture other effects of advertising (such as increases in the dispersion of the valuations or reductions in the price-elasticity of the advertised product's demand), and to explore welfare implications in a fairly general benchmark.[7]

[7] Importantly, although the current model clearly lays out a more general setting and uses weaker assumptions, there is a technical difference between the two models that makes them not directly comparable in terms of inclusion between their sets of assumptions. While the current article considers a general family of distributions for the valuations of the advertised product, they are required to have a common support. Without this requirement, the platform's profit function might feature discontinuities that would affect qualitatively the results in the general case. On the other hand, the restriction to uniform distributions of valuations in Gonzalez-Guerra & Jimenez-Martinez (2016) leads by construction

The motivation of these two articles comes from the analysis of price discrimination where a monopolist offers a menu of different qualities of some product, which originated with the works of Mussa & Rosen (1978) and Maskin & Riley (1984), and by the large literature that explores the effects of network externalities on economic decisions, which initiated with Farrell & Saloner (1985) and Katz & Shapiro (1985).

Considering that advertising generates exposure costs to consumers dates back to the seminal article of Becker & Murphy (1993). The notion of informative advertising, as well as its effects on demand rotation, used in this article builds largely on the insights provided by Johnson & Myatt (2006). Regarding this modeling assumption, there is also a deep connection with the structure of the endogenous information decision problem explored by Amir & Lazzati (2016) in the context of (common value) Bayesian supermodular games proposed by Van-Zandt & Vives (2007).[8] As in supermodular structures, consumers' demand decisions and valuations of the advertised product are complements in the current article. Given this, as Johnson & Myatt (2006) shows, the assumption that the inverse demand of the advertised product rotates clockwise implies that the platform optimally wishes to provide either fully informative or totally non-informative advertising. Although they study a problem with a different motivation, Amir & Lazzati (2016) consider an information structure that is convex in the supermodular order, which is conceptually similar to the assumption on demand rotation effects induced by higher advertising informative quality.

The technical side of the current article relates to the prolific literature on random networks that started with the influential work of Erdös & Renyi (1959). The notion of random networks used in the article meets the assumptions of the canonical *configuration model*, which was proposed by Bender & Canfield (1978) and subsequently applied by a number of articles in the social networks area.[9] The article's interpretations of the role of the hazard rate function of the network degree distribution rely heavily on the recent micro-founded model provided by Shin (2016). Using a dynamic benchmark, Shin (2016)

to varying supports in the considered family of distributions. Specifically, the family of distributions considered in Gonzalez-Guerra & Jimenez-Martinez (2016) does not satisfy the key condition imposed by Assumption 1 of the current article.

[8] See also Dessein et al. (2016) within the organizational literature.

[9] See, e.g., Bollobás (2001), Newman et al. (2001), Jackson & Yariv (2007), Galeotti & Goyal (2009), and Fainmesser & Galeotti (2016)).

provides a full characterization result wherein increasing hazard rates arise if an only if nodes are less likely (or equally likely) to engage in new links as their degrees increase.

Although less closely related, the current article also complements a prolific literature on how firms optimally spread information over social networks. The literature on optimal advertising in the presence of adoption externalities initiated with the insights of Butters (1977) and Grossman & Shapiro (1994). Also, optimal targeting and advertising strategies through local *word of mouth communication* have been investigated by a number of articles since the seminal contributions of Ellison & Fudenberg (1995) and Bala & Goyal (1998).[10] Using random networks, Leduc et al. (2017) have recently proposed a model of sequential product adoption where the monopolist offers referrals payments. Interestingly, their results on optimal pricing and optimal profits can be related to the degree distribution of the underlying random network. The current article departs from those contributions in two respects. First, the novel form of second-degree discrimination with advertising that this article proposes is not present in those works. Secondly, the current model does not consider that the information about the advertised product flows through the network depending on how consumers are linked. While information is transmitted publicly to all free version adopters, the only role of the network is to provide consumers with the incentives for their purchasing decisions, depending on the differences between the network externalities.

Finally, Gramstad (2016) has recently addressed questions similar to the ones explored here by considering a monopolist that allows consumers to choose from a menu of differentiated products in the presence of local externalities. In particular, a consumer benefits from the consumption of a connection (either direct or indirect) only if they purchase the same product. While Gramstad (2016) explores the role of the network structure in optimal pricing when consumers may choose between different versions of a product, as the current article does, there are important differences between both approaches. The

[10] For instance, Galeotti & Goyal (2009) relate optimal marketing strategies, both under *word of mouth communication* and adoption externalities, to the characteristics of the random network. Under the assumption that consumers only inform their neighbors if they themselves purchase the product, Campbell (2013) proposes a dynamic model of optimal pricing and advertising in random networks where information diffusion is endogenously generated. More recently, Fainmesser & Galeotti (2016), building upon the insights on perfect discrimination by Candogan et al. (2012) and Bloch & Quérou (2013), explore pricing and welfare implications when consumers are heterogenous with respect to their influence abilities.

main one is that a mechanism such as advertising, which influences consumers' valuations, is absent in Gramstad (2016)'s analysis. In addition, the size of the network externality in the current model depends only on the version chosen by the consumer himself and not on the versions chosen by his neighbors. Thus, the current model considers a single market for a (two-version) service, whereas Gramstad (2016) focuses more on segmented markets along the network.

The rest of the article is organized as follows. Section 2 proposes a new taxonomy of second-degree discrimination through "versioning" with advertising and lays out the model. Section 3 describes the model's main results on optimal pricing and profits. Section 4 explores some effects of "versioning" with advertising on consumer surplus. Section 5 provides the main applications of the model and Section 6 concludes. The proofs omitted in the text are relegated to the Appendix.

2 The Model

There is a unit mass of consumers, indexed by $i \in [0, 1]$, that can purchase two unrelated commodities, a consumption *product* and a two-version online *service*. The product is offered by a monopolist that produces $z \geq 0$ units, at a marginal cost $c > 0$, and sets a price $p \geq 0$ for each unit. The monopolist does not price discriminate and, thus, the price p is common to all consumers. The two-version service consists of a *premium version* and a *free version*, and it is provided by an online platform, at no cost. Each consumer has a unit demand for the service and I will use exchangeably the expressions "purchase" or "adopt" a version of the service. I will refer to the policy where the platform finds optimal to serve both versions of the service as "versioning."[11] With "versioning," the platform follows a second-degree discrimination policy where the consumers self-select themselves to adopt one version or the other. Premium version adopters pay a price $q \geq 0$ for each unit of the service. Free version adopters pay nothing for the service but, attached to the service, they receive ads about the consumption product. Advertising generates an

[11] Formally, "versioning" will correspond to an interior optimal choice whereas serving only either the premium version or the free version will be associated to corner optimal choices.

(exogenous) exposure cost $\psi > 0$ to free version adopters.[12] Throughout the article, the notation $a = 0$ or $a = 1$ will indicate, respectively, that ads about the product are either absent or present. As a compensation from its advertising activity, the platform receives a fraction $\alpha \in (0, 1]$ of the profits from the product sales.[13]

A consumer is willing to pay up to ω for a single unit of the product and up to θ for a single unit of the service. Conditional on receiving an advertising level $a \in \{0, 1\}$, the valuation for the product ω is drawn from a distribution $F_a(\omega)$ with support on some interval $(\underline{\omega}, \overline{\omega}) \subset \mathbb{R}_+$.[14] Each distribution $F_a(\omega)$, for $a \in \{0, 1\}$, is assumed to be twice continuously differentiable and to have a strictly positive density $f_a(\omega)$. The valuations of the product are independent across consumers. The valuation of the service θ is randomly drawn from some interval $(\underline{\theta}, \overline{\theta}) \subset \mathbb{R}_{++}$. The valuations of the service are independent across consumers as well. The two commodities are totally unrelated and, therefore, the valuations ω and θ are assumed to be independent from each other.

I assume that the exposure cost is very small relative to the valuation of the service, $\psi \in (0, \underline{\theta})$, which enables us to focus on situations where all consumers find optimal to purchase at least one version of the service (that is, at least the free version). Given this, the analysis then explores the conditions under which consumers might decide to purchase the premium version rather than the free one.

The Network and Consumer Preferences

The consumers are embedded in an (exogenous) complex social network.[15] The network allows consumers to interact locally with respect to their consumptions (only) of the service. The consumption of the service exhibits a local (positive) network effect: a consumer's utility from (any version of) the service increases as his neighbors increase

[12]Such type of costs from exposure to advertising have been commonly considered by the literature on advertising. See, e.g., Stigler & Becker (1977) and Becker & Murphy (1993).

[13]Parameter α could be interpreted as the level of integration between the monopolist and the platform. The value $\alpha = 1$ reflects that both companies are totally integrated, which is the case in some real-world instances, such as, e.g., IBM and the Weather Channel, Under Armour and MapmyRun.

[14]The upper limit $\overline{\omega}$ is allowed to tend to infinity.

[15]The social network includes the links that are provided by the platform as part of its service but a consumer's neighbors can include connections derived from other online platforms, or from informal relations such as family, friendship, or working relations. In this sense, the platform allows the consumers to form links but the platform does not design the network.

their consumptions. These externalities simply capture the idea that the total utility from using the service is positively related to the number of neighbors who are using it.[16]

The network is assumed to be stochastically generated to capture its complexity and the uncertainty that the platform and the consumers may have about its architecture, as well as to allow for dynamic interpretations where the network evolves over time. The platform and the consumers are uncertain about the specific architecture of the network but they commonly know the stochastic process that generates it. There is a set $[\underline{n}, \overline{n}] \subseteq \mathbb{R}_+$ of possible neighborhood sizes, or *degrees in the social network*.[17] Let $n_i \in [\underline{n}, \overline{n}]$ denote a possible *degree for consumer* i. The *degree distribution of the social network* is given by a twice continuously differentiable distribution $H_s(n)$, with a strictly positive density $h_s(n)$ over the support $[\underline{n}, \overline{n}]$. The parameter $s \in [\overline{s}, \underline{s}]$ indexes a family of degree distributions.

Let x_i be the probability that consumer i buys the premium version of the service and let $x_s(q)$ be the fraction of premium version adopters at price q, conditioned on the the degree distribution $H_s(n)$. Also, in consonance with earlier notation, let z_i denote consumer i's consumption of the product. By considering a setting where each consumer adopts one of the versions of the service (which is ensured by the assumption $\psi < \underline{\theta}$), we trivially obtain that the average consumption of some version of the service of a consumer's neighbor equals one.[18] Therefore, the number n_i specifies the average consumption of the service of consumer i's neighbors, conditioned on having degree n_i. In addition to these considerations, the preference specification in (1) makes use of the *degree independence*

[16]In practice, these externalities often take the form of informational gains (e.g., weather forecast, traffic monitoring, news services, or review and rating services), collaborative gains (e.g., joint entrepreneurial projects or online gaming), gains from being able to interact with a higher number of people (e.g., exercise tracking, or dating services), or gains from facilitating transactions (e.g. second hand trade).

[17]In some applications, one might consider $\underline{n} = 0$. Also, the set $[\underline{n}, \overline{n}]$ could be unbounded as well and, in particular, $\overline{n}$ could tend to infinity in some applications.

