# Persuasion under "Aspect-Restricted" Experimentation*

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**Abstract**

This article explores information design in two-aspect-uncertainty environments under the assumption that the Sender is (exogenously) restricted to choosing only one of the aspects to design experiments over it. The equilibrium concept used incorporates a "backwards-induction" requirement (for the initial aspect choice) to the Bayes–correlated equilibrium notion typically used in the information design literature. Optimal experimentation is driven by the marginal priors over the separate aspects, the joint priors about the state, and the players' preferences. Through the new information it discloses, optimal experimentation seeks to alleviate the original conflict of interests. For the two-action case, the optimal aspect choice and any optimal experiment are "tailor-designed" according to the preferences of the Receiver. The results provide a rationale for Senders deliberately selecting aspects in order to meet Receivers' tastes when they are constrained to selecting subsets of aspects from all the relevant aspects.

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# 1. Introduction

Experimentation[1] is central to advice and influence decision-making when individuals care about uncertain variables. Bayesian persuasion (Brocas and Carrillo, 2007; Rayo and Segal, 2010; Kamenica and Gentzkow, 2011; Gentzkow and Kamenica, 2017a,b)—or, more generally,[2] information design (Bergemann and Morris, 2013, 2016; Taneva, 2018)—formalizes the design of experiments as the (ex ante) selection of information structures that specify the (new) information that will be subsequently disclosed. Canonical information design models assume that commitment is both *full*[3] and *complete*.[4] Thus, for environments where the relevant uncertainty is multi-dimensional, the standard assumption is that Senders/information-designers can pick any rule that maps *all* dimensions of the state into a probability distribution over action recommendations.[5] The typical assumptions give a lot of flexibility to experimentation, placing little restrictions on the possibilities to design experiments and, therefore, to persuade Receivers/decision-makers.

In practice, situations abound where the relevant uncertainty has a high number of separate dimensions—or *aspects*—which are very distinct conceptually one from another. Undecided voters want to consider data released by experiments such as political campaigns, financial stress tests, income inequality studies, misconduct and corruption investigations, income growth forecasts, opinion rating systems, political science research, immigration data, audits of public institutions, crime indicators, or experimental evidence on climate change. Some natural limitations, though, are: (1) a single experiment cannot usually provide data about all aspects jointly, (2) a given Sender is rarely capable of designing separate experiments about all aspects, and (3) voters are unable to pay attention to the data released by

[1] For concreteness, this paper will use the term *experimentation* to refer to, e.g., scientific research, journalistic investigation, forensic tests, witnesses' reports, fact-based studies, expert consultation, audits, polls, trials, medical tests, or lab experimentation.

[2] Bayesian persuasion gives us a particular formulation of the more general information design problem for situations with a single Sender and a single Receiver who has no private information. The current paper builds more closely upon the developments on information design and thus relies more on its terminology.

[3] Commitment is *full* when it is entirely binding, with probability one. For those cases where there is some probability that commitment is not binding, the information design literature generally uses the terms *partial* or *limited* commitment.

[4] I reserve the term *complete* commitment to capture the consideration that commitment can be imposed over all dimensions of a multi-dimensional aspect.

[5] Using a Revelation Principle argument, this is equivalent to map the state of the world into a probability distribution over signals which, in turn, provide new information for the Receivers to choose their preferred actions. Using signals instead of action recommendations is the formulation most typical in Bayesian persuasion models.

experiments about all aspects. Owing to technological bounds,[6] restrictions on the ability of Receivers to process, or use, all the data from experimentation, or simply time constraints, it maybe unfeasible to end up with experiments over *all* aspects before decision-making is due. Senders aware of such limitations need to select deliberately a subset of aspects for experimentation from much broader sets of relevant aspects.

This article is concerned about natural (exogenous) restrictions to the complete commitment assumption for multi-dimensional environments. Under which conditions can a Sender who is constrained to selecting a subset of aspects persuade through experimentation? How would a Sender favor one aspect against another to optimally design experimentation when he is restricted to picking just one out of two aspects?[7] In consonance with the pertinent literature, the term *complete commitment power* will capture in this paper that commitment power is available over *all* the relevant dimensions. In addition, I propose the term *"aspect-restricted" commitment* to describe situations where commitment power is full but only available over a (strict) subset of the dimensions.

The approach is abstract but the underlying ideas are of substantial relevance. Consider the consulting CEO (Sender/information-designer) of an automobile company who advises the stock-holders Board of the firm (Receiver/decision-maker). The Board must decide whether or not to launch the company's flagship electric model into a new market. The relevant state of the world $\theta = (x, y)$ consists of two aspects about the consumers of this market, their future income ($x$) and their environmental concerns ($y$). Although experiments can usually be designed separately about income growth or about environmental concerns, it seems less feasible that a common data–generation process be able to disclose information about both aspects jointly. The model proposed here assumes that the Board can use the data from experimentation about any single one of the two aspects before the decision of whether or not to launch the electric model is due. The assumption of *aspect-restricted commitment* considers that the expert cannot select information structures about the pair $(x, y)$. Instead, the expert is restricted to selecting information structures for any of the two variables, income growth ($x$) or environmental concerns ($y$), before the Board is due to make its decision.

A key contribution of the paper is to identify the features of marginal priors over each aspect, of joint priors about the state, and of the players' preferences that characterize

[6] For instance, restrictions to sample size or high costs of conducting research.

[7] Considering a choice of one out of two available aspects gives us a particularly stylized setup.

optimal experimentation when the Sender is constrained to selecting a single aspect from a two-dimensional state of the world (Theorem 1, Theorem 2, Corollary 1, and the analysis of Section 7). A general insight is that experimentation over one aspect can persuade the Receiver only if—despite the conflict of interests between the two parties—it is able to release pieces of data conditional on which the interests are aligned. Notably, the pieces of data that mitigate the conflict of interests are not required to coincide. Consider a given action that can be picked by the Receiver in her decision-making problem. If there exist aspect realizations—which need not coincide across the two parties—of a given aspect conditional on which (i) the Sender receives no disutility and (ii) the Receiver prefers such a given action over any alternative, then we say that experimentation over the aspect makes the players "agree on that such an action is a suitable choice." Theorem 1 and Theorem 2 also identify the necessary and sufficient conditions that ensure internal compatibility among the incentive-compatibility conditions of an aspect-restricted information design problem. This is a relevant consideration in environments with a high number of possible actions as experimentation may in general be unable to recommend any action owing to contradictions among the incentive-compatible recommendations that arise. The analysis relies here on foundational work about the consistency of linear inequalities (Stokes, 1931; Dines, 1936).

Suppose that the CEO of the automobile company wants the Board to launch the electric model independently of the state, whereas the Board wants to do so only if the state belongs to a certain "acceptance" set. Suppose that for some aspect, either income growth, environmental concerns, or both, experimentation can release at least one piece of data conditional on which the Board wishes to launch the new model. Then, the CEO will optimally design experiments that recommend launching the electric model with probability one conditional on such a piece of data. For the canonical two-action model, provided that experimentation makes the players "agree on that one given action is suitable," the optimally selected experiments will also recommend such an action (with probability between zero and one) for realizations different from the ones that make the Receiver to prefer such an action. These recommendations will be based on pieces of data that maximize a certain ratio that captures the degree to which the Receiver gets harmed by choosing the action that she dislikes conditional on observing such aspect realizations.

For the general model with multiple actions, each (aspect-restricted) information design problem requires a particular algorithm to solve the associated linear programming problem. This limits severely any analysis that relies upon closed-form value functions for optimal experimentation over each of the separate aspects. This is why general results regarding the features of the primitives that make the Sender prefer one aspect over the other are unfeasible, unless we impose further restrictions to the model. One way to tackle these difficulties is to restrict attention to the two-action case. For the two-action model in which the Sender wants experimentation to recommend always a particular action, optimal experimentation is based on two features of the primitives. First, the Sender favors the aspect such that its *marginal* priors place relatively high probabilities on aspect realizations that lead the Receiver to prefer the action preferred also by the Sender. Secondly, for aspect realizations under which the Receiver indeed dislikes the Sender's preferred action, the Sender favors the aspect such that the *joint* priors place relatively high probabilities on state realizations conditional on which such an action harms the least the Receiver.

Since the Sender prioritizes the aspect whose possible realizations yield the highest (ex ante) probability that the incentives of the parties be (partially) aligned, a core message that emerges from the analysis is that the optimal (restricted) experimentation seeks to mitigate the original conflict of interest. For the two-action case in which the Sender always prefers one given action—regardless of the state—, the natural interpretation is that optimal experimentation is in particular "tailored-designed" to fit exclusively the tastes of the Receiver.

It should be noted that the model's main insights can be readily extrapolated to the traditional case with complete commitment. The logic behind information design over a given aspect coincides entirely with that of the canonical information design setup. In this sense, this paper aspires to provide a comprehensive analysis of some features of the information design approach, also under the standard assumption of commitment over the entire state of the world. The contributions of the aspect-restricted commitment assumption lie in investigating what can be achieved under (exogenous) restrictions to the amount of aspects for experimentation—relative to complete commitment—and in obtaining criteria to help us understand how (restricted) information-designers decide among different aspects.

Section 2 outlines the model and Section 3 illustrates its functioning through a number of examples. Section 4 introduces the equilibrium concept and compares it to the traditional framework under complete commitment. The main results are presented in Section 5. Section 6 takes on a duality approach to study the aspect-restricted information design problem. Section 7 focuses on the canonical two-action case. Section 8 applies the model to provide a plausible rationale for slant in media persuasion. The closest related literature is discussed in Section 9, and Section 10 concludes. All the proofs are relegated to the Appendix.

## 2. Model

This section lies out the framework. "Experimentation over a given aspect" will be formalized using the Bayesian persuasion/information design approach.[8] The model will assume *full commitment* in experimentation decisions about each separate aspect.

There are two players, $i = S, R$, a potentially informed *(S)ender* (he) and an uninformed *(R)eceiver* (she). The Receiver must choose an *action* $a$ from a set $A = \{a_0, \ldots, a_j, \ldots, a_n\}$ (i.e., $|A| = n + 1$). Both players $i = S, R$ care about action $a$ and about a two-dimensional *state of the world* $\theta \equiv (x, y) \in \Theta \equiv X \times Y$. In particular, $X \equiv \{x_1, \ldots, x_l, \ldots, x_m\}$ and $Y \equiv \{y_1, \ldots, y_l, \ldots, y_m\}$.[9] The term $\mathcal{K}$ will be used to indicate a generic set $\mathcal{K} \in \{X, Y\}$ of all the possible realizations $\kappa_l$ of a separate *aspect* $\kappa \in \{x, y\}$ of the state $\theta$. For a given aspect $\kappa$, notation $-\kappa$ will identify the remaining aspect—i.e., $\{-\kappa\} \equiv \{x, y\} \setminus \{\kappa\}$—and, similarly, the term $-\mathcal{K}$ will identify the remaining set of aspect realizations—i.e., $\{-\mathcal{K}\} \equiv \{X, Y\} \setminus \{\mathcal{K}\}$.

The players are uncertain about the state of the world and have common *joint priors* $\psi \in \Delta_{++}(\Theta)$ about $\theta$. Notation $\psi_x \in \Delta_{++}(X)$ and $\psi_y \in \Delta_{++}(Y)$ will indicate the *marginal priors* over the respective aspect. Also, $\psi(\cdot \mid \kappa_l) \in \Delta(-\mathcal{K})$ will denote the *conditional probability over aspect $-\kappa$ given realization $\kappa = \kappa_l$, according to priors*. There is no uncertainty about how the aspects are related to each other: both Sender and Receiver have perfect information about the family of conditional probabilities $\tau_\kappa \equiv \{\psi(\cdot \mid \kappa_l) \in \Delta(-\mathcal{K}) \mid \kappa_l \in \mathcal{K}\}$, for any $\kappa \in \{x, y\}$, that relates the two relevant aspects.

The preferences of the Receiver and the Sender are described by (ex post) utility functions, respectively, $u : A \times \Theta \to \mathbb{R}$ and $v : A \times \Theta \to \mathbb{R}$.

**Novel Assumption.**—The novel element of the proposed framework is that the Sender is constrained to choosing (fully committed) information structures, or experiments, over any single one of the two aspects, either $x$ or $y$, before decision-making is due. The term *aspect-restricted commitment* describes this consideration. This assumption can be motivated by either technological bounds or limitations on the Sender's ability to process the data disclosed by experiments over a number of different aspects. Conceivably, Senders are (exogenously) constrained in many environments to select deliberately a subset of aspects

[8] Brocas and Carrillo (2007), Rayo and Segal (2010), Kamenica and Gentzkow (2011), and the literature reviewed by Bergemann and Morris (2019).

[9] The sets $X$ and $Y$ are assumed to have the same cardinality $m$ simply for notational convenience. This assumption does not play any role in the model's implications.

for experimentation from broader sets of relevant aspects. The model is developed in terms of a two-dimensional state for simplicity.[10] Restricting experimentation to any single one of two available aspects aims at laying out the most simple setup that captures more generally the idea that the Sender needs to be selective in his choice from all the relevant aspects.

Notwithstanding, being endowed with some information about the dependencies between the two aspects—which in the model takes the form of assuming perfect information about the families of conditional probabilities $\tau_\kappa$—, combined with experimentation over a single aspect, is in fact able to disclose information over the two aspects.[11]

**Time Line.**—The timing of the disclosure game is as follows. First, $S$ chooses a single aspect $\kappa \in \{x, y\}$ and a (fully committed) information structure, or experiment, over such a selected aspect. $R$ observes the experimentation choice made by $S$. Nature chooses a true value of the state $\theta \in \Theta$ according to the prior $\psi$. Then, the chosen information structure makes action recommendations to $R$ based on the new information that it discloses (only) about the selected aspect $\kappa$. After observing the recommendations of the selected information structure, $R$ chooses her action.

Using the leading example from the Introduction, the consulting CEO of the automobile company is able to select either economic experimentation about income growth or sociologic/marketing experimentation about environmental concerns before the Board must make its decision. The Board decides then whether or not to launch the company's electric model, based on the outcome of the selected experiment.

**Aspect-Restricted Information Design.**—Following the Revelation Principle arguments of the information design approach (Bergemann and Morris, 2019), I consider that the disclosure of information from experimentation takes the form of direct "action recommendations."[12] The recommendations provided by the selected experiments become public and cannot be subsequently concealed or distorted. As mentioned earlier, the setup considers

[10] The model's logic and main implications, though, hold qualitatively for a general multi-dimensional state with a finite number of dimensions. The assumption of aspect-constrained commitment can then be more generally stated as the Sender being constrained to select a (strict) subset of all available dimensions for experimentation.

[11] Using the semantics of information design, each (aspect-restricted) information structure over a single aspect, together with full knowledge about how one aspect is related to another, induces a joint information structure over the two-dimensional state of the world.

[12] This simplifies the analysis without loss of generality while it allows to avoid an explicit treatment of how indirect signals induce action recommendations.

- Sender (i) selects one of the aspects and (ii) chooses an experiment over such an aspect
- Receiver observes Sender's experimentation choice
- Nature chooses a state of the world
- Experiment makes action recommendations based on the disclosed new information about the selected aspect
- Receiver observes the recommendations made by the selected experiment
- Receiver chooses action and both players obtain their payoffs

Table 1: Timing of the Information Disclosure Game

*full commitment* over each separate aspect of uncertainty.

Let me review briefly the key elements of the traditional information design approach to appreciate better how the proposed model builds upon (and differs from) the standard framework. The information design approach rests on the concept of decision rule (Bergemann and Morris, 2013, 2016). A *decision rule (under complete commitment)* is a mapping $\sigma : \Theta \to \Delta(A)$, where $\sigma(a \mid \theta)$ specifies the probability according to which the selected information structure recommends action $a$ if the true realization of the state is $\theta$. Let $\Sigma$ be the set of all decision rules, or experiments, available over both dimensions of uncertainty. A decision rule (under complete commitment) $\sigma$ is said to satisfy the incentive-compatibility, or *obedience*, condition if, for each action $a \in A$, we have

$$\sum_{\theta} \sigma(a \mid \theta)\psi(\theta)\big[u(a,\theta) - u(a',\theta)\big] \geq 0 \qquad \forall\, a' \in A. \tag{1}$$

The condition in Eq. (1) is a formulation of the obedience criterion required by Bergemann and Morris (2013, 2016)—for the case with a single Receiver who has no private information—to define a *Bayes correlated equilibrium*.[13]

Nevertheless, the current paper considers that information design is restricted over any single separate aspect of uncertainty. Building on the information design approach, I define a *decision rule (under aspect-restricted commitment) over aspect* $\kappa \in \{x, y\}$ as a mapping

[13] Equivalently, the requirements in Eq. (1) characterize the behavior of an information-designer in the key concavification problem explored by Kamenica and Gentzkow (2011) for Bayesian persuasion.

$\sigma_\kappa : \mathcal{K} \to \Delta(A)$. The interpretation of a decision rule under aspect-restricted commitment $\sigma_\kappa$ is that if the true realization of aspect $\kappa$ is $\kappa_l \in \mathcal{K}$, then the selected information structure recommends action $a \in A$ with probability $\sigma_\kappa(a \mid \kappa_l)$. Let $\Sigma_\kappa$ be the set of all available decision rules restricted to aspect $\kappa$. Also, let me use $(\kappa, \sigma_\kappa) \in \{x, y\} \times \Sigma_\kappa$ to denote an *experimentation choice*, which identifies an aspect $\kappa \in \{x, y\}$ chosen for experimentation, as well as a decision rule $\sigma_\kappa$ over aspect $\kappa$.

There is a clear analogy with the notion of decision rule under complete commitment. However, an aspect-restricted decision rule makes recommendations based only on partial information about the state. Compared to complete commitment, less information is disclosed under aspect-restricted commitment. Exactly as in the complete commitment benchmark, though, the approach rests on the key consideration that the Sender does not need to know the true realization of the respective aspect $\kappa$. The commitment assumption only requires that the Sender can condition the decision rule $\sigma_\kappa$ on the realization of aspect $\kappa$.[14]

To fix ideas about how a decision rule under aspect-restricted commitment discloses credible information in the proposed setup, suppose that $S$ makes an experimentation choice $(\kappa, \sigma_\kappa)$. Then, the aspect-restricted decision rule $\sigma_\kappa$ satisfies the incentive-compatibility, or obedience, condition if for each action $a_j \in A$ such that $\sigma_\kappa(a_j \mid \kappa_l) > 0$ for some $\kappa_l \in \mathcal{K}$, we have:

$$\sum_{\theta} \sigma_\kappa(a_j \mid \kappa)\, \psi(\theta) \big[u(a_j, \theta) - u(a, \theta)\big] \geq 0 \qquad \forall\, a \neq a_j. \tag{2}$$

Notice that the condition in Eq. (2) above is simply an adjusted version of the key obedience condition in Eq. (1). The essential distinction is that the action recommendation in the proposed benchmark is based only on the realization of a particular dimension $\kappa \in \{x, y\}$ of the state of the world $\theta$.