[18]Formally, under the assumptions of the *configuration model*, the expected consumption of some version of the service of a consumer i's neighbor can be computed as

$$\frac{\int_{\underline{n}}^{\overline{n}} m h_s(m) \big[\mathbb{P}(z_i = 1 \,|\, n_i = m) + \mathbb{P}(z_i = 0 \,|\, n_i = m)\big] dm}{\int_{\underline{n}}^{\overline{n}} n h_s(n) dn} = 1$$

because, given the assumption $\psi < \underline{\theta}$, we trivially have $\mathbb{P}(z_i = 1 \,|\, n_i = m) + \mathbb{P}(z_i = 0 \,|\, n_i = m) = 1$ for each degree $m \in [\overline{n}, \underline{n}]$.

assumption in order to guarantee that the only relevant information about the network for each consumer is his degree.[19] The expected utility of a consumer i, conditioned on having degree n_i, is then specified as

$$u(z_i, x_i \,|\, n_i) = \underbrace{z_i(\omega - p)}_{\text{product}} \;+\; \underbrace{x_i\big[\theta - q + (1+\beta)n_i\big]}_{\text{premium version of the service}} \;+\; \underbrace{(1-x_i)\big[\theta - \psi + n_i\big]}_{\text{free version of the service}}. \quad (1)$$

The expression in (1) captures the presence of local (positive) network externalities. A consumer's utility raises by an amount of 1, when he adopts the free version of the service, or by an amount $1+\beta$, when he adopts the premium version, for each unit of the service consumed by his neighbors, regardless of the version that the neighbors adopt. I assume that $\beta > 0$, which can be viewed formally as a single crossing condition for the two types of service adopters. Thus, the term β describes the presence of an *externality premium*. This seems to be the consistent with most real-world online services where the premium version allows consumers to enjoy the network externalities to a greater extent compared to the free version.

From the preference specification in (1), it follows that the fraction of consumers that purchase the premium version of the service at price q, conditional on the degree distribution $H_s(n)$, is given by

$$\begin{aligned} x_s(q) &= \mathbb{P}(x_i = 1 \,|\, s) = \mathbb{P}\big(\theta - q + (1+\beta)n_i \geq \theta - \psi + n_i \,|\, s\big) \\ &= \mathbb{P}\left(n_i \geq \frac{q-\psi}{\beta} \,\Big|\, s\right) = 1 - H_s\left(\frac{q-\psi}{\beta}\right). \end{aligned} \quad (2)$$

For price q, let $n(q) = \frac{q-\psi}{\beta}$ be the cutoff degree such that $x_i = 0$ if $n_i \leq n(q)$ whereas $x_i = 1$ if $n_i > n(q)$. From the expression in (2) above, we observe that the fraction of consumers that purchase the premium version naturally decreases in its price q. The sensitivity of the demand $x_s(q)$ of the premium version with respect to its price depends on the random process that generates the network and on the externality premium. In particular, we have $x_s'(q) = -\left(\frac{1}{\beta}\right) h_s\big(n(q)\big)$. Finally, note that all consumers purchase the free version of the service (i.e., $x_s(q) = 0$) when its price satisfies $q \geq \overline{q}$, where the upper

[19]The *degree independence assumption* states that the nodes of the network regard their shared links as independently chosen from the random network. This is a very common assumption in the literature on random networks and has been used, among others, by Jackson & Yariv (2007), Galeotti et al. (2010), Fainmesser & Galeotti (2016), and Shin (2016).

bound $\overline{q}$ on the service price is given by $\overline{q} = \beta\overline{n} + \psi$. Thus, in addition to the natural requirement that $q \in (\underline{\theta}, \overline{\theta})$, we need to consider that $q \leq \overline{q}$ throughout the analysis.

Hazard rate analysis, which has traditionally been used to account for "survival rates" in epidemiology and finance, is useful here to capture key features of how the random network evolves in dynamic interpretations of the model. Besides, in the current benchmark, the main insights on the platform's optimal pricing will be related to the hazard rate of the degree distribution. The *hazard rate function* of the random network with distribution degree $H_s(n)$ is the function on $[\underline{n}, \overline{n}]$ defined as

$$r_s(n) = \frac{h_s(n)}{1 - H_s(n)}. \tag{3}$$

For dynamic interpretations where the network evolves along several periods, the function $r_s(n)$ gives us the probability that a randomly selected consumer has approximately n neighbors in a subsequent period,[20] conditioned on his current neighborhood size being no less than n.

Advertising the Product

Leaving aside strategic or dynamic issues, the literature on advertising has traditionally considered three types of ads: *persuasive*, *complementary*, and *informative* ads (e.g., Bagwell (2007) or Renault (2015)). Persuasive and complementary ads change the consumer's preferences for the advertised product.[21] On the other hand, informative advertising influences the consumers' knowledge, or perceptions, of the product features. While the general effects of the three types of advertising are rather similar, in the sense that all of them ultimately affect the consumers' tastes for the product, they are conceptually different and might lead to slightly different implications. In particular, persuasive and complementary advertising always raise individual demands, which leads to an upwards shift of the inverse demand function,[22] whereas informative ads need not do so because

[20]Formally, for the continuous distribution case, $r_s(n)$ is the probability that the number of neighbors of a randomly selected consumer lies in the interval $(n - \varepsilon, n + \varepsilon)$, for $\varepsilon > 0$ sufficiently small.

[21]Specifically, advertising is persuasive when it raises the consumers' propensity to pay for the advertised product and it is complementary when the consumption of the product is complementary to that of the ads.

[22]In some of the pioneering models of the complementary advertising view, such as Stigler & Becker (1977) and Becker & Murphy (1993), the final effects on demand are ambiguous. However, this does

consumers might learn that their tastes are indeed not well suited to the product's characteristics.

This article considers that ads are informative and that they improve the consumers' knowledge of their own tastes for the product. More specifically, building on the micro foundations provided by Johnson & Myatt (2006), I assume that advertising rotates clockwise the distribution of the product valuations or, equivalently, the inverse demand of the product. At price p, conditional on receiving an advertising level $a \in \{0, 1\}$, a fraction $z_a(p) = \mathbb{P}(\omega \geq p \mid a) = 1 - F_a(p)$ of consumers purchase the product. Therefore, $P_a(z_a) = F_a^{-1}(1 - z_a)$ gives us the inverse demand function of the consumers that receive advertising level a. The following assumption describes how the consumers' valuations (or, equivalently, the inverse demand curve) are influenced by advertising in this model.

Assumption 1 (Rotation Effects of Advertising[23]). *There exists a single $\omega^R \in (\underline{\omega}, \overline{\omega})$ such that*

$$\omega \gtreqless \omega^R \Leftrightarrow F_0(\omega) \gtreqless F_1(\omega).$$

Equivalently, there exists a single $z^R \in (0, 1)$ such that

$$z \gtreqless z^R \Leftrightarrow P_0(z) \gtreqless P_1(z).$$

Under Assumption 1 above, advertising rotates clockwise the distribution of valuations (or, equivalently, the inverse demand function) in a way such that both functions cross exactly once.[24] In intuitive terms, this article assumes that advertising makes the distribution of valuations "more disperse" or "more heterogenous."

Example 1. *To grasp better the intuition of the proposed effect of rotation of demand, consider an application where the monopolist knows the true valuation of the product ω*

not follow from the consumers' preferences for the product but from the fact that these models usually incorporate exposure to advertising costs as well.

[23]The conditions described by Assumption 1 are totally analogous to those stated in Definition 1 of Johnson & Myatt (2006). Johnson & Myatt (2006), though, consider instead that the inverse demand of the product is parameterized by a continuous set of parameters and allow the crossing points to vary with the parameter. The conditions stated in Assumption 1 give us the analog for the case with two parameters where there is a fixed crossing point.

[24]The model's main implications would continue to follow qualitatively if there were more than a single rotation point, provided that we assumed further that each density $f_a(\omega)$ is unimodal.

but the consumers only know that ω is drawn from some distribution $Q(\omega)$, supported on $(\underline{\omega}, \overline{\omega})$, that determines their priors. Suppose that advertising provides consumers with some (noisy) signal y about their valuations ω. The signal yields either the true value of the valuation $y = \omega$, with probability $(1+a)/(2+a)$, or some independent drawn y from the distribution $Q(\omega)$, with probability $1/(2+a)$. Then, upon receiving advertising level $a \in \{0,1\}$ and observing signal realization y, the posterior expectation $E[\omega\,|y,a\,]$ of the valuation follows the simple Bayesian updating rule

$$E[\omega\,|y,a\,] = \left(\frac{1+a}{2+a}\right) y + \left(\frac{1}{2+a}\right) E_Q[\omega].$$

Thus, with no advertising, a consumer assigns probability $1/2$ to his valuation of the product being either the value stated by the signal $\omega = y$ or his prior expectation $E_Q[\omega]$, according to distribution $Q(\omega)$. In the presence of advertising, however, a consumer obtains additional information and turns to assign probability $2/3$ to the valuation $\omega = y$ and $1/3$ to his prior expectation $E_Q[\omega]$. Although advertising does not fully reveal the true realization of the valuation ω in this application, it allows consumers to put higher probability on such a realization. In this way, advertising helps consumers to learn about the true value of their own valuations of the product. Then, if the monopolist sells z units of the product at some common price, it will be purchased by the consumers receiving a signal value greater than $y = Q^{-1}(1-z)$. As a consequence, for an advertising level $a \in \{0,1\}$, the monopolist sets a price

$$P_a(z) = \left(\frac{1+a}{2+a}\right) Q^{-1}(1-z) + \left(\frac{1}{2+a}\right) E_Q[\omega].$$

Accordingly, the distribution of the product valuation, conditioned on an advertising level $a \in \{0,1\}$, is given by

$$F_a(\omega) = Q\left(\frac{(2+a)\omega - E_Q[\omega]}{1+a}\right).$$

Hence, we obtain an inverse demand function, and an associated distribution of valuations, that satisfy Assumption 1, where the rotation points are $z^R = 1 - Q\big(E_Q[\omega]\big)$ and $\omega^R = E_Q[\omega]$.[25]

[25] Considering that consumers are uncertain about their own valuations of the product and that advertising helps them to learn about such valuations, Johnson & Myatt (2006) (Subsection IIIB) follow the

The condition stated in Assumption 1 does not place strong restrictions on the role of advertising and allows for a very flexible class of implications. By improving the consumers' information about their valuations of the product, advertising may persuade some consumers to purchase the product while discouraging others. In particular both increases in the variance of the valuations and reductions in the price-elasticity of the product are particular cases captured by Assumption 1. The density function $f_a(\omega)$ under advertising ($a = 1$) reflects more dispersion of the product valuations than without advertising ($a = 0$). The relation between $F_0(\omega)$ and $F_1(\omega)$ imposed by Assumption 1 is a bit more general that *second-order stochastic dominance* as it does not require *mean-preserving*.[26]

Profit Functions

Let $\pi_a(\omega) = (\omega - c)[1 - F_a(\omega)]$ (or, equivalently, $\pi_a(p) = (p - c)z_a(p)$) denote the profits from the product sales for a level of advertising $a \in \{0, 1\}$. Given a degree distribution $H_s(n)$ and a price $q \in (\underline{\theta}, \overline{\theta})$ (with $q \leq \overline{q}$) of the service, the monopolist sets a price $p_s^* \in (\underline{\omega}, \overline{\omega})$ for the product so as to maximize the profits from the product sales

$$\pi\big(p, q\big) = x_s(q)\pi_0(p) + \big[1 - x_s(q)\big]\pi_1(p). \tag{4}$$

Price discrimination is not allowed for the advertised product so that all consumers face a common price p, regardless of the version of the service that they adopt. Because advertising affects (some of) the consumers' willingness to pay for the product, the profits from the product sales depend on the fraction $x_s(q)$ of premium version adopters. For a

insights of Lewis & Sappington (1994) on informative advertising to provide a totally analog application of their theory of demand rotations where informative advertising rotates clockwise the inverse demand of the advertised product. In their application, though, the parameter a is considered to lie in a bounded interval instead. In addition, as the price $P_a(z)$ that the platform sets for the product (for a given quantity $z \in [0, 1]$) and the advertising level $a \in [0, 1]$ are complements, the advertising structure considered in this application is closely related to the literature on supermodular games. The information acquisition problem explored by Amir & Lazzati (2016) in the context of (common value) Bayesian supermodular games provides the key result of convexity of the value of information. As in Amir & Lazzati (2016), one can regard the proposed signaling application as a structure that exploits supermodularity results for a problem with an endogenous informative choice.