Importantly, the Revelation Principle arguments provided by Bergemann and Morris (2016) in the proof of their Theorem 1 apply entirely to the definition of decision rule $\sigma_\kappa$ under aspect-restricted commitment, for each given aspect $\kappa$.[15] The key argument is that any signal that an aspect-restricted information structure can disclose gives rise to an action in equilibrium that can be equivalently labeled as a signal associated to such an action. Therefore, following the Revelation Principle result in Theorem 1 of Bergemann and Morris

[14] The premises behind the idea of a decision rule $\sigma_\kappa$ can be intuitively phrased as: the two players commonly known that the Sender is both (i) able to "commission" any possible experiment over aspect $\kappa$ and (ii) unable to affect in any way the data subsequently released by the chosen experiment.

[15] More generally, the required Revelation Principle arguments are also given by Aumann (1987) in the context of correlated equilibrium.

(2016), I assume without loss of generality that experimentation over any separate aspect $\kappa$ discloses only action recommendations that are obeyed by the Receiver or, in other words, that satisfy the incentive-compatibility condition in Eq. (2).

Notwithstanding, the key Revelation Principle implication of the classical information design models does not hold in general in the *partial*, or *limited*, commitment literature. The partial commitment literature initiated with Bester and Strausz (2001) and the Revelation Principle is indeed challenged here. The current paper, nonetheless, assumes that the Sender is not limited in his commitment power over any chosen aspect. Therefore, the standard Revelation Principle does not fail in this case because $S$ cannot exploit the selected decision rule to his advantage as it is the case under *partial*, or *limited*, commitment power. Accordingly, $R$ anticipates that any (possibly indirect) signal disclosed by the information structure corresponds to truthful reporting. $S$ and $R$ commonly know that the recommendations from the selected information structures are completely binding, despite being based only on partial information about the state. Therefore, although we can consider in principle that experimentation over a given aspect offers any communication mechanism (perhaps indirect, or through signal realizations), we can further resort without loss of generality to a "direct communication" mechanism that recommends actions to $R$. Then, we only need to verify that $R$ is given the right incentives to obey the recommendations from aspect-restricted experimentation, as expressed in Eq. (2) above.

# 3. Leading Examples

Consider the situation, spelled out in the Introduction, in which the CEO of the automobile company wants to persuade the Board to launch the electric model into the new market. The following ingredients will be common in the three examples analyzed in this Section 3.

Aspect $x$ describes income growth and aspect $y$ captures environmental concerns. There are nine possible states of the world, with $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$. In particular, let $\theta_1 = (x_1, y_1)$, $\theta_2 = (x_1, y_2)$, $\theta_3 = (x_1, y_3)$, $\theta_4 = (x_2, y_1)$, $\theta_5 = (x_2, y_2)$, $\theta_6 = (x_2, y_3)$, $\theta_7 = (x_3, y_1)$, $\theta_8 = (x_3, y_2)$, and $\theta_9 = (x_3, y_3)$. The marginal priors about the two aspects of the state are given by $\psi_x(x_1) = 3/6$, $\psi_x(x_2) = 1/6$ and $\psi_x(x_3) = 2/6$, and by $\psi_y(y_1) = \psi_y(y_2) = \psi_y(y_3) = 1/3$.

There are two possible actions, $A = \{a_0, a_1\}$. Action $a_0$ is interpreted as launching the electric model in the new market and action $a_1$ as rejecting such a proposal. The CEO wants to persuade the Board to accepting the proposal always, regardless of the true value of the state: $v(a_0, \theta) = 1$ and $v(a_1, \theta) = 0$ for each $\theta \in \Theta$. The Board wants to accept only if the state belongs to the "acceptance set" $\Theta_0 = \{\theta_3, \theta_6, \theta_8, \theta_9\}$. In particular, consider that $u(a_1, \theta) = 0$ for each $\theta \in \Theta$, whereas $u(a_0, \theta) = 1/3$ if $\theta \in \Theta_0$, and $u(a_0, \theta) = -1$ if $\theta \in \Theta_1$.[16] The set of states, the acceptance set $\Theta_0$, and the marginal priors over the separate aspects for this example are shown in Fig. 1.

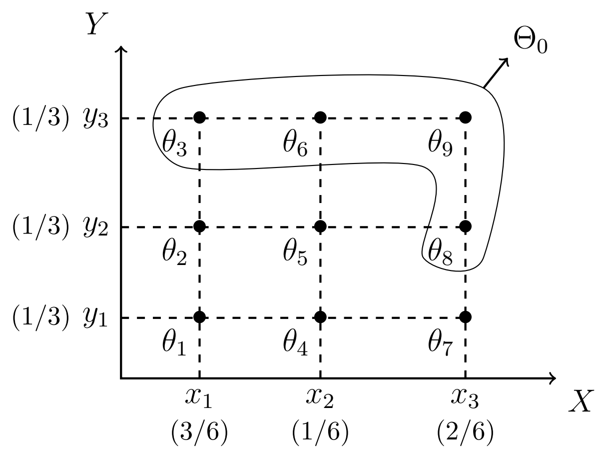


**Figure 1** – Leading Examples: Set of States and Marginal Priors.

To introduce different examples, the marginal distributions $\psi_\kappa$ specified above over the two aspects $\kappa$ are kept fixed, whereas the conditional distributions $\psi(y \mid x)$ are allowed to vary. This Section 3 considers then three different examples according to three possible (Bayes-consistent) families $\tau_x$ of conditional distributions $\tau_x = \{\psi(\cdot \mid x) \in \Delta(Y) \mid x \in X\}$ that may relate the two relevant aspects. Note that we can use a matrix $\big(\psi(y \mid x)\big)_{(y,x)\in Y\times X}$ to capture a family $\tau_x$ of conditional distributions. In the following examples, experimentation over each aspect $\kappa \in \{x, y\}$ can be described by setting three parameters for each of the two aspects. For $l = 1, 2, 3$, use the short-hand notations $z^\kappa_{0l} \equiv \sigma_\kappa(a_0 \mid \kappa_l) \in [0, 1]$ for $\kappa \in \{x, y\}$.

**Example I. The Two Aspects Are Independent.**—Consider a family $\tau_x$ of conditional distributions described by the matrix of conditional probabilities

$$\begin{pmatrix} \psi(y_1 \mid x_1) & \psi(y_1 \mid x_2) & \psi(y_1 \mid x_3) \\ \psi(y_2 \mid x_1) & \psi(y_2 \mid x_2) & \psi(y_2 \mid x_3) \\ \psi(y_3 \mid x_1) & \psi(y_3 \mid x_2) & \psi(y_3 \mid x_3) \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}.$$

[16] Preferences in this example are analogous versions (for a two-dimensional state of the world) of those in the leading "courtroom example" of Kamenica and Gentzkow (2011)'s influential contribution on Bayesian persuasion and "investment example" of Bergemann and Morris (2019)'s survey on information design.

Under the family $\tau_x$, income growth and environmental concerns are independent of each other. The Bayesian plausible prior $\psi$ in this case is given by

$$\psi(\theta_1) = \psi(\theta_2) = \psi(\theta_3) = 3/18;$$
$$\psi(\theta_4) = \psi(\theta_5) = \psi(\theta_6) = 1/18;$$
$$\psi(\theta_7) = \psi(\theta_8) = \psi(\theta_9) = 2/18.$$

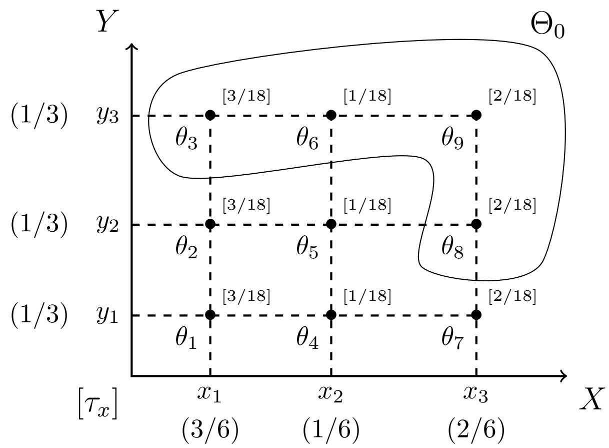


**Figure 2** – Example I: Priors about $\theta$ under $\tau_x$.

*I-A. Experimentation over Income Growth.*—Let us first analyze what experimentation over aspect $x$ can attain. Take an experimentation choice $(x, \sigma_x)$. The (ex ante) expected utility that the CEO receives is given by

$$(3/6)z^x_{01} + (1/6)z^x_{02} + (2/6)z^x_{02},$$

and the incentive-compatibility requirement on information design specified in Eq. (2) turns into

$$15z^x_{01} + 5z^x_{02} + 2z^x_{03} \leq 0.$$

Therefore, all that the CEO can do in this case when trying to maximize his expected utility is to select ${z^x_{01}}^* = {z^x_{02}}^* = {z^x_{03}}^* = 0$. In this case, experiments over income growth are unable to influence the Board towards acceptance and the CEO obtains a zero expected utility.