[26] When both distributions $F_0(\omega)$ and $F_1(\omega)$ have the same mean, the relation described by Assumption 1 implies that $F_1(\omega)$ second-order dominates $F_0(\omega)$.

price of the product p, let

$$\Delta(p) \equiv \pi_1(p) - \pi_0(p)$$

be the difference in profits from the product sales when advertising is present with respect to the situation without advertising. The function $\Delta(p) : (\underline{\omega}, \overline{\omega}) \to \mathbb{R}$ can thus be interpreted as an indicator of the market conditions of the product which summarizes how advertising changes the profitability of the product sales at each price p.

Importantly, considering only two possible degrees of advertising is without loss of generality in the current benchmark. Johnson & Myatt (2006) consider that demand rotates clockwise for the case where the parameter a is drawn from a bounded interval and, under (the appropriate version of) Assumption 1, show that (the appropriate version of) the profit function in (4) is quasi-convex in a. It follows from their key insight (Proposition 1) that, if the monopolist were allowed in the current setting to choose an advertisement level $a \in [0, 1]$, then it would optimally choose either $a = 0$ or $a = 1$.

For tractability reasons, this article will restrict attention to the fairly general class of inverse demand functions such that the profit functions $\pi_a(p)$, for $a \in \{0, 1\}$, and $\pi\big(p, q\big)$, for each given price of the service $q \in (\underline{\theta}, \overline{\theta})$ (with $q \leq \overline{q}$) have a unique interior maximum. In addition, to make the problem of discrimination through "versioning" interesting we need to assume that advertising indeed allows the monopolist to achieve higher profits than in the absence of it.[27] The following assumption will be maintained throughout the article.

Assumption 2. *For the advertising levels $a \in \{0, 1\}$, given a price of the service $q \in (\underline{\theta}, \overline{\theta})$, with $q \leq \overline{q}$, the distributions $F_a(\omega)$ of consumer valuations (or, equivalently, the demand inverse functions $z_a(p)$) of the product are such that:*
(i) each profit function $\pi_a(\omega)$ has a unique interior maximum $\omega_a^* \in (\underline{\omega}, \overline{\omega})$ for $a \in \{0, 1\}$, the profit function $\pi\big(p, q\big)$ has a unique interior maximum $p_s^* \in (\underline{\omega}, \overline{\omega})$,[28] and
(ii) $\pi_1(\omega_1^*) > \pi_0(\omega_0^*)$.

[27] Otherwise, the problem would be trivial as the monopolist would prefer to provide no advertising and, thus, the platform would offer only the premium version of the service.

[28] Requirement (i) of Assumption 2 could be alternatively replaced by the stronger condition that each function $\pi_a(\omega)$, $a \in \{0, 1\}$, be strictly concave.

Given an optimal price $p_s^* \in (\underline{\omega}, \overline{\omega})$ of the product, the objective of the platform is then to choose a price $q_s^* \in (\underline{\theta}, \overline{\theta})$, with $q_s^* \leq \overline{q}$, for the service so as to maximize its profits

$$\begin{aligned} \Pi(p_s^*, q) &= q x_s(q) + \alpha \pi\big(p_s^*, q\big) \\ &= \big[q - \alpha \Delta(p_s^*)\big] x_s(q) + \alpha \pi_1(p_s^*). \end{aligned} \tag{5}$$

Formally, the optimal choices (p_s^*, q_s^*) correspond to a *subgame perfect Nash equilibrium* of the (perfect information) game where the monopolist chooses first the price of the product and then the platform selects a price for the service. Existence of equilibrium is guaranteed because the profits specified in (4) and (5) are continuous functions on compact convex sets. Importantly, using backwards induction, the optimal choices of both prices are determined by solving the platform's decision problem $\max_{(p,q)\in(\underline{\omega},\overline{\omega})\times(\underline{\theta},\overline{\theta})} \Pi(p,q)$ under the restriction that $q \leq \overline{q}$.[29] Using the expression in (5) for the platform's profits, we can write the value function of the platform's problem as

$$\Pi^*(s) \equiv \max_{(p,q)\in(\underline{\omega},\overline{\omega})\times(\underline{\theta},\overline{\theta})} \big\{ \big[q - \alpha\Delta(p)\big] x_s(q) + \alpha \pi_1(p) \ : \ q \leq \overline{q} \big\}\, . \tag{6}$$

This will be our key expression to explore the how the degree distribution of the social network affects the platform's optimal profits.

3 Optimal Pricing and Profits

A rise in the price q of the service has two effects on the platform's profits. On the one hand, higher prices naturally increase the revenue for each unit sold of (the premium version of) the service. On the other hand, as more consumers purchase the free version, the advertising activity of the consumption product increases. More advertising could lead to an increase in the product sales, which would be beneficial for the platform through the compensation that it receives from the product sales. In short, the platform must solve the dilemma of whether making its profits from the sales of the premium version of the service or from its advertising activity through the free version. The conflict between these two effects are captured by the term $q-\alpha\Delta(p)$ in the optimal profits specified in (6).

[29] As $\alpha > 0$, the monopolist and the platform have common interests with respect to the choice of the price p of the advertised product. Because of its second-mover advantage, though, by solving its decision problem, the platform determines the optimal profits of the monopolist.

The final implications on the platform's profits are, in principle, ambiguous. On the one hand, they depend on the shape of the inverse demand functions of the product ($z_a(p)$) and the production cost (c) (i.e., they depend on the market conditions of the product). Interestingly, in the current benchmark, the final effects on the platform's profits also rely substantially on the network that connects the consumers ($H_s(n)$), and on the externality premium that it enables (β).

The first result on optimal pricing deals with an interesting class of optimal corner choices. Proposition 1 establishes that the platform prefers to provide only the premium version of the service, rather than discriminate through "versioning," for relatively low prices of the product. A direct implication of Assumption 1 on the role of advertising is that the profit functions $\pi_0(p)$ and $\pi_1(p)$ cross exactly once, at the rotation valuation $p = \omega^R$. Assumption 1 directly implies that, for each given valuation ω,

$$(\omega - c)[1 - F_0(\omega)] \gtreqless (\omega - c)[1 - F_1(\omega)] \Leftrightarrow \omega \lesseqgtr \omega^R.$$

In other words, $\pi_0(p) > \pi_1(p)$ if and only if $p < \omega^R$. Therefore, $\Delta(p_s^*) < 0$ for optimal prices of the product $p_s^* \in (\underline{\omega}, \omega^R)$. Then, by using the expression in (5) for the platform's profits, it follows that the platform's optimal choice entails $x_s = 1$. Under the fairly general role of advertising imposed by Assumption 1, the profits from the product sales under advertising are lower than without advertising for relatively low prices of the product. In these cases, the platform prefers to offer only the premium version of the service. On the other hand, for optimal prices of the product $p_s^* \in (\omega^R, \overline{\omega})$, we know that $\Delta(p_s^*) > 0$. From the form of profits of the platform described by (5), we observe that providing both versions of the service is optimal only if $p_s^* > \omega^R$. The result in Proposition 1 follows directly from the previous arguments.

Proposition 1. *Consider a random social network with degree distribution $H_s(n)$. Under assumptions 1 and 2, the platform optimally chooses to provide only the premium version of the service for each price of the product $p_s^* \in (\underline{\omega}, \omega^R)$.*

Thus, the platform may find optimal to pursue a discrimination policy through "versioning" only for prices of the product higher than ω^R. Notably, this result only requires that advertising rotates inverse demand around some price higher than the lowest valuation $\underline{\omega}$ (Assumption 1). In other words, offering only the premium version is the platform optimal choice for relatively low prices of the advertised product if one simply considers that advertising raises the dispersion of the product's valuations. Figure 4 displays a situation where the proportion of premium version adopters x_s is not optimal for the platform to pursue "versioning." Here the platform best-replies by offering only the premium version. On the other hand, Figure 5 displays a situation where the platform prefers to pursue "versioning" for the proportion x_s of premium version adopters.

The model's implications on discrimination through "versioning" follow when attention is restricted to interior optimal choices. By combining the assumptions that the cost from exposure to advertising is relatively small, $\psi < \underline{\theta}$, and that each density $h_s(n)$ is strictly positive on the support $[\underline{n}, \overline{n}]$, we know that interior optimal prices of the service $q_s^* \in \left(\underline{\theta}, \min\left\{\overline{\theta}, \overline{q}\right\}\right)$ lead to $x_s(q_s^*) \in (0,1)$. In other words, the platform is in fact constrained to implement discrimination through "versioning" when we restrict attention to interior optimal prices. Lemma 1 gives us the necessary and sufficient conditions for interior optimal prices. Moreover, Lemma 1 provides the key condition that ensures that the platform optimally pursues "versioning" rather than serving only the free version of the service.

Despite the sufficient condition provided by Proposition 1, the structure of the model is not appropriate to compare in general situations where the platform pursues "versioning" with others where it chooses to offer only the premium version. The assumption of relatively small advertising exposure costs ($\psi < \underline{\theta}$) guarantees that all consumers at least prefer the free version rather than no purchasing any version of the service whatsoever. This enables us to compare the case where the platform serves only the free version and, accordingly, *all* consumers adopt the free version, with other situations where the platform offers both versions. However, the model's assumptions do not guarantee that *all* consumers would purchase the premium version if this were the only version offered in the market. The fraction of consumers that would purchase the service if only the premium version were served, $\tilde{x}_s(q)$, would depend only on the relation between the valuation θ and the service price q. In the current model, such a fraction of consumers $\tilde{x}_s(q)$ would be different from the fraction of premium version adopters $x_s(q)$ under "versioning." For this reason, the comparison between the associated profits to the platform would not be appropriate.

Lemma 1. *Assume 1 and 2. Consider a random social network with degree distribution $H_s(n)$ and suppose that $p_s^* \in (\omega^R, \overline{\omega})$ is an optimal price of the consumption product. Then, if the interior service price $q_s^* \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$ corresponds to an optimal choice by the platform, the following first-order condition must be satisfied:*

$$\underbrace{q_s^*}_{MR_{premium}} = \underbrace{\alpha\Delta(p_s^*)}_{MR_{product}} + \underbrace{\frac{\beta}{r_s\big(n(q_s^*)\big)}}_{MU_{premium}}. \tag{7}$$

If, in addition to condition (7), the interior service price $q_s^* \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$ satisfies the second-order condition

$$\big[H_s\big(n(q_s^*)\big) - 1\big] h_s'\big(n(q_s^*)\big) \leq 2\big[h_s\big(n(q_s^*)\big)\big]^2, \tag{8}$$

then it corresponds to an optimal choice by the platform. Moreover, the platform optimally chooses an interior optimal price q_s^* rather than serving only the free version of the service if the condition

$$\alpha\big[\pi_1(\omega_1^*) - \pi_1(p_s^*)\big] < \frac{\beta\big[1 - H_s\big(n(q_s^*)\big)\big]}{r_s\big(n(q_s^*)\big)} \tag{9}$$

is satisfied.

As the difference of profits $\pi_1(\omega_1^*) - \pi_1(p_s^*)$ is bounded, we observe from the requirement in (9) that, under the conditions specified in (7) and (8), the platform prefers to pursue "versioning" rather than serving only the free version of the service for sufficiently high values of the externality premium β. This is intuitive because higher values of the externality premium make more attractive the premium version.

As mentioned earlier, the results on the quasi-convexity of the profits from the product sales provided by Johnson & Myatt (2006) guarantee that considering only a binary choice $a \in \{0, 1\}$ for advertising suffices to capture a very general class of problems with informative advertising. Given this binary specification, condition (7) of Lemma 1 provides an intuitive interpretation of the platform's incentives to choose its optimal (interior) prices for the service. Condition (7) states that the marginal revenue from the premium version sales, q_s^*, must be equal to the revenue increase in the product sales derived by switching from no advertising to advertising, $\alpha\Delta(p_s^*)$, plus the term $\beta/r_s\big(n(q_s^*)\big)$. Here, $\beta/r_s\big(n(q_s^*)\big)$ describes the marginal utility that accrues to consumers from the externality gain when they switch from the free to the premium version, normalized by the conditional probability that the cutoff degree $n(q_s^*)$ in the social network is maintained.[30] In short, the platform optimal's price of the service q_s^* depends additively on the change in the profits induced by advertising and on the network features which, in the current model, are summarized by the externality premium β and the conditional probability $r_s(n)$ that the degree of a randomly chosen consumer increases as the network grows.

Lemma 1 provides a straight-forward proof of the key result of Proposition 2, which establishes that *first order stochastic dominance* over the degree distribution of the social network follows if and only if the dominating random network yields higher optimal profits (provided that attention is restricted to interior optimal prices).[31] The following definition states the classical notion of *first-order stochastic dominance* in terms of local changes in the distributions.

Definition 1. *The family of degree distributions* $\{H_s(n)\}$ *is* ordered by first-order stochastically dominance (FOSD–ordered) *if, for each given* $n \in [\underline{n}, \overline{n}]$*, we have* $\partial H_s(n)/\partial s < 0$*.*

The proof of Proposition 2 follows from two basic observations. First, recall from (2) that, for a given price $q \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$, the fraction of premium version adopters is $x_s(q) = 1 - H_s\big(n(q)\big)$ so that $\partial x_s(q)/\partial s = -\partial H_s\big(n(q)\big)/\partial s$. It then follows that $\partial x_s(q)/\partial s > 0$ if and only if the family of degree distributions $\{H_s\}$ is FOSD–ordered. Secondly, given the previous observation, we note from the expression for the value function of the platform's problem in (6) that $\partial \Pi^*(s)/\partial s > 0$ if and only if $q - \alpha\Delta(p) > 0$.

[30] This expression for the increase in the consumers' utility depends only on the features of the underlying random network. By using the first-order condition to the optimization problem of the monopolist in (4), the local change in utility $\beta/r_s\big(n(q_s^*)\big)$ can be alternatively written as

$$\frac{\beta}{\left[1 - \frac{\pi_0'(p_s^*)}{\pi_1'(p_s^*)}\right] h_s\big(n(q_s^*)\big)}.$$

This expression makes use instead of the unconditional probability of having a degree $n(q_s^*)$ and of some additional features of the product market, which are summarized by the slopes of the profit functions from the product sales at the optimal price.

[31] The analysis in Gonzalez-Guerra & Jimenez-Martinez (2016) provides a sufficient condition for higher optimal profits in terms of the hazard dominance criterion, which is stronger than first order dominance.

Therefore, from Lemma 1, we observe that the inequality above is guaranteed when we restrict attention to interior optimal prices (p_s^*, q_s^*) because $\beta/r_s\big(n(q_s^*)\big) > 0$.

Proposition 2. *Suppose that the family of degree distributions $\{H_s(n)\}$ gives rise to interior optimal prices $p_s^* \in (\omega^R, \overline{\omega})$ and $q_s^* \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$. Then, the platform's optimal profits $\Pi^*(s)$ are strictly increasing in $s \in [\underline{s}, \overline{s}]$ if and only if $\{H_s(n)\}$ is FOSD–ordered.*

Consider two degree distributions $H_s(n)$ and $H_{s'}(n)$ such that $H_s(n)$ *FOSD* $H_{s'}(n)$. It follows that $H_s(n) \leq H_{s'}(n)$ for each $n \in [\underline{n}, \overline{n}]$. The intuitive message for a dynamic interpretation of random networks, where the average neighborhood size increases over time, is that the probability $\mathbb{P}(n > m \mid s)$ is higher than $\mathbb{P}(n > m \mid s')$. When m raises as the network grows over time, consumers are more likely to raise the number of their future neighbors under $H_s(n)$ than under $H_{s'}(n)$. Then, given the externality premium β, consumers value in average the premium version of the service relatively more under $H_s(n)$ than under $H_{s'}(n)$. This raises the platform's revenue from the service sales without affecting any of the fundamentals of the revenue from the advertising activity. In short, the degree distribution $H_s(n)$ gives the platform more flexibility in its discrimination problem, which allows for higher optimal profits, relative to the optimal profits achievable under distribution $H_{s'}(n)$.

Application of the *implicit function theorem* to the first-order condition in (7) of Lemma 1 allows us to study the relation between local changes in the optimal interior prices of the product and the service. Suppose that, starting from some interior optimal pair (p_s^*, q_s^*), the price of the service increases locally. Then, Proposition 3 can be used to obtain testable implications as to whether the monopolist will best-reply by either increasing or decreasing the price of the product.

Proposition 3. *Consider a random social network with degree distribution $H_s(n)$ and suppose that $p_s^* \in (\omega^R, \overline{\omega})$ and $q_s^* \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$ are interior optimal prices of the consumption product and the service. Suppose that the optimal price of the service q_s^* increases locally. Under Assumptions 1 and 2, it follows that*

$$\frac{\partial p_s^*}{\partial q_s^*} = \frac{1 + \frac{r_s'(n(q_s^*))}{[r_s(n(q_s^*))]^2}}{\alpha\big[\pi_1'(p_s^*) - \pi_0'(p_s^*)\big]}. \tag{10}$$

Moreover, if q_s^* corresponds to an interior optimal choice, then it must be the case that $\pi_0'(p_s^*) < 0$ and, in addition, either (a) $\pi_1'(p_s^*) > 0$, or (b) $\pi_1'(p_s^*) < \pi_0'(p_s^*)$.

To obtain unambiguous implications from Proposition 3, we must explore the relation existing between the slopes $\pi_0'(p_s^*)$ and $\pi_1'(p_s^*)$, for each optimal price $p_s^* \in (\omega^R, \overline{\omega})$. Assumption 1, though, is too general so as to place such relevant restrictions on the slopes of the profit functions. Assumption 3 imposes further structure on the role of advertising by considering that advertising decreases the textbook price-elasticity of the product demand.[32] Let $\varepsilon_a(p) = -pz_a'(p)/z_a(p)$ be the price-elasticity of the product demand under advertising level $a \in \{0, 1\}$.

Assumption 3 (Decreasing Elasticity)**.** *For each $p \in (\underline{\omega}, \overline{\omega})$, advertising lowers the price-elasticity of the product demand, $\varepsilon_1(p) < \varepsilon_0(p)$.*

Using the definition of price-elasticity of the product demand and the expression of the profits from the sales of the product under advertisement level a, we can write

$$\pi_a'(p) = z_a(p)\left[1 - \left(1 - \frac{c}{p}\right)\varepsilon_a(p)\right]. \tag{11}$$

By Assumption 1, it follows that $z_0(p_s^*) < z_1(p_s^*)$ for each $p_s^* \in (\omega^R, \overline{\omega})$. Then, if we further impose Assumption 3, it follows from the expression in (11) that $\pi_0'(p_s^*) < \pi_1'(p_s^*)$. This relation gives us the case (a) of Proposition 3 if, in addition, we have $\pi_1'(p_s^*) > 0$. From the expression in (11), we obtain that the slope of the profit function $\pi_1(p)$ is positive at price p_s^* if and only if $\varepsilon_1(p_s^*) < p_s^*/(p_s^* - c)$. These arguments allow us to obtain the following corollary to Proposition 3.

Corollary 1. *Consider a random social network with degree distribution $H_s(n)$ and suppose that $p_s^* \in (\omega^R, \overline{\omega})$ and $q_s^* \in \left(\underline{\theta}, \min\left\{\overline{\theta}, \overline{q}\right\}\right)$ are interior optimal prices of the consumption product and the service such that $\varepsilon_1(p_s^*) < p_s^*/(p_s^* - c)$. Suppose that the optimal price of the service q_s^* increases locally. Under Assumptions 2 and 3, it follows that*

$$\frac{\partial p_s^*}{\partial q_s^*} \gtreqless 0 \Leftrightarrow r_s'\big(n(q_s^*)\big) \gtreqless -\left[r_s\big(n(q_s^*)\big)\right]^2.$$

[32] As shown by Johnson & Myatt (2006), Assumption 3 gives us a stronger condition that implies the rotation effect described by Assumption 1.

When advertising lowers the price-elasticity of the product demand and such price-elasticity is sufficiently low under advertising, optimal prices move in opposite directions only if the hazard rate of the degree distribution is decreasing around the cutoff degree $n(q_s^*)$ at the optimal price. In addition, the corresponding (negative) slope of the hazard rate must be sufficiently high (in absolute value).

The hazard rate $r_s(n)$ of the degree distribution provides an interesting measure when we interpret the social network as a collection of consumers' degrees that evolve dynamically according to some stochastic law. Suppose that, up to a certain period t, there are no neighborhoods in the social network of size less than some minimum level $\widehat{n}$. Then, $r_s(\widehat{n})$ can be interpreted as the odds that, in a subsequent period $t+1$, consumers have a number of links approximately equal to such a former minimum value $\widehat{n}$. Using this dynamic interpretation, an increasing hazard rate indicates that if the average neighborhood size increases over time, then it becomes very likely to have a reversal in this growing tendency because at $t+1$ the consumers tend to have a number of neighbors around the minimum value $\widehat{n}$, which was already achieved in the previous period t. Decreasing hazard rates imply higher probabilities that consumers have degrees in the future that diverge from the current minimum degree, thus making it more likely that the network maintains its minimum neighborhood size as it grows.