*I-B. Experimentation over Environmental Concerns.*—Let us now explore what can be attained through experimentation over aspect $y$. Take an experimentation choice $(y, \sigma_y)$.

Upon considering the incentive-compatibility requirement on information design specified in Eq. (2), the problem of the CEO can now be rewritten as

$$\max_{\{z^y_{01}, z^y_{02}, z^y_{03} \in [0,1]\}} \ (1/3)z^y_{01} + (1/3)z^y_{02} + (1/3)z^y_{03}$$
$$\text{s.t.}: \ z^y_{03} \geq 3z^y_{01} + (5/3)z^y_{02}.$$

In this case, the CEO maximizes his expected utility by choosing ${z^y_{03}}^* = 1$, ${z^y_{01}}^* = 0$, and ${z^y_{02}}^* = 3/5$. Then, the CEO attains an ex ante expected utility of 8/15.

**Example II. The Two Aspects Depend on Each Other.**—Let us consider another family $\tau'_x$ of conditional distributions described by the matrix of conditional probabilities

$$\begin{pmatrix} \psi(y_1 \mid x_1) & \psi(y_1 \mid x_2) & \psi(y_1 \mid x_3) \\ \psi(y_2 \mid x_1) & \psi(y_2 \mid x_2) & \psi(y_2 \mid x_3) \\ \psi(y_3 \mid x_1) & \psi(y_3 \mid x_2) & \psi(y_3 \mid x_3) \end{pmatrix} = \begin{pmatrix} 2/3 & 0 & 0 \\ 1/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Note that income growth and environmental concerns are correlated under the family of conditional distributions $\tau'_x$. The Bayesian plausible prior $\psi$ in this case is given by

$$\psi(\theta_1) = 6/18, \quad \psi(\theta_2) = 3/18, \quad \psi(\theta_3) = 0;$$
$$\psi(\theta_4) = \psi(\theta_6) = 0, \quad \psi(\theta_5) = 3/18;$$
$$\psi(\theta_7) = \psi(\theta_8) = 0, \quad \psi(\theta_9) = 6/18.$$

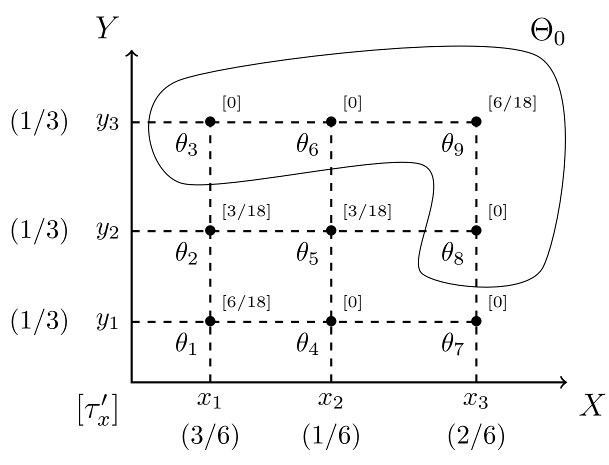


**Figure 3** – Example II: Priors about $\theta$ under $\tau'_x$.

*II-A. Experimentation over Income Growth.*—Take an experimentation choice $(x, \sigma_x)$. Upon considering the incentive-compatibility requirement on information design specified in Eq. (2), the problem of the CEO can now be rewritten as

$$\max_{\{z_{01}^x, z_{02}^x, z_{03}^x \in [0,1]\}} \; (3/6)z_{01}^x + (1/6)z_{02}^x + (2/6)z_{03}^x$$
$$\text{s.t.}: \; 2z_{03}^x \geq 9z_{01}^x + 3z_{02}^x.$$

In this case, the CEO maximizes his expected utility by choosing ${z_{03}^x}^* = 1$, and any ${z_{01}^x}^*, {z_{02}^x}^* \in [0, 1]$ such that $(9/2){z_{01}^x}^* + (3/2){z_{02}^x}^* = 1$. The expected utility that the CEO attains is 4/9.

*II-B. Experimentation over Environmental Concerns.*— Take an experimentation choice $(y, \sigma_y)$. Using the incentive-compatibility requirement for information design specified in Eq. (2), the problem of the CEO can now be written as

$$\max_{\{z_{01}^y, z_{02}^y, z_{03}^y \in [0,1]\}} \; (1/3)z_{01}^y + (1/3)z_{02}^y + (1/3)z_{03}^y$$
$$\text{s.t.}: \; z_{03}^y \geq 3z_{01}^y + 3z_{02}^y.$$

In this case, the CEO maximizes his expected utility by selecting ${z_{03}^y}^* = 1$, and any ${z_{01}^y}^*, {z_{02}^y}^* \in [0, 1]$ such that $3{z_{01}^y}^* + 3{z_{02}^y}^* = 1$. His attained ex ante expected utility is 4/9.

**Example III. The Two Aspects Depend on Each Other.**—As a final example, consider another family $\tau_x''$ of conditional distributions described by the matrix of conditional probabilities

$$\begin{pmatrix} \psi(y_1 \mid x_1) & \psi(y_1 \mid x_2) & \psi(y_1 \mid x_3) \\ \psi(y_2 \mid x_1) & \psi(y_2 \mid x_2) & \psi(y_2 \mid x_3) \\ \psi(y_3 \mid x_1) & \psi(y_3 \mid x_2) & \psi(y_3 \mid x_3) \end{pmatrix} = \begin{pmatrix} 3/6 & 1/6 & 1/6 \\ 2/6 & 4/6 & 1/6 \\ 1/6 & 1/6 & 4/6 \end{pmatrix}.$$

In this example, income growth and environmental concerns are also correlated under the family of conditional distributions $\tau_x''$. The Bayesian plausible prior $\psi$ in this case is given by

$$\psi(\theta_1) = 9/36, \quad \psi(\theta_2) = 6/36, \quad \psi(\theta_3) = 3/36;$$
$$\psi(\theta_4) = \psi(\theta_6) = 1/36, \quad \psi(\theta_5) = 4/36;$$
$$\psi(\theta_7) = \psi(\theta_8) = 2/36, \quad \psi(\theta_9) = 8/36.$$

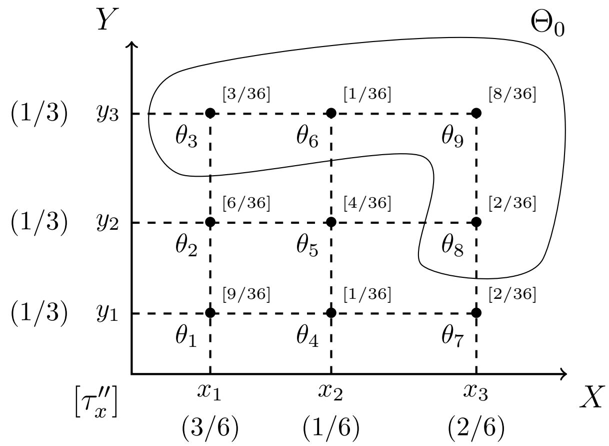


**Figure 4** – Example III: Priors about $\theta$ under $\tau''_x$.

*III-A. Experimentation over Income Growth.*—Take an experimentation choice $(x, \sigma_x)$. Upon considering the incentive-compatibility requirement on information design specified in Eq. (2), the problem of the CEO can now be rewritten as

$$\max_{\{z^x_{01}, z^x_{02}, z^x_{03} \in [0,1]\}} (3/6)z^x_{01} + (1/6)z^x_{02} + (2/6)z^x_{03}$$
$$\text{s.t.}: \; 2z^x_{03} \geq 21z^x_{01} + 7z^x_{02}.$$

In this case, the CEO maximizes his expected utility by choosing ${z^x_{03}}^* = 1$, and any ${z^x_{01}}^*, {z^x_{02}}^* \in [0,1]$ such that $(21/2){z^x_{01}}^* + (7/2){z^x_{02}}^* = 1$. The CEO attains an expected utility of 8/21.

*III-B. Experimentation over Environmental Concerns.*— Take an experimentation choice $(y, \sigma_y)$. Using the incentive-compatibility requirement for information design specified in Eq. (2), the problem of the CEO can now be written as

$$\max_{\{z^y_{01}, z^y_{02}, z^y_{03} \in [0,1]\}} (1/3)z^y_{01} + (1/3)z^y_{02} + (1/3)z^y_{03}$$
$$\text{s.t.}: \; 3z^y_{03} \geq 9z^y_{01} + 7z^y_{02}.$$

In this case, the CEO maximizes his expected utility by selecting ${z^y_{03}}^* = 1$, ${z^y_{02}}^* = 3/7$, and ${z^y_{01}}^* = 0$. His attained ex ante expected utility is now 10/21.

Given the optimal experiments described in these three examples, it is intuitive to see that the optimal aspect choice for those examples can be summarized as: (a) if the two aspects are independent (thus, the two aspects are related according to $\tau_x$), then the CEO optimally selects experimentation over environmental concerns; (b) if the aspects are correlated according to $\tau'_x$, then the CEO optimally selects experimentation over any of the two aspects; (c) if the aspects are correlated according to $\tau''_x$, then the CEO optimally selects experimentation over environmental concerns.

These three examples have developed the functioning of the equilibrium notion in the proposed framework. Also, these examples contain the elements that drive the model's main insights. The next sections introduce formally the equilibrium notion and the model's results.

# 4. Equilibrium

The equilibrium notion is specified from the Sender's perspective. The Receiver's optimal behavior is mechanical. Information design places incentive-compatibility conditions, in terms of additional information, for the Receiver to follow the recommendations of experimentation over the selected aspect. Following the information design approach, I assume that the Sender considers, for each of the two separate aspects $x$ and $y$, the set of all possible outcomes that could arise (as Bayes correlated equilibria) from (aspect-restricted) decision rules that satisfy the incentive-compatibility conditions stated in Eq. (2). Then, for each of the two separate aspects $\kappa \in \{x, y\}$, and for the respective set of incentive-compatible (aspect-restricted) decision rules, the Sender seeks for rules that maximize his (ex ante) expected utility. Let

$$V_\kappa(\sigma_\kappa) \equiv \sum_a \sum_\theta \sigma_\kappa(a \mid \kappa)\psi(\theta)v(a, \theta) \tag{3}$$

denote the Sender's (ex ante) expected utility from an experimentation choice $(\kappa, \sigma_\kappa)$.

In addition, the equilibrium notion considers that the constrained Sender chooses optimally the selected aspect through "backwards-induction" reasoning. First, for each aspect $\kappa$, the Sender wants to consider decision rules $\sigma_\kappa$ that maximize his (ex ante) expected utility $V_\kappa(\sigma_\kappa)$ in Eq. (3) subject to the constraints imposed by the incentive-compatibility conditions in Eq. (2). Secondly, upon consideration of what aspect-restricted information design can attain for each of the two separate aspects, the Sender chooses the aspect that gives him the higher expected utility. Therefore, I add a classical (sequential rationality) perfection requirement to the notion of Bayes correlated equilibria to propose the equilibrium notion used in this paper. All these considerations where illustrated by our three examples of Section 3.

An *(aspect-restricted) information disclosure strategy* is a list $\phi = ((x, \sigma_x), (y, \sigma_y); \kappa)$ that identifies experimentation choices $(x, \sigma_x)$ and $(y, \sigma_y)$ for the two aspects, and a given aspect $\kappa$, which is actually selected for experimentation.

**Definition 1.** An *equilibrium* of the described information disclosure game is an information disclosure strategy $\phi^* = ((x, \sigma_x^*), (y, \sigma_y^*); \kappa^*)$ such that:

(i) *(ex ante) optimality constrained to incentive-compatibility*: for each aspect $\kappa \in \{x, y\}$, the decision rule $\sigma_\kappa^*$ solves the problem

$$\begin{aligned} \max_{\sigma_\kappa \in \Sigma_\kappa} \quad & V_\kappa(\sigma_\kappa) \equiv \sum_a \sum_\theta \sigma_\kappa(a \mid \kappa)\psi(\theta)v(a,\theta) \\ \text{s.t.:} \quad & (IC) \ \text{ for each } a_j \in A, \\ & \sum_\theta \sigma_\kappa(a_j \mid \kappa)\psi(\theta)\big[u(a_j,\theta) - u(a,\theta)\big] \geq 0 \qquad \forall\, a \neq a_j; \end{aligned} \tag{4}$$

(ii) *optimal aspect choice*: the Sender selects aspect $\kappa^*$ (over aspect $-\kappa$) for experimentation whenever

$$V_{\kappa^*}(\sigma_{\kappa^*}^*) \equiv \sum_a \sum_\theta \sigma_{\kappa^*}^*(a \mid \kappa)\psi(\theta)v(a,\theta) \geq \sum_a \sum_\theta \sigma_{-\kappa}^*(a \mid -\kappa)\psi(\theta)v(a,\theta) \equiv V_{-\kappa}(\sigma_{-\kappa}^*).$$

Equilibria of the proposed information disclosure game will in general be multiple. The optimization problem of $S$ has the form of a linear programming problem and, as usual in this class of problems, one can obtain multiple optimal decision rules that satisfy the required incentive-compatibility condition.[17] Thus, in a way totally analogous to the traditional information design benchmark, multiple aspect-restricted decision rules will typically solve $S$'s information design problem in Eq. (4). In fact, our leading examples featured this kind of multiplicity.

**Aspect-Restricted *vs* Complete Information Design.**—The recommendations of a decision rule $\sigma_\kappa$ over any fixed aspect $\kappa$ can also be made by a decision rule under complete commitment $\sigma$. This is very intuitive. All that a decision rule $\sigma$ needs to do in order to offer the same contingent action recommendation as a decision rule with aspect-restricted

[17] In geometric terms, the usual tangency between $S$'s indifference hyperplane, according to each directional slope, and the respective incentive-compatibility constraint gives this result.

commitment $\sigma_\kappa$ is simply to not condition its recommendations on the remaining random variable $-\kappa$. An information-designer with the ability to design information over the entire state of the world $\theta$ is less (incentive-compatible) constrained. In general, this higher degree of flexibility allows the Sender to attain higher (ex ante) utility. The formal arguments are detailed in Observation 1 below.

**Observation 1.** *Let us compare what can be attained by (i) a Sender who is restricted to choosing experiments over a single aspect (say aspect $\kappa = x$) and (ii) another Sender who can choose experiments conditional on the the entire state of the world $\theta = (x, y)$. The problem of the unconstrained Sender consists of picking decision rules (under complete commitment) $\sigma \in \Sigma$ that satisfy the incentive-compatibility condition expressed earlier in Eq. (1).*

*Interestingly, the Radon-Nikodym Theorem allows us to apply the definition of conditional probability to construct a decision rule under complete commitment $\sigma$ from any given decision rule under aspect-restricted commitment $\sigma_x$. This can be done by suitably selecting a family of conditional probability distributions, which must satisfy a Bayesian plausibility condition. Specifically, to construct $\sigma$ from a given $\sigma_x$, we can resort to a family of conditional distributions $\Lambda[\sigma, \sigma_x] \equiv \{\lambda(\cdot \mid a, x) \in \Delta(Y) \mid a \in A, x \in X\}$ such that each $\lambda \in \Lambda[\sigma, \sigma_x]$ satisfies the Bayesian plausibility condition: $\lambda(y \mid a, x) = \sigma(a \mid \theta)[1/\sigma_x(a \mid x)]\psi(y \mid x)$ for each $a \in A$ and each $\theta = (x, y) \in \Theta$. Then, if we start from some optimal decision rule $\sigma_x^* \in \Sigma_x$ under aspect-restricted experimentation, the incentive-compatibility constraint that a Sender with complete commitment power faces can be rewritten as*

$$\forall\, a_j \in A, \quad \sum_{\theta} \lambda(y \mid a_j, x)\sigma_x^*(a_j \mid x)\psi_x(x)\big[u(a_j, \theta) - u(a, \theta)\big] \geq 0 \quad \forall\, a \neq a_j,$$

*where $\lambda \in \Lambda[\sigma, \sigma_x]$. Therefore, instead of choosing directly a decision rule $\sigma^* \in \Sigma$, the Sender with the ability to choose experimentation simultaneously informative about both dimensions of uncertainty can resort to pick a decision rule $\sigma_x^* \in \Sigma_x$ and a family of conditional distributions $\lambda^* \in \Lambda[\sigma, \sigma_x]$ so as to solve the problem:*

$$\begin{aligned} &\max_{\{\lambda, \sigma_x\}} \sum_a \sum_\theta \lambda(y \mid a, x)\sigma_x^*(a \mid x)\psi_x(x)v(a, \theta) \\ &\textit{s.t.: for each } \ a_j \in A, \\ &\sum_\theta \lambda(y \mid a_j, x)\sigma_x^*(a_j \mid x)\psi_x(x)\big[u(a_j, \theta) - u(a, \theta)\big] \geq 0 \quad \forall a \neq a_j. \end{aligned} \tag{5}$$

*We observe that a Sender in the traditional information design has more flexibility relative* *to an information-designer in the proposed approach of aspect-restricted commitment. In particular, upon selecting $\lambda^*(y \mid a_j, x) = \psi(y \mid x)$ for each $a_j \in A$, and then choosing $\sigma_x^*$ so as to solve the problem in Eq. (4), a Sender in the complete commitment world has obviously also the ability to solve the problem in Eq. (5).*[18] *On the contrary, a Sender in the aspect-restricted world cannot solve the full commitment problem in Eq. (5) simply because he does not have the ability to choose the family of distributions $\lambda^* \in \Lambda[\sigma, \sigma_x]$.*

Let us use the particulars of our automobile company story to illustrate what experimentation committed simultaneously over both income and environmental concerns could attain.

## 4.1. What Could Complete Commitment Attain?

As noted in Observation 1, the set of (ex ante) expected utilities attainable under aspect-restricted experimentation is a subset of the set achievable under completely committed experimentation. Let us use the details of Examples I and II of Section 3. In other words, let us consider the two families of conditional distributions $\tau = \tau_x, \tau'_x$. Suppose now that the CEO can commit on experiments informative both about income growth ($x$) and environmental concerns ($y$), just as a Sender in the canonical information design framework can do. Set the short-hand notation $\sigma_h^\tau \equiv \sigma^\tau(a_0 \mid \theta_h) \in [0,1]$ to identify the probability according to which the decision rule selected by the CEO recommends to launch the new model given that the state of the world is $\theta_h$, for $h = 1, \dots, 9$.