Those dynamic interpretations have recently received quite appealing formal microeconomic foundations by Shin (2016). Under the assumptions of the canonical *configuration model* to formalize how the random graph is generated,[33] Shin (2016) (Proposition 2) shows that increasing hazard rates follow if and only if a node is less likely to form additional new links as his degree increases. On the other hand, with decreasing hazard rates, the degree of a randomly chosen node becomes arbitrarily large as the average

[33]The *configuration model* was originally developed by Bender & Canfield (1978) and, since then, it has been extensively used by a number models, such as Bollobás (2001), Newman et al. (2001), Jackson & Yariv (2007), Fainmesser & Galeotti (2016), and Shin (2016), among many others. A nice discussion of the configuration model, and of its relation with other random graph models, is provided by Jackson (2008). The idea behind the configuration model is that each consumer i with degree n_i gets randomly linked to a set of size n_i of other consumers according to a weighted uniform distribution where the weights are determined by the corresponding degrees n_j of the consumers in the remaining sample. As already mentioned, the preference specification in (1) encompasses the assumptions of the configuration model because, under the assumption that $\psi < \underline{\theta}$, all consumers purchase some version of the service. In other words, the expected proportion of consumers that purchase some version of the service equals one, as captured by (1), when the configuration model is formally used in our setting.

neighborhood size increases (Shin (2016), Proposition 3). Thus, by combining the result in Corollary 1 with the insights provided by Shin (2016) (Proposition 2), we obtain the implication that if the network evolves in a way such that a node is less likely to form additional new links as his degree increases, then necessarily optimal prices move in the same direction. On the other hand, the random network must evolve in a way such that a node becomes very likely to form new links as his degree increases for optimal prices to move in opposite directions.

The final part of this section explores a class of optimal corner choices where the platform prefers to offer only the free version of the service. Proposition 4 provides a sufficient condition under which the platform prefers to offer only the free version of the service.

Proposition 4. *Consider a random social network with degree distribution $H_s(n)$. Under Assumptions 1 and 2, the platform finds optimal to provide only the free version of the service if the following condition holds*

$$\alpha\big[\pi_1(\omega_1^*) - \pi_1(\omega^R)\big] \geq \frac{\beta[1 - H_s(n(q_s^{\mathrm{pm}}))]}{r_s(n(q_s^{\mathrm{pm}}))},$$

where q_s^{pm} is the (interior) optimal price that the platform sets for the service when it provides only the premium version.

The sufficient condition provided by Proposition 4 unveils a rather counterintuitive mechanism that may lead the platform to ultimately provide only the free version of the service. As we can observe from the expression of the platform's profits given by (5), the platform benefits from relying on the free version of the service when the market features of the product makes the advertising activity more profitable relative to the sales of the premium version. The mechanism suggested by Proposition 4, though, is quite different. Suppose, on the contrary, that the network structure and the externality premium lead to that the profits from the premium version sales always increase in its price. This would induce the platform to optimally rise the price of the premium version until all consumers choose to adopt the free version. If we let $\Omega_s(n) = [1 - H_s(n)]/r_s(n)$, then it follows that $r_s'(n) > 0$ ensures $\Omega_s'(n) < 0$. Notice that the right-hand side of the inequality provided by Proposition 4 can be written as $\beta\Omega\big(n(q_s^{\mathrm{pm}})\big)$, where the cutoff degree $n(q)$ increases with q.

Therefore, for degree distributions with increasing hazard rate functions, we observe that the right-hand side of the inequality in the sufficient condition provided by Proposition 4 decreases as the optimal service price (for the case where only the premium service being served) increases. In other words, higher prices for the service make it easer for the platform to ultimately offer only the free version when the degree distribution displays an increasing hazard rate. The intuition here is that rising the price of the service drives a large proportion of consumers to adopt the free version, thus making it profitable for the platform to rely on the revenue from its advertising activity. Nonetheless, having an increasing hazard rate works only as a sufficient requirement in this argument. As the key applications of Section 5 show, the implication that higher prices of the service can facilitate that the platform ultimately offers only the free version can also follow under degree distributions with decreasing or constant hazard rates.

4 Consumer Surplus

This section explores the effects of optimal pricing on consumer surplus. In the current setting, *consumer surplus at prices* (p, q) is given by the expression

$$CS(p,q) = \int_{i\in[0,1]} \int_{n_i \in [\underline{n}, \overline{n}]} u(z_i, x_i \,|\, n_i) dn_i \, di.$$

The following lemma derives a useful expression for the consumer surplus.

Lemma 2. *Under the preference specification in (1), consumer surplus at prices* (p, q) *can be written as*

$$CS(p,q) = 1 + z_1(p) + x_s(q)\big[1 + z_0(p) - z_1(p)\big]. \tag{12}$$

An immediate insight follows from the form of the consumer surplus obtained in (12). For a given pair of prices $p \in (\omega^R, \overline{\omega})$ and $q \in \big(\underline{\theta}, \min\big\{\overline{\theta}, \overline{q}\big\}\big)$, it follows from (12) that

$$\frac{\partial CS(p,q)}{\partial s} = -\frac{\partial x_s(q)}{\partial s}\big[1 + z_0(p) - z_1(p)\big].$$

In the expression above, we have $\big[1+z_0(p)-z_1(p)\big] > 0$ for each $p \in (\omega^R, \overline{\omega})$ as $z_1(p) \in [0, 1]$ by construction. On the other hand, recall from (2) that the fraction of premium version adopters is $x_s(q) = 1 - H_s\big(n(q)\big)$ so that $\partial x_s(q)/\partial s = -\partial H_s\big(n(q)\big)/\partial s$. It then follows that $\partial x_s(q)/\partial s > 0$ if and only if the family of degree distributions $\{H_s(n)\}$ is FOSD–ordered. These implications remain valid for any interior optimal prices that satisfy the first-order condition (7) obtained in Lemma 1. The previous arguments provide a proof of the intuitive result in Proposition 5. In the current benchmark, FOSD always decreases consumer surplus when we restrict attention to interior optimal prices.

Proposition 5. *Suppose that the family of degree distributions* $\{H_s(n)\}$ *gives rise to interior optimal prices* $p_s^* \in (\omega^R, \overline{\omega})$ *and* $q_s^* \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$. *Then, consumer surplus* $CS(p_s^*, q_s^*)$ *is strictly decreasing in* $s \in [\underline{s}, \overline{s}]$ *if and only if* $\{H_s(n)\}$ *is FOSD–ordered.*

The result provided by Proposition 5 gives us a welfare implication that naturally complements the one obtained in Proposition 2. First-order stochastic dominance of the degree distribution that generates the random network leads to higher optimal profits for the platform and to lower consumer surplus values.

Proposition 1 established that relatively low optimal prices of the product induce the platform to offer only the premium version whereas it may be profitable to pursue "versioning" for higher optimal prices of the product. Proposition 6 shows that consumer surplus is always lower when the platform pursues "versioning" with advertising compared to the cases where it optimally serves only the premium version of the service. In a nutshell, when the conditions of the advertised product market makes it profitable for the platform to rely (at least to some extent) on advertising, consumers are worse off.

Proposition 6. *Consider two random social networks with degree distributions* $H_s(n)$ *and* $H_{s'}(n)$ *that induce, respectively, optimal prices* p_s^* *and* $p_{s'}^*$ *of the product. Assume Assumptions 1 and 2. Suppose that* $p_s^* < \omega^R$ *so that the platform optimally chooses to serve only the premium version of the service and that* $p_{s'}^* > \omega^R$ *is a price of the product such that the platform optimally chooses to serve both versions of the service,* $x_{s'}(q_{s'}^*) \in (0, 1)$, *for an optimal price of the service* $q_{s'}^*$. *Then, consumer surplus when the platform offers only the premium version exceeds consumer surplus under "versioning," that is,* $CS(p_s^*) > CS(p_{s'}^*, q_{s'}^*)$.

The comparison between different levels of consumer surplus for the cases of "versioning" and serving only the free version of the service is more ambiguous. "Versioning" with advertising allows higher consumer surplus than serving only the free version either if it entails a lower price for product or if the proportion of premium version adopters under "versioning" is sufficiently low.

Proposition 7. *Consider two random social networks with degree distributions $H_s(n)$ and $H_{s'}(n)$ that induce, respectively, optimal prices $p_s^* > \omega^R$ and $p_{s'}^* > \omega^R$ of the product. Assume Assumptions 1 and 2. Suppose that p_s^* induces the platform to optimally serve only the free version of the service and that $p_{s'}^*$ induces the platform to optimally serve both versions of the service, $x_{s'}(q_{s'}^*) \in (0,1)$, for an optimal price of the service $q_{s'}^*$. Then, consumer surplus when the platform offers both versions of the services exceeds consumer surplus when it serves only the free version of the service, $CS(p_{s'}^*, q_{s'}^*) > CS(p_s^*)$, either if (a) $p_{s'}^* < p_s^*$, or (b) $p_{s'}^* > p_s^*$ and, at the same time, the proportion of premium version adopters satisfies*

$$x_{s'}(q_{s'}^*) < \frac{z_1(p_s^*) - z_1(p_{s'}^*)}{1 + z_0(p_{s'}^*) - z_1(p_{s'}^*)}.$$

Both conditions (a) and (b) in Proposition 7 are rather intuitive. "Versioning" allows for higher consumer surplus if either it leads to lower prices of the product or if it is associated to a relatively high proportion of free version adopters.

The final part of this section explores how changes in optimal prices affect consumer welfare. Suppose that, for a random social network with degree distribution $H_s(n)$, there exists a unique pair (p_s^*, q_s^*) of interior optimal prices. Then, let $CS(s) = CS(p_s^*, q_s^*)$ denote the consumer surplus associated with the degree distribution $H_s(n)$ at such optimal prices. Suppose now that, starting from the pair (p_s^*, q_s^*) of interior optimal prices, the price of the service q_s^* increases locally. Then, using the expression for the consumer surplus in (12), we obtain

$$\begin{aligned} \frac{\partial CS(s)}{\partial q_s^*} = \left(\frac{\partial p_s^*}{\partial q_s^*}\right) \Big[x_s(q_s^*) z_0'(p_s^*) + [1 - x_s(q_s^*)] z_1'(p_s^*) \Big] \\ + x_s'(q_s^*) \big[1 + z_0(p_s^*) - z_1(p_s^*) \big]. \end{aligned} \tag{13}$$

As $z_a'(p_s^*) < 0$ for each $a \in \{0,1\}$, $x_s'(q_s^*) < 0$, and $1 + z_0(p_s^*) - z_1(p_s^*) > 0$, we observe from the expression in (13) that an increase in the price of the service may increase consumer surplus only if it induces a sufficiently high decrease $\partial p_s^*/\partial q_s^* < 0$ in the price of the product. Intuitively, consumer welfare could only increase upon a rise in the price of the service if a higher proportion of consumers that receive ads leads the monopolist to optimally decrease the price of the product. By using the insight provided by expression (10) in Proposition 3, Proposition 8 gives us the required condition on the induced change $\partial p_s^*/\partial q_s^*$ under which a rise in the price of the service is welfare improving for consumers.