If there is no correlation between income growth and environmental concerns (Example I, where $\tau = \tau_x$), then the problem that the CEO must solve to decide about experimentation simultaneously over both aspects is:

$$\max_{\{\sigma_1^{\tau_x}, \dots, \sigma_9^{\tau_x}\}} \frac{3}{18}\left[\sigma_1^{\tau_x} + \sigma_2^{\tau_x} + \sigma_3^{\tau_x}\right] + \frac{1}{18}\left[\sigma_4^{\tau_x} + \sigma_5^{\tau_x} + \sigma_6^{\tau_x}\right] + \frac{2}{18}\left[\sigma_7^{\tau_x} + \sigma_8^{\tau_x} + \sigma_9^{\tau_x}\right]$$
$$\text{s.t.:} \qquad 3\sigma_3^{\tau_x} + 3\sigma_6^{\tau_x} + 2\sigma_8^{\tau_x} + 2\sigma_9^{\tau_x} \geq 9\sigma_1^{\tau_x} + 9\sigma_2^{\tau_x} + 3\sigma_4^{\tau_x} + 3\sigma_5^{\tau_x} + 6\sigma_7^{\tau_x}.$$

The incentive-compatibility restriction above is simply the expression of the obedience condition that Bergemann and Morris (2013, 2016, 2019) propose to explore information design. In this case, the CEO would optimally choose $\sigma_3^{\tau_x *} = \sigma_6^{\tau_x *} = \sigma_8^{\tau_x *} = \sigma_9^{\tau_x *} = 1$, any

[18] Importantly, by choosing $\lambda^*(y \mid a, x) = \psi(y \mid x)$, the Sender will not necessarily solve the information design problem under full commitment (in Eq. (5)) but he will certainly solve the corresponding problem under aspect-restricted commitment (in Eq. (4)).

$\sigma_1^{\tau_x *}, \sigma_2^{\tau_x *} \in [0,1]$ such that $\sigma_1^{\tau_x *} + \sigma_2^{\tau_x *} = 8/9$, and $\sigma_4^{\tau_x *} = \sigma_5^{\tau_x *} = \sigma_7^{\tau_x *} = 0$. The CEO would accordingly receive an expected utility of 16/27.

On the other hand, if there is some correlation between income growth and environmental concerns (Example II, where $\tau = \tau_x'$), then the problem that the CEO must solve to decide about experimentation simultaneously over both aspects is:

$$\max_{\{\sigma_1^{\tau_x'}, \dots, \sigma_9^{\tau_x'}\}} \frac{6}{18}\sigma_1^{\tau_x'} + \frac{2}{18}\sigma_2^{\tau_x'} + \frac{3}{18}\sigma_5^{\tau_x'} + \frac{6}{18}\sigma_9^{\tau_x'}$$

$$\text{s.t.:} \qquad 2\sigma_9^{\tau_x'} \geq 6\sigma_1^{\tau_x'} + 3\sigma_2^{\tau_x'} + 3\sigma_5^{\tau_x'}.$$

In this case, the CEO would optimally choose $\sigma_9^{\tau_x' *} = 1$, and either (i) any $\sigma_2^{\tau_x' *}, \sigma_5^{\tau_x' *} \in [0,1]$ such that $\sigma_2^{\tau_x' *} + \sigma_5^{\tau_x' *} = 2/3$ and then $\sigma_1^{\tau_x' *} = 0$, or (ii) $\sigma_1^{\tau_x' *} = 1/3$ and then $\sigma_2^{\tau_x' *} = \sigma_5^{\tau_x' *} = 0$. The CEO would accordingly receive an expected utility of 4/9.

Unsurprisingly, the CEO would receive exactly the same expected utility under complete commitment than under aspect-restricted commitment if the two aspects are correlated according to $\tau_x'$. From the set of states displayed in Fig. 4 (for $\tau = \tau_x'$), we can appreciate that the incentives of the CEO and the Board are perfectly aligned under experimentation over environmental concerns (the $y$-aspect), regardless of what experimentation about income growth (the $x$-aspect) can disclose. It follows that experimentation only over environmental concerns enables the CEO to place an incentive-compatibility condition that allows the same expected utility as the one allowed under experimentation over both aspects combined. However, we also observe that, if the two aspects are independent ($\tau = \tau_x$), then experimentation over income growth alone delivers an expected utility (16/30) lower than what simultaneous experimentation over both aspects allows (16/27), as could be expected in general for a relatively more constrained information-designer.

# 5. Main Results

**Preliminary Definitions.**—It is convenient to introduce first some elements that capture relevant features of the priors and the players' preferences. Consider a given aspect $\kappa \in \{x, y\}$ and a fixed realization $\bar{\kappa} \in \mathcal{K}$ of such an aspect $\kappa$. For each player $i$, let me define the functions $\beta_\kappa^i(\cdot \mid \bar{\kappa}) : A \to \mathbb{R}$ as

$$\beta_\kappa^R(a \mid \bar{\kappa}) \equiv \sum_{-\kappa_l} \psi(\theta) u(a, \theta) \quad \text{and} \quad \beta_\kappa^S(a \mid \bar{\kappa}) \equiv \sum_{-\kappa_l} \psi(\theta) v(a, \theta) \qquad \text{for } \theta = (\bar{\kappa}, -\kappa_l). \tag{6}$$

The number $\beta^i_\kappa(a \mid \bar{\kappa})$ describes the extent to which player $i$ likes (or dislikes) action $a$ according to the priors, provided that there is perfect information that the realization of aspect $\kappa$ is $\bar{\kappa}$. Importantly, the definition of function $\beta^i_\kappa(\cdot \mid \bar{\kappa})$ requires that knowing that $\kappa = \bar{\kappa}$ is the *only* additional information beyond the priors that the players have. In particular, $\beta^i_\kappa(\cdot \mid \bar{\kappa})$ considers that the players receive no further information about the remaining aspect $-\kappa$. Then, if the only additional information beyond the priors that the players have is that the realization of aspect $\kappa$ is $\bar{\kappa}$, the expected utility of player $i$ from action $a$ increases with $\beta^i_\kappa(a \mid \bar{\kappa})$.

Let $\overline{A}^i_\kappa(\bar{\kappa}) \equiv \arg\max_{a \in A} \beta^i_\kappa(a \mid \bar{\kappa})$ be the set of actions most preferred by player $i$, conditional on knowing only that the realization of aspect $\kappa$ is $\bar{\kappa}$. To avoid nontrivial implications, the analysis will restrict attention to environments where, conditional on knowing only any given realization $\kappa_l$ of aspect $\kappa$, the Receiver is not indifferent among all available actions. Thus, we want to rule out the uninteresting cases where $A = \overline{A}^R_\kappa(\kappa_l)$ for any realization $\kappa_l \in \mathcal{K}$ of any aspect $\kappa \in \{x, y\}$.

The sets $\overline{A}^i_\kappa(\kappa_l)$ $(i = S, R)$ capture features about the players preferences over actions conditional on a particular realization $\kappa_l$ of aspect $\kappa$. In addition, we will be also interested in capturing some information about the Receiver's preferences across all possible realizations of any aspect $\kappa$. To this end, let us first consider an aspect $\kappa \in \{x, y\}$ and fix some action $a_j \in A$. Then, for each alternative action $a \neq a_j$, the term

$$\delta^{a_j,a}_{\bar{\kappa}} \equiv \beta^R_\kappa(a_j \mid \bar{\kappa}) - \beta^R_\kappa(a \mid \bar{\kappa}) = \sum_{-\kappa_l} \psi(\theta)\big[u(a_j,\theta) - u(a,\theta)\big] \quad \text{for } \theta = (\bar{\kappa}, -\kappa_l) \tag{7}$$

measures the expected utility change to the Receiver from choosing action $a_j$ rather than action $a$ if the only additional information beyond the priors that she has is that the realization of aspect $\kappa$ is $\kappa_l$. Secondly, let me specify then the set $\mathcal{B}_\kappa(a_j)$, which includes $n$ vectors, as

$$\mathcal{B}_\kappa(a_j) \equiv \Big\{ (\delta^{a_j,a}_{\kappa_l})_{\kappa_l \in \mathcal{K}} \in \mathbb{R}^m \;\Big|\; a \neq a_j \Big\}. \tag{8}$$

Note that the set of vectors $\mathcal{B}_\kappa(a_j)$ depends only on the Receiver's preferences and on the priors about the state. In particular, $\mathcal{B}_\kappa(a_j)$ captures the net gains (or loses) to the Receiver from action $a_j$ over each alternative action $a$, provided that the Receiver obtains perfect information only about aspect $\kappa$. Unsurprisingly, the set $\mathcal{B}_\kappa(a_j)$ is indicative of how hard would be for experimentation over aspect $\kappa$ to recommend action $a_j$ in an incentive-compatible way.

Let $co(\mathcal{B}_\kappa(a_j))$ denote the *convex hull* of the set of vectors $\mathcal{B}_\kappa(a_j)$.[19]

## 5.1. Optimal Aspect-Restricted Information Design

The two (equivalent) observations provided by Theorem 1 and Theorem 2 below characterize the existence of equilibria for the proposed game such that experimentation over a given aspect $\kappa$ is able to recommend (in an incentive-compatible sense) a given action $a_j$ with positive probability. The characterization results of Theorem 1 and Theorem 2 build upon foundational results in linear programming—in particular, upon work from the 1930's on the consistency of systems of linear inequalities (Stokes, 1931; Dines, 1936). The proofs of these and of all the remaining results of the paper are relegated to the Appendix.