Proposition 8. *Consider a random social network with degree distribution $H_s(n)$ and suppose that there exists a unique pair (p_s^*, q_s^*) of interior optimal prices. Under Assumptions 1 and 2, a local increase in the price q_s^* of the service increases consumer surplus $CS(s)$ if and only if the condition*

$$\frac{\frac{\beta}{r_s(n(q_s^*))}\left[1+\frac{r_s'(n(q_s^*))}{[r_s(n(q_s^*))]^2}\right]}{\alpha\left[\pi_0'(p_s^*)-\pi_1'(p_s^*)\right]} > \xi(p_s^*, q_s^*)$$

is satisfied, where

$$\xi(p_s^*, q_s^*) = \frac{\left[(p_s^*-c)+\pi_0(p_s^*)-\pi_1(p_s^*)\right]}{z_0(p_s^*)+[1/x_s(q_s^*)-1]z_1(p_s^*)} > 0$$

for each pair (p_s^*, q_s^*) of interior optimal prices.

Obviously, when an increase in the price of the service induces a rise in the price of the product, both changes in prices have a negative impact on consumer welfare. An increase in the price of the service could be welfare improving for consumers only if it triggers a reduction in the price of the product. Furthermore, the size of such a price reduction must be sufficiently high. Proposition 8 gives us the requirement on the magnitude that the induced reduction in the price of the product must satisfy. When advertising reduces the price-elasticity of the product demand (Assumption 3), and the induced price-elasticity is sufficiently low under advertising, a rise of the service price can induce a welfare improvement only if the hazard rate is (at least locally) decreasing and, in addition, it satisfies $|r_s'(n(q_s^*))| > \left[r_s(n(q_s^*))\right]^2$.

5 Applications

This section presents two applications of the model to degree distributions which typically match well the data available on many complex social networks. While there is no conclusive evidence as to which degree distributions describe best specific real-world networks, many empirical studies suggest that most complex social networks are fairly well captured by either scale-free/power law patterns or exponential degree distributions (see, e.g., Barabási & Albert (1999), Clauset et al. (2009), Stephen & Toubia (2009) or Ugander et al. (2011)). In both applications provided in this section, the sufficient condition provided by Proposition 4 leads to that rises in the price of the service (for the case where only the premium version is served) facilitates that the platform ultimately finds optimal to provide only the free version of the service.[34]

Power Law Degree Distribution

Empirical evidence suggests that most real-world online and Internet-based social networks are *scale-free*. Therefore, if we consider that such networks are randomly generated, then the corresponding degree distribution must follow a *power law*. Also, power law degree distributions are particularly suitable to model the formation of networks that follow a preferential attachment pattern. For a degree support $[\underline{n}, +\infty)$, with $\underline{n} > 0$ a *power law degree distribution with parameter* $\sigma > 1$, is given by

$$H_s(n) = 1 - \underline{n}^{\sigma-1} n^{-(\sigma-1)}.$$

Most empirical estimates propose values for the parameter σ that lie in the interval $(2, 3)$. The corresponding hazard rate function is $r_s(n) = (\sigma - 1)/n$, which decreases in n. Following the insights provided by Shin (2016) (Proposition 3), the degree of a randomly chosen consumer becomes arbitrarily large as the average degree of the network increases. The result provided by Lemma 1 leads to that interior optimal prices q_s^* of the service

[34] Recently, an increasing number of platforms (such as Twitter, YouTube, Facebook, or Google, just to mention a few of the most prominent ones) had adhered to this strategy. Such platforms have often reported on the media to regard potential expansions of the social network as beneficial for their businesses because they allow them to increase considerably its audience for ads.

must satisfy the condition

$$q_s^* = \left(\frac{\sigma-1}{\sigma-2}\right)\alpha\Delta(p_s^*) - \frac{\psi}{\sigma-2}.$$

As for the second-order condition stated in (8) of Lemma 1, we have

$$h_s(n) = (\sigma-1)\underline{n}^{\sigma-1}n^{-\sigma} \quad \text{and} \quad h_s'(n) = -\sigma(\sigma-1)\underline{n}^{\sigma-1}n^{-(\sigma+1)}.$$

Therefore, we obtain that

$$\begin{aligned}\big[H_s\big(n(q_s^*)\big)-1\big]h_s'\big(n(q_s^*)\big) &= \Big[-\underline{n}^{\sigma-1}n^{-(\sigma-1)}\Big]\Big[-\sigma(\sigma-1)\underline{n}^{\sigma-1}n^{-(\sigma+1)}\Big] \\ &= \sigma(\sigma-1)\underline{n}^{2(\sigma-1)}n^{-2\sigma}\end{aligned}$$

and

$$2\big[h_s\big(n(q_s^*)\big)\big]^2 = 2(\sigma-1)^2\underline{n}^{2(\sigma-1)}n^{-2\sigma},$$

so that the second-order requirement in (8) translates into $2(\sigma-1) > \sigma$ for the power law degree distribution. Such a requirement is satisfied for $\sigma > 2$.

Notably, the optimal price of the service does not depend on the size of the externality premium in the scale-free network pattern. For $\sigma > 2$, it follows that the optimal price of the service increases in the difference $\Delta(p_s^*)$ between the optimal profits (with and without advertising) from the product sales. As to the relation between the optimal prices of the product and the service, notice that

$$1 + \frac{r_s'(n)}{[r_s(n)]^2} = \frac{\sigma-2}{\sigma-1}.$$

Therefore, the result in Proposition 3 leads to that, for $\sigma > 2$, the optimal prices of both commodies move in the same direction ($\partial p_s^*/\partial q_s^* > 0$) if and only if $\pi_1'(p_s^*) > \pi_0'(p_s^*)$. If, in addition, we make use of the stronger Assumption 3 on the effect of advertising on the price-elasticity of the product, Corollary 1 indicates that $\partial p_s^*/\partial q_s^* > 0$ for $\sigma > 2$ because, for those parameter values, we have

$$r_s'(n) = -\frac{\sigma-1}{n^2} > -\frac{(\sigma-1)^2}{n^2} = -[r_s(n)]^2.$$

Thus, a rise in the price of the service always reduces consumer welfare for random networks that are generated by a power law degree distribution with parameter $\sigma > 2$.

We observe that the model's predictions on optimal pricing depend crucially on whether the parameter value σ exceeds or not 2.

Finally, by using the result of Proposition 4, it follows that the platform prefers to offer only the free version of the service if the sufficient condition

$$\alpha\big[\pi_1(\omega_1^*) - \pi_1(\omega^R)\big] \geq \frac{\beta^{\sigma-1}\underline{n}^{\sigma-1}}{(\sigma-1)(q_s^{\text{pm}} - \psi)^{\sigma-2}}$$

is satisfied. Therefore, as the right-hand side of the expression above is decreasing in q_s^{pm} for any parameter value $\sigma > 1$, higher optimal prices q_s^{pm} makes it easier for the sufficient condition provided by Proposition 4 to hold.

Exponential Degree Distribution

A typical degree distribution with constant hazard rate function is the *exponential degree distribution*. For a degree support $[0, +\infty)$, an *exponential degree distribution with parameter* $\sigma > 0$ is given by

$$H_s(n) = 1 - e^{-n/\sigma}.$$

The corresponding hazard rate function is $r_s(n) = 1/\sigma$. The result provided by Lemma 1 leads to that interior optimal prices q_s^* of the service satisfy the condition

$$q_s^* = \beta\sigma + \alpha\Delta(p_s^*).$$

As for the second-order condition stated in (8), notice that $h_s(n) = (1/\sigma)e^{-n/\sigma}$ and $h_s'(n) = -(1/\sigma^2)e^{-n/\sigma}$. Therefore, we obtain that

$$\big[H_s\big(n(q_s^*)\big) - 1\big]h_s'\big(n(q_s^*)\big) = -e^{-n/\sigma}\Big[-\frac{1}{\sigma^2}e^{-n/\sigma}\Big] = \frac{1}{\sigma^2}e^{-2n/\sigma}$$

and

$$2\big[h_s\big(n(q_s^*)\big)\big]^2 = \frac{2}{\sigma^2}e^{-2n/\sigma},$$

so that the second-order requirement in (8) is automatically satisfied.

We obtain that the optimal price of the service increases in the difference $\Delta(p_s^*)$ between the optimal profits (with and without advertising) from the product sales. In addition, the optimal price of the service increases with the size of the externality premium β. Under Assumption 3 on the effect of advertising on the price-elasticity of the product, Corollary 1 indicates that $\partial p_s^*/\partial q_s^* > 0$ for low values of the price-elasticity of $z_1(p)$ because

$$r_s'(n) = 0 > -1/\sigma^2 = -[r_s(n)]^2.$$

A rise in the price of the service always reduces consumer welfare for random networks that are generated by an exponential degree distribution.

Finally, by using the result of Proposition 4, it follows that the platform prefers to offer only the free version of the service if the sufficient condition

$$\alpha\big[\pi_1(\omega_1^*) - \pi_1(\omega^R)\big] \geq \sigma e^{\frac{\psi - q_s^{\text{pm}}}{\sigma\beta}}$$

is satisfied. Thus, as the right-hand size of the expression above decreases in q_s^{pm}, higher optimal prices q_s^{pm} facilitates that the sufficient condition provided by Proposition 4 holds.

6 Concluding Remarks

The growth of online and assorted technologies over social networks has made it possible for platforms to implement new discrimination strategies by using advertising. In this article, I have investigated a new taxonomy of second-degree discrimination over random networks that is implemented through two versions of a service, one of which is served with advertising about some other unrelated product. Under the assumption that advertising helps consumers to learn about their own valuations of the product so that it makes such valuations "more heterogenous" or "more disperse," I have related the optimal choice of the platform, and its welfare implications, to some features of the product market and to the degree distribution that generates the network. The platform pursues "versioning" only if the profits from advertising are sufficiently high relative to the case of no advertising, which translates into sufficiently high prices for the advertised product. When the difference in profits is not sufficiently high, the platform chooses to serve only the premium version of the service. Implementing "versioning" always lowers consumer surplus relative to situations where the platform finds optimal to serve only the premium version. When "versioning" is optimally chosen by the platform, first-order stochastic dominance of the degree distribution that generates the network always leads to higher optimal profits to the platform and to lower values of the consumer surplus. Also, whether the optimal prices of the advertised product and the service move or not in the same direction depends crucially on the shape of the hazard rate function of the degree distribution.

There are different directions in which the current model could be extended. First, it would be very interesting to explore the consequences of having several platforms competing in a common social network. Nevertheless, the results here provided continue to be compelling if, for most practical situations, we believe that there is indeed sufficient service differentiation and/or that different platforms do not actually serve a significant number of common links (i.e., there is a certain degree of network differentiation across platforms). Secondly, the framework here proposed assumes that the only factor that determines the size of the network externalities is the degree of the nodes. Yet, other important features, such as clustering or assortativity, could influence also the consumption externalities. For example, the externality could be larger when common neighbors are shared or when the links are formed between consumers with some similar characteristics. Finally, the analysis has ignored the fact that the links could be used to transmit information about the characteristics of the advertised product. This information transmission would interact (either in a complementary or substitutive way) with the information distributed through formal advertising. This allows for interesting interactions between the formal advertising served by the platform and the *world of mouth* advertising through the social network. In particular, for review and rating services, including this possibility into the analysis seems relevant.

While the proposed framework is fairly general, it seems applicable to study empirical regularities about how real-world platforms operate in their provision of online services through social networks. This model's implications could be useful for conducting empirical research and of interest for practitioners in the area.

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Appendix: Omitted Proofs

Proof of Lemma 1. Consider a given a degree distribution $H_s(n)$ and an optimal price for the consumption product $p_s^* \in (\omega^R, \overline{\omega})$. Using the profit specification in (5), we know that the platform's goal is to choose a price for the service $q_s^* \in (\underline{\theta}, \overline{\theta})$, with $q \leq \overline{q}$, so as to maximize its profits

$$\Pi(p_s^*, q) = \big[q - \alpha\Delta(p_s^*)\big] x_s(q) + \alpha\pi_1(p_s^*).$$

The first-order condition for interior optimal prices to this problem is

$$\frac{\partial \Pi(p_s^*, q_s^*)}{\partial q} = x_s(q_s^*) + \big[q_s^* - \alpha\Delta(p_s^*)\big] x_s'(q_s^*) = 0.$$

As the density $h_s(n)$ is strictly positive in the support $[\underline{n}, \overline{n}]$, interior optimal prices $q_s^* \in \big(\underline{\theta}, \min\big\{\overline{\theta}, \overline{q}\big\}\big)$ ensure that $x_s(q_s^*) \in (0,1)$. Therefore, by dividing the first-order condition above over $x_s(q_s^*)$, we obtain that if q_s^* is an interior optimal price then, necessarily,

$$\big[q_s^* - \alpha\Delta(p_s^*)\big] \left[-\frac{x_s'(q_s^*)}{x_s(q_s^*)}\right] = 1.$$

From the preference specification in (1), combined with the assumptions on the degree distribution and the cost from exposure to adverting is relatively low ($\psi < \underline{\theta}$), it follows that $x_s(q_s^*) = 1 - H_s\big(n(q_s^*)\big)$ and $x_s'(q_s^*) = -\left(\frac{1}{\beta}\right) h_s\big(n(q_s^*)\big)$. Furthermore, from the definition of hazard rate function of the degree distribution in (3), we obtain

$$-\frac{x_s'(q_s^*)}{x_s(q_s^*)} = \frac{\left(\frac{1}{\beta}\right) h_s\big(n(q_s^*)\big)}{1 - H_s\big(n(q_s^*)\big)} = \left(\frac{1}{\beta}\right) r_s\big(n(q_s^*)\big).$$

Then, an interior optimal price for the service q_s^* must satisfy the first-order condition

$$q_s^* = \alpha\Delta(p_s^*) + \frac{\beta}{r_s\big(n(q_s^*)\big)}.$$

As for the second-order condition for interior optimal prices to the platform's problem, we obtain

$$\frac{\partial^2 \Pi(p_s^*, q_s^*)}{\partial q^2} = 2x_s'(q_s^*) + \big[q_s^* - \alpha\Delta(p_s^*)\big] x_s''(q_s^*) \leq 0.$$

Using the first-order condition derived earlier, the condition above can be rewritten as

$$2x'_s(q_s^*) + \left[-\frac{x_s(q_s^*)}{x'_s(q_s^*)}\right] x''_s(q_s^*) \leq 0 \Leftrightarrow x_s(q_s^*)x''_s(q_s^*) \leq 2\big[x'_s(q_s^*)\big]^2.$$

Then, by using the equalities $x_s(q_s^*) = 1 - H_s\big(n(q_s^*)\big)$, $x'_s(q_s^*) = -\left(\frac{1}{\beta}\right) h_s\big(n(q_s^*)\big)$, and $x''_s(q_s^*) = -\left(\frac{1}{\beta}\right)^2 h'_s\big(n(q_s^*)\big)$, we obtain the expression

$$\big[H_s\big(n(q_s^*)\big) - 1\big]h'_s\big(n(q_s^*)\big) \leq 2\big[h_s\big(n(q_s^*)\big)\big]^2$$

for the required second-order condition.

Finally, to ensure that the platform optimally chooses an interior optimal price q_s^* rather than serving only the free version of the service, we need to impose the condition that the optimal profits to the platform from serving only the free version, $\alpha\pi_1(\omega_1^*)$, do not exceed the optimal profits from such an interior optimal price q_s^*. By combining the first-order condition for interior optimal prices with the expression for the platform's profits derived in (5), it follows that the optimal profits for an interior optimal price are given by

$$\Pi^*(s) = \frac{\beta\big[1 - H_s\big(n(q_s^*)\big)\big]}{r_s\big(n(q_s^*)\big)} + \alpha\pi_1(p_s^*).$$

Therefore, to guarantee that the corner choice where the platform serves only the free version does not yield higher profits than the interior choice q_s^*, we need to impose the condition

$$\alpha\big[\pi_1(\omega_1^*) - \pi_1(p_s^*)\big] < \frac{\beta\big[1 - H_s\big(n(q_s^*)\big)\big]}{r_s\big(n(q_s^*)\big)},$$

as stated. ∎

Proof of Proposition 3. Consider the function defined as

$$\Gamma\big(p_s(q_s), q_s\big) = q_s - \frac{\beta}{r_s\big(n(q_s)\big)} - \alpha\big[\pi_1(p_s) - \pi_0(p_s)\big],$$

so that $\Gamma\big(p_s^*(q_s^*), q_s^*\big) = 0$ gives us the first-order condition (7) that was obtained in Lemma 1. It follows that

$$\frac{\partial\Gamma}{\partial p_s} = -\alpha\big[\pi'_1(p_s) - \pi'_0(p_s)\big]$$

and

$$\frac{\partial \Gamma}{\partial q_s} = 1 + \frac{r_s'\big(n(q_s)\big)}{\left[r_s\left(n(q_s)\right)\right]^2}.$$

Then, by applying the *implicit function theorem*, we obtain

$$\frac{\partial p_s^*}{\partial q_s^*} = -\frac{\partial \Gamma\big(p_s^*(q_s^*), q_s^*\big)/\partial q_s}{\partial \Gamma\big(p_s^*(q_s^*), q_s^*\big)/\partial p_s} = \frac{1 + \frac{r_s'(n(q_s^*))}{[r_s(n(q_s^*))]^2}}{\alpha\big[\pi_1'(p_s^*) - \pi_0'(p_s^*)\big]}.$$

Moreover, for an interior optimal price for the service $q_s^* \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$, the first-order condition to the problem $\max_{p \in (\omega^R, \overline{\omega})} \pi(p, q_s^*)$, where $\pi(p, q)$ are the profits from the product sales specified in (4), leads to

$$\begin{aligned} \frac{\partial \pi(p_s^*, q_s^*)}{\partial p} =& x_s(q_s^*)\pi_0'(p_s^*) + \big[1 - x_s(q_s^*)\big]\pi_1'(p_s^*) = 0 \\ &\Rightarrow x_s(q_s^*) = \frac{\pi_1'(p_s^*)}{\pi_1'(p_s^*) - \pi_0'(p_s^*)}. \end{aligned} \tag{14}$$

As the density $h_s(n)$ is strictly positive in the support $[\underline{n}, \overline{n}]$, interior optimal prices $q_s^* \in \big(\underline{\theta}, \min\{\overline{\theta}, \overline{q}\}\big)$ imply that $x_s(q_s^*) \in (0, 1)$. Therefore, by using the condition in (14), which must be satisfied by the fraction of premium version adopters, we obtain that $x_s(q_s^*) < 1 \Rightarrow \pi_0'(p_s^*) < 0$. Then, the requirement $x_s(q_s^*) > 0$ leads to either (a) $\pi_1'(p_s^*) > 0$ or (b) $\pi_1'(p_s^*) < \pi_0'(p_s^*)$. ∎

Proof of Proposition 4. The stated sufficient condition follows by considering a price for the product, not necessarily optimal, which induces an upper bound on the platform's profits when it pursues discrimination through "versioning." As the platform finds optimal to offer the free version only if $\pi_1(p) \geq \pi_0(p)$, then, by picking $p = \omega^R$, it follows from the expression in (5) for the platform's profits that

$$\nu(q) = \text{supp}_{p \in (\omega^R, \overline{\omega})} \Pi(p, q) = q x_s(q) + \alpha \pi_1(\omega^R)$$

gives us the supremum of $\Pi(p, q)$ when only the price of the product is allowed to vary, under the restriction that the platform benefits from providing the free version of the service. Then, the argument proceeds by comparing the optimal profits to the platform when it provides only the free version of the service, $\alpha \pi_1(\omega_1^*)$, with the above derived upper bound on the profits from pursuing "versioning." By applying the first-order condition for interior optima to the problem $\max_{q\in(\underline{\theta},\min\{\overline{\theta},\overline{q}\})} \nu(q)$, it follows that

$$\nu(q_s^{\text{pm}}) = \frac{\beta[1 - H_s(n(q_s^{\text{pm}}))]}{r_s(n(q_s^{\text{pm}}))} + \alpha\pi_1(\omega^R),$$

where the price q_s^{pm} corresponds to the optimal choice of the platform when it offers only the premium version of the service. The sufficient condition provided in the statement of the proposition follows by requiring that the optimal profits from offering only the free version of the service are no less than the upper bound obtained above. ∎

Proof of Lemma 2. Using the preference specification in (1), consumer surplus at prices (p, q) can be written as

$$\begin{aligned} CS(p,q) = &\int_{i\in[0,1]} \int_{n_i\in[\underline{n},\overline{n}]} z_i(\omega - p)dn_i\, di \\ &+ \int_{i\in[0,1]} \int_{n_i\in[\underline{n},\overline{n}]} \Big[x_i\big[\theta - q + (1+\beta)n_i\big] + (1-x_i)\big[\theta - \psi + n_i\big]\Big]dn_i\, di. \end{aligned}$$

The proof proceeds by decomposing each of the two terms obtained above. First, notice that

$$\begin{aligned} &\int_{i\in[0,1]} \int_{n_i\in[\underline{n},\overline{n}]} z_i(\omega - p)dn_i\, di \\ &\qquad = \mathbb{P}(n_i > n(q)\,|\,s)\mathbb{P}(\omega \geq p\,|\,a=0) + \mathbb{P}(n_i \leq n(q)\,|\,s)\mathbb{P}(\omega \geq p\,|\,a=1) \\ &\qquad = x_s(q)z_0(p) + \big[1 - x_s(q)\big]z_1(p). \end{aligned}$$

Secondly, notice that

$$\begin{aligned} &\int_{i\in[0,1]} \int_{n_i\in[\underline{n},\overline{n}]} \Big[x_i\big[\theta - q + (1+\beta)n_i\big] + (1-x_i)\big[\theta - \psi + n_i\big]\Big]dn_i\, di \\ &\qquad = \mathbb{P}\big(n \geq n(q)\,\big|\,s\big) + \mathbb{P}(\theta + n > \psi\,|\,s). \end{aligned}$$

Finally, given the two formulas derived above, the expression stated in (12) follows from the result that $\mathbb{P}\big(n \geq n(q)\,\big|\,s\big) = 1 - x_s(q)$ and by making use of the fact that the assumption $\psi < \underline{\theta}$ directly implies $\mathbb{P}(\theta + n > \psi\,|\,s) = 1 - H_s(\psi - \theta) = 1$ for each $\theta \in (\underline{\theta}, \overline{\theta})$ because $\underline{n} \geq 0$. ∎