**Theorem 1.** *Consider a given aspect $\kappa \in \{x, y\}$. There exists an information disclosure equilibrium $\phi^* = ((x, \sigma_x^*), (y, \sigma_y^*); \kappa^*)$ of the proposed game where experimentation over aspect $\kappa$ recommends a given action $a_j \in A$, conditional on realization $\kappa_l$, with (strictly) positive probability if and only if the following three conditions are satisfied:*

*(i)* $\beta_\kappa^S(a_j \mid \kappa_l) > 0$,

*(ii) for each alternative action $a \neq a_j$, there is some some realization $\kappa(a) \in \mathcal{K}$ of aspect $\kappa$ such that $\delta_{\kappa(a)}^{a_j,a} > 0$, and*

*(iii) the vector of zeroes of size $m$, $\underline{0}_m \equiv (0, \dots, 0) \in \mathbb{R}^m$, is not an interior point of the convex hull of the set $\mathcal{B}_\kappa(a_j)$—i.e., $\underline{0}_m \notin int(\ co(\mathcal{B}_\kappa(a_j))\ )$.*

Conditions (i) and (ii) of Theorem 1 provide necessary requirements for a given action $a_j$ to be recommended with positive probability by any optimal experiment over a given aspect $\kappa$. For $a_j \in A$, note that the set $\hat{\mathcal{K}}(a_j)$ defined as

$$\hat{\mathcal{K}}(a_j) \equiv \{\kappa_l \in \mathcal{K} \mid \delta_{\kappa_l}^{a_j,a} \geq 0 \quad \forall a \in A\} \tag{9}$$

captures the realizations of aspect $\kappa$ such that knowing only that $\kappa = \kappa_l$ induces $R$ to prefer action $a_j$ over *any* alternative. Then, for an action $a_j$ to be recommended with positive probability by optimal experimentation over aspect $\kappa$, the players' utility functions $u$ and $v$, and the priors $\psi$, must be such that (i) knowing only that $\kappa = \kappa_l$ makes action $a_j$ valuable to $S$—i.e., $\beta_\kappa^S(a_j \mid \kappa_l) > 0$—and (ii) the set $\hat{\mathcal{K}}(a_j)$ is nonempty. Notably, the aspect

[19] Recall that the convex hull of a set of vectors is the greatest common subset of all the convex sets that contain the set of vectors.

realization(s) $\kappa_l$ that lead(s) to $\beta^S_\kappa(a_j \mid \kappa_l) > 0$ need(s) not belong to the set $\hat{\mathcal{K}}(a_j)$. In other words, the players must agree on that action $a_j$ is suitable conditional on some pieces of data from experimentation but they do not need to agree on the particular pieces of data that make such an action a suitable choice.

When the primitives of the model are such that conditions (i) and (ii) of Theorem 1 are satisfied, we say that there exist pieces of data from experimentation over aspect $\kappa$ that lead $S$ and $R$ to "agree on that $a_j$ is a suitable action to choose." Note that for the players to agree on a particular action being suitable conditional on experimentation, $S$ is not required to prefer such an action over the alternatives. In this sense, the role of $S$'s preferences in the agreement is arguably less relevant as we only need $S$ not to receive disutility from such an action. The role of $R$'s preferences, though, is crucial for the agreement because the selected experiment must make such an action preferred over *any* alternative conditional on some aspect realization.

An intuitive insight of information design is that it becomes harder to persuade as the number of available actions increases. Condition (iii) of Theorem 1 deals with the possibility of having conflicting incentive-compatible conditions. Obviously, ending up with conflicting incentive-compatible conditions is facilitated when the number of available actions increases. To illustrate geometrically this message, as well as the necessary and sufficient conditions given by Theorem 1, consider an example where experimentation is conducted over aspect $\kappa = x$. and suppose that $X = \{x_1, x_2\}$—i.e., $m = 2$. Consider that $\beta^S_\kappa(a_0 \mid x_2) > 0$ so that $S$ wishes that his experimentation choice be able to recommend action $a_0$ conditional on $x_2$ in a way that makes $R$ follow the recommendation. Take first a possible set of actions $A = \{a_0, a_1, a_2\}$—i.e., $n = 2$. Fig. 5 shows a situation where, from the results of Theorem 1,

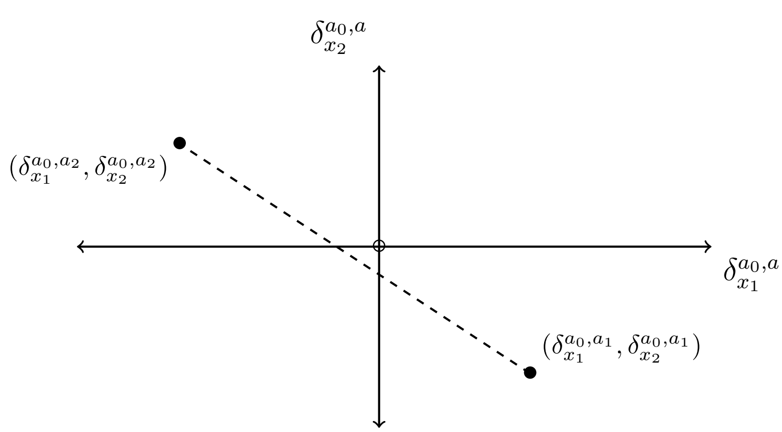


**Figure 5** – Theorem 1: Two alternative actions

experimentation is able to induce $R$ to choose action $a_0$ with probability one. In this case, the set $\mathcal{B}_x(a_0)$ is given by

$$\mathcal{B}_x(a_0) = \{(\delta_{x_1}^{a_0,a_1}, \delta_{x_2}^{a_0,a_1}), (\delta_{x_1}^{a_0,a_2}, \delta_{x_2}^{a_0,a_2})\}$$

and thus $co(\mathcal{B}_x(a_0))$ is geometrically given by the dashed segment that connects points $(\delta_{x_1}^{a_0,a_1}, \delta_{x_2}^{a_0,a_1})$ and $(\delta_{x_1}^{a_0,a_2}, \delta_{x_2}^{a_0,a_2})$. Since $\overline{A}_x^S(x_2) = \{a_0\}$, it follows that the Sender wishes to select optimally $\sigma_x(a_0 \mid x_2) = 1$. Then, for an experiment that includes the feature $\sigma_x(a_0 \mid x_2) > 0$ to be (incentive-compatible) feasible, the following system of inequalities must be satisfied

$$\begin{aligned} &\delta_{x_1}^{a_0,a_1}\sigma_x(a_0 \mid x_1) + \delta_{x_2}^{a_0,a_1}\sigma_x(a_0 \mid x_2) \geq 0; \\ &\delta_{x_1}^{a_0,a_2}\sigma_x(a_0 \mid x_1) + \delta_{x_2}^{a_0,a_2}\sigma_x(a_0 \mid x_2) \geq 0. \end{aligned} \tag{10}$$

Upon the appropriate relabeling of terms, the system of inequalities in Eq. (10) gives us the incentive-compatible conditions expressed in Eq. (4) for the case where $\sigma_x(a_0 \mid x_2) > 0$ in the optimal information design choice. Note first that if $\sigma_x(a_0 \mid x_2) > 0$ in the chosen experiment, then both inequalities in Eq. (10) can be satisfied only if (i) at least one of the terms $\delta_{x_1}^{a_0,a_1}$ or $\delta_{x_2}^{a_0,a_1}$ is positive and (ii) at least one of the terms $\delta_{x_1}^{a_0,a_2}$ or $\delta_{x_2}^{a_0,a_2}$ is positive. From Fig. 5, we observe that $\delta_{x_1}^{a_0,a_1} > 0$ and $\delta_{x_2}^{a_0,a_2} > 0$ so that the two necessary conditions (i) and (ii) are met. However, the system of inequalities in Eq. (10) could still be inconsistent or, in other words, one of the inequalities could contradict the other one. Foundational developments in convex hulls and linear inequalities provide necessary and sufficient conditions for such a system of inequalities to be consistent. In particular, the conditions provided by Dines (1936) (Theorem 2) show that the system in Eq. (10) is consistent if and only if $\underline{0}_2 \notin int(\ co(\mathcal{B}_x(a_0))\ )$. Fig. 5 illustrates that this is the case in this example.

Secondly, consider another situation where we add one more action $a_3$ to the previous set of actions $A$ and thus consider a different set $A' = \{a_0, a_1, a_2, a_3\}$—i.e., now $n = 3$. Fig. 6 shows a situation where, using Theorem 1, we observe that experimentation is not able to induce $R$ to choose action $a_0$ with positive probability. In this case, we have

$$\mathcal{B}_x(a_0) = \{(\delta_{x_1}^{a_0,a_1}, \delta_{x_2}^{a_0,a_1}), (\delta_{x_1}^{a_0,a_2}, \delta_{x_2}^{a_0,a_2}), (\delta_{x_1}^{a_0,a_3}, \delta_{x_2}^{a_0,a_3})\}$$

and thus $co(\mathcal{B}_x(a_0))$ corresponds to the triangular area delimited by the dashed segments that connects the there points $(\delta_{x_1}^{a_0,a_1}, \delta_{x_2}^{a_0,a_1})$, $(\delta_{x_1}^{a_0,a_2}, \delta_{x_2}^{a_0,a_2})$, and $(\delta_{x_1}^{a_0,a_3}, \delta_{x_2}^{a_0,a_3})$. Suppose that