Proof of Proposition 6. Consider two degree distributions $H_s(n)$ and that $H_{s'}(n)$ that induce, respectively, two optimal prices for the product p_s^* and $p_{s'}^*$ such that $p_s^* < \omega^R$ and

$p^*_{s'} > \omega^R$. It follows from Proposition 1 that price p^*_s induces the platform to optimally serve only the premium version. Suppose that price $p^*_{s'}$ induces the platform to optimally serve both versions of the service. Let $q^*_{s'}$ be the optimal price of the service induced by the optimal price of the product $p^*_{s'}$. Notice first that, as $p^*_s < \omega^R$, Assumption 1 implies that $z_0(p^*_s) > z_1(p^*_s)$. Secondly, as $z_1(p) = 1 - F_1(p)$ decreases in p, then we have $z_1(p^*_s) > z_1(p^*_{s'})$. Using the expression obtained in (12) for the consumer surplus, it then follows from the relation $z_0(p^*_s) > z_1(p^*_{s'})$ established above that

$$\begin{aligned} CS(p^*_s) - CS(p^*_{s'}, q^*_{s'}) &= \Big[2 + z_0(p^*_s)\Big] - \Big[1 + z_1(p^*_{s'}) + x_{s'}(q^*_{s'})\big[1 + z_0(p^*_{s'}) - z_1(p^*_{s'})\big]\Big] \\ &= \big[z_0(p^*_s) - z_1(p^*_{s'})\big] + 1 - x_{s'}(q^*_{s'})\big[1 + z_0(p^*_{s'}) - z_1(p^*_{s'})\big] > 0 \end{aligned}$$

because $\big[1 + z_0(p^*_{s'}) - z_1(p^*_{s'})\big] < 1$. ∎

Proof of Proposition 7. Consider two degree distributions $H_s(n)$ and that $H_{s'}(n)$ that induce, respectively, two optimal prices for the product $p^*_s > \omega^R$ and $p^*_{s'} > \omega^R$. Suppose that price p^*_s induces the platform to optimally serve only the free version and that price $p^*_{s'}$ induces the platform to optimally serve both versions of the service. Let $q^*_{s'}$ be the optimal price of the service induced by the optimal price of the product $p^*_{s'}$. Using the expression obtained in (12) for the consumer surplus, it then follows that

$$\begin{aligned} CS(p^*_s) - CS(p^*_{s'}, q^*_{s'}) &= \Big[1 + z_1(p^*_s)\Big] - \Big[1 + z_1(p^*_{s'}) + x_{s'}(q^*_{s'})\big[1 + z_0(p^*_{s'}) - z_1(p^*_{s'})\big]\Big] \\ &= \big[z_1(p^*_s) - z_1(p^*_{s'})\big] - x_{s'}(q^*_{s'})\big[1 + z_0(p^*_{s'}) - z_1(p^*_{s'})\big]. \end{aligned}$$

As $z_1(p) = 1 - F_1(p)$ decreases in p, if (a) $p^*_{s'} < p^*_s$, then it follows directly that $CS(p^*_{s'}, q^*_{s'}) > CS(p^*_s)$. On the other hand, if (b) $p^*_{s'} > p^*_s$, we observe that $CS(p^*_{s'}, q^*_{s'}) > CS(p^*_s)$ if and only if

$$x_{s'}(q^*_{s'}) < \frac{z_1(p^*_s) - z_1(p^*_{s'})}{1 + z_0(p^*_{s'}) - z_1(p^*_{s'})},$$

as stated. ∎

Proof of Proposition 8. By plugging the first-order condition to the monopolist's problem stated in (4), the expression in (13), which gives us the change in consumer surplus induced by a local change in the optimal price of the service, can be rewritten as

$$\begin{aligned} \frac{\partial CS(s)}{\partial q^*_s} = &- \left(\frac{\partial p^*_s}{\partial q^*_s}\right) \frac{x_s(q^*_s) z_0(p^*_s) + [1 - x_s(q^*_s)] z_1(p^*_s)}{(p^*_s - c)} \\ &+ x'_s(q^*_s)\big[1 + z_0(p^*_s) - z_1(p^*_s)\big]. \end{aligned}$$

Therefore, $\partial CS(s)/\partial q_s^* > 0$ if and only if $\partial p_s^*/\partial q_s^* < 0$ with

$$-\left(\frac{\partial p_s^*}{\partial q_s^*}\right) > \frac{-x_s'(q_s^*)(p_s^*-c)\big[1+z_0(p_s^*)-z_1(p_s^*)\big]}{x_s(q_s^*)z_0(p_s^*)+[1-x_s(q_s^*)]z_1(p_s^*)} = \frac{-\frac{x_s'(q_s^*)}{x_s(q_s^*)}\big[(p_s^*-c)+\pi_0(p_s^*)-\pi_1(p_s^*)\big]}{z_0(p_s^*)+[(1/x_s(q_s^*))-1]z_1(p_s^*)}. \tag{15}$$

Recall that the definition of the hazard rate function of the degree distribution in (3), together with the results obtained earlier in (2) on the optimal fraction of premium version adopters, leads to that

$$-\frac{x_s'(q_s^*)}{x_s(q_s^*)} = \left(\frac{1}{\beta}\right) r_s\left(n(q_s^*)\right).$$

Then, using the expression provided by equation (10) of Proposition 3 for the change $\partial p_s^*/\partial q_s^*$, we obtain that the inequality in (15) above is satisfied if and only if

$$\frac{\frac{\beta}{r_s(n(q_s^*))}\left[1+\frac{r_s'(n(q_s^*))}{[r_s(n(q_s^*))]^2}\right]}{\alpha\big[\pi_0'(p_s^*)-\pi_1'(p_s^*)\big]} > \xi(p_s^*, q_s^*),$$

where

$$\xi(p_s^*, q_s^*) = \frac{\big[(p_s^*-c)+\pi_0(p_s^*)-\pi_1(p_s^*)\big]}{z_0(p_s^*)+[(1/x_s(q_s^*))-1]z_1(p_s^*)}.$$

Finally, we know that $z_0(p_s^*)+[(1/x_s(q_s^*))-1]z_1(p_s^*) > 0$ and $\big[(p_s^*-c)+\pi_0(p_s^*)-\pi_1(p_s^*)\big] = (p_s^*-c)\big[1+z_0(p_s^*)-z_1(p_s^*)\big] > 0$ so that $\xi(p_s^*, q_s^*) > 0$ for each pair of interior optimal prices. ∎

Figures

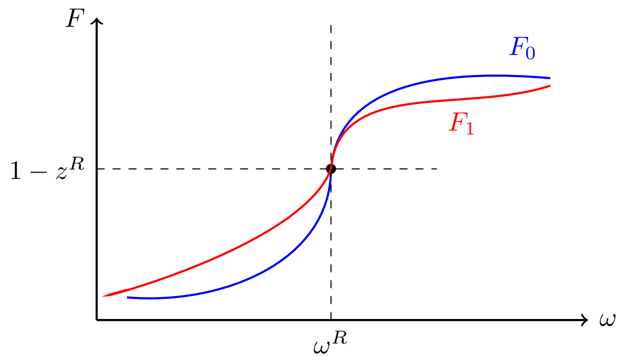

Figure 1: Rotation of Distribution of Valuations

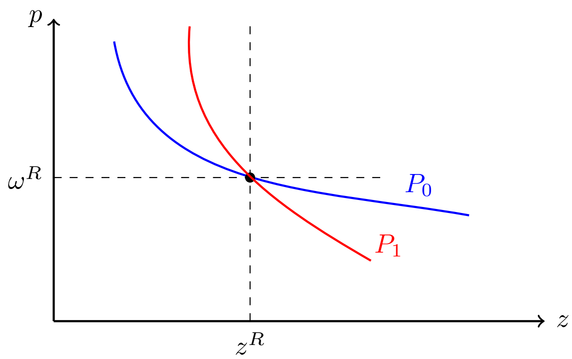

Figure 2: Rotation of Inverse Demand

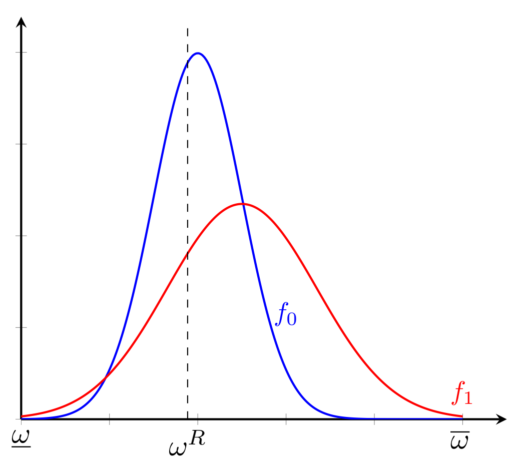

Figure 3: Advertising and Dispersion of the Product Valuations

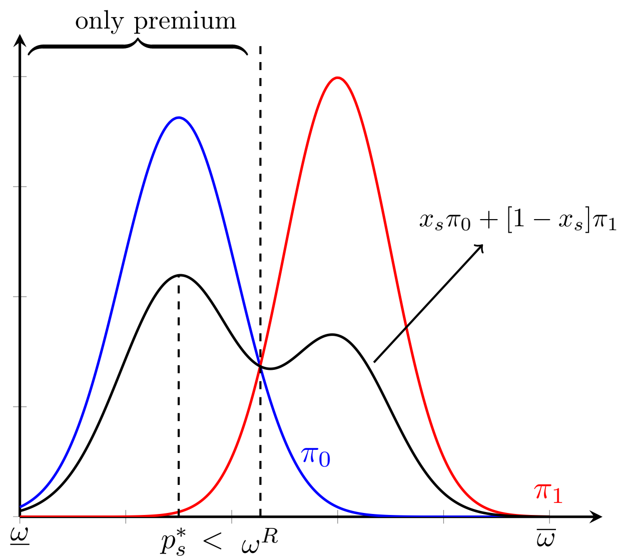

Figure 4: Profits from the Product Sales

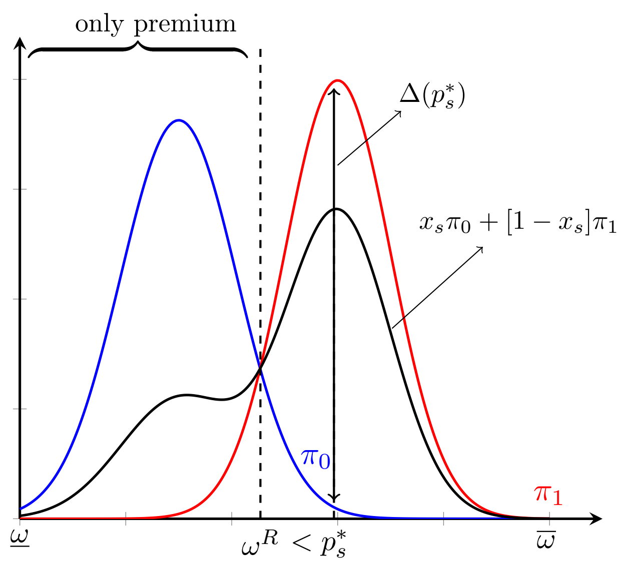

Figure 5: Profits from the Product Sales