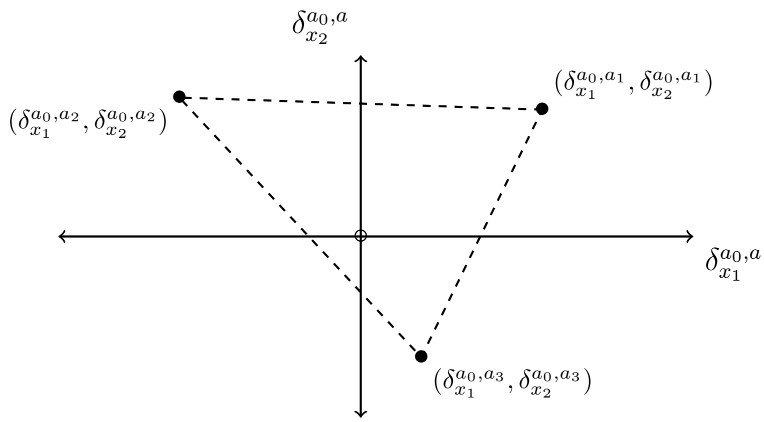


**Figure 6** – Theorem 1: Three alternative actions

we continue to have $\overline{A}_x^S(x_2) = \{a_0\}$, so that that the Sender wishes to select optimally $\sigma_x(a_0 \mid x_2) = 1$. Now, for an experiment that includes the feature $\sigma_x(a_0 \mid x_2) > 0$ to be (incentive-compatible) feasible, the following system of inequalities must be satisfied

$$\begin{aligned} \delta_{x_1}^{a_0,a_1}\sigma_x(a_0 \mid x_1) + \delta_{x_2}^{a_0,a_1}\sigma_x(a_0 \mid x_2) \geq 0; \\ \delta_{x_1}^{a_0,a_2}\sigma_x(a_0 \mid x_1) + \delta_{x_2}^{a_0,a_2}\sigma_x(a_0 \mid x_2) \geq 0; \\ \delta_{x_1}^{a_0,a_3}\sigma_x(a_0 \mid x_1) + \delta_{x_2}^{a_0,a_3}\sigma_x(a_0 \mid x_2) \geq 0. \end{aligned} \tag{11}$$

Note first that since $\delta_{x_1}^{a_0,a_1} > 0$, $\delta_{x_2}^{a_0,a_1} > 0$, $\delta_{x_2}^{a_0,a_2} > 0$, and $\delta_{x_1}^{a_0,a_3} > 0$, it follows that the three inequalities in Eq. (11) could in principle be satisfied when $\sigma_x(a_0 \mid x_2) > 0$. Yet, intuitively, having an additional action makes it harder to provide $R$ with the required incentive to choose $a_0$ over any alternative. In particular, for the example illustrated by Fig. 6, we observe that $\underline{0}_2 \in int(\ co(\mathcal{B}_x(a_0))\ )$. Therefore, using the insights of Theorem 2 of Dines (1936), it follows that the system in Eq. (11) is inconsistent and, thus, $S$ is not able in this case to pick an experiment that recommends action $a_0$ with positive probability. Enlarging the set of available actions of a decision problem makes it harder for an experiment to persuade $R$. Compared to a system as the one in Eq. (10), it is more difficult for a system of inequalities such as the one in Eq. (11) to meet its internal consistency requirements.

Alternatively to condition (iii) of Theorem 1, which characterizes when the required incentive-compatible conditions are consistent—yet in a totally analogous way—, we can follow a (slightly) different approach. Take a given aspect $\kappa \in \{x, y\}$ and a given action

$a_j \in A$. Consider the following (homogenous) linear system of equalities:

$$\sum_{a \neq a_j} \delta_{\kappa_l}^{a_j,a} \, q_\kappa(a_j, a) = 0 \qquad\qquad (l = 1, \ldots, m). \tag{12}$$

The system in Eq. (12) above consists of $m$ equations with $n$ unknowns, $q_\kappa(a_j, a) \in \mathbb{R}$, for $a \neq a_j$. Then, as an alternative to condition (iii) of Theorem 1, we can study the signs of the solutions $q^*_\kappa(a_j, a)$ to the system in Eq. (12) to establish whether experimentation over a selected aspect is capable of making recommendations to the Receiver such that she receives the right informational incentives to choose action $a_j$. The logic behind the result in Theorem 2 (iii) below rests in a duality argument to the result of Theorem 1 (iii).

**Theorem 2.** *Consider a given aspect $\kappa \in \{x, y\}$. There exists an information disclosure equilibrium $\phi^* = ((x, \sigma^*_x), (y, \sigma^*_y); \kappa^*)$ of the proposed game where experimentation over aspect $\kappa$ recommends action $a_j \in A$, conditional on realization $\kappa_l$, with (strictly) positive probability if and only if the following three conditions are satisfied:*

*(i)* $\beta^S_\kappa(a_j \mid \kappa_l) > 0$,

*(ii) for each alternative action $a \neq a_j$, there is some realization $\kappa(a) \in \mathcal{K}$ of aspect $\kappa$ such that $\delta^{a_j,a}_{\kappa(a)} > 0$, and*

*(iii) the system of linear equations in Eq. (12) has at least one solution $q^*_\kappa(a_j) \equiv (q^*_\kappa(a_j, a))_{a \neq a_j}$ such that $q^*_\kappa(a_j, a) < 0$ for some $a \neq a_j$.*

The following Corollary 1 to Theorem 2 provides interesting sufficient conditions for optimal experimentation to be able to recommend a particular action $a_j$ that is beneficial for $S$ conditional on a particular piece of data $\kappa_l$ from experimentation. The required condition is that such a given action be the most beneficial one for $R$ conditional on some piece of data $\kappa_s$ which, interestingly, needs not coincide with $\kappa_l$.

**Corollary 1.** *Consider a given aspect $\kappa \in \{x, y\}$ and suppose that some action $a_j$ is beneficial for the Sender conditional on some realization $\kappa_l$ of the aspect, $\beta^S_\kappa(a_j \mid \kappa_l) > 0$. If action $a_j$ is one of the most beneficial actions for the Receiver conditional on some realization $\kappa_s$ of the aspect, i.e., $a_j \in \overline{A}^R_\kappa(\kappa_s)$, then action $a_j$ is recommended with strictly positive probability, conditional on realization $\kappa_l$, by experimentation over aspect $\kappa$ at any equilibrium $\phi^* = ((x, \sigma^*_x), (y, \sigma^*_y); \kappa^*)$ of the information disclosure game.*

# 6. A Duality Approach

Interesting implications can be drawn by studying the dual problem of the information design problem specified in Eq. (4). The analysis of this Section 6 builds closely upon the approach developed by Galperti and Perego (2018) for the traditional information design setup under complete commitment. To rewrite $S$'s optimization problem as a primal problem for each given aspect $\kappa \in \{x, y\}$, it is convenient to reinterpret the action recommendations of the selected experiments as joint distributions over actions and aspect realizations. Thus, for each (aspect-restricted) decision rule $\sigma_\kappa$, let $\gamma_\kappa(a, \kappa_l) \equiv \sigma_\kappa(a \mid \kappa_l)\psi_\kappa(\kappa_l)$ be the *distribution over actions and aspect realizations*, or *outcome*, induced by the decision rule $\sigma_\kappa$ and the marginal prior $\psi_\kappa$. Then, the primal linear programming problem, which I now denote by $\mathcal{P}$, described earlier in Eq. (4) can be conveniently expressed as

$$\begin{aligned}
\mathcal{P} \equiv \max_{\{\gamma_\kappa(a,\kappa_l)\}} \quad & \sum_a \sum_{\kappa_l} \gamma_\kappa(a, \kappa_l) \sum_{-\kappa_l} \psi(-\kappa_l \mid \kappa_l)\, v(a, (\kappa_l, -\kappa_l)) \\
& \text{s.t.:} \qquad \forall a_j, a \in A: \\
(IC) \quad & \sum_{\kappa_l} \gamma_\kappa(a_j, \kappa_l) \sum_{-\kappa_l} \psi(-\kappa_l \mid \kappa_l) \Big[u(a_j, (\kappa_l, -\kappa_l)) - u(a, (\kappa_l, -\kappa_l))\Big] \geq 0; \\
& \sum_a \gamma_\kappa(a, \kappa_l) = \psi_\kappa(\kappa_l); \\
& \gamma_\kappa(a, \kappa_l) \geq 0.
\end{aligned} \tag{13}$$

To derive the dual problem of $\mathcal{P}$, we need to consider two sets of new *control variables*. In particular, for a given aspect $\kappa \in \{x, y\}$, let $p_\kappa : \mathcal{K} \to \mathbb{R}$ and let $\mu_\kappa : A \times A \to \mathbb{R}$ be the functions that describe the control variables of the dual problem for experimentation over aspect $\kappa$. Using the terms introduced in Eq. (6) and Eq. (7), the dual, denoted by $\mathcal{D}$, to the primal problem $\mathcal{P}$ in Eq. (13) is then

$$\begin{aligned}
\mathcal{D} \equiv \min_{\{p_\kappa(\kappa_l)\}} \quad & \sum_{\kappa_l} \psi_\kappa(\kappa_l) p_\kappa(\kappa_l) \\
& \text{s.t.:} \qquad \forall a_j \in A, \quad \forall \kappa_l \in \mathcal{K}: \\
(D) \quad & \psi_\kappa(\kappa_l) p_\kappa(\kappa_l) \geq \beta_\kappa^S(a_j \mid \kappa_l) + \sum_{a \neq a_j} \delta_{\kappa_l}^{a_j, a} \mu_\kappa(a_j, a); \\
& \mu_\kappa(a_j, a) \geq 0.
\end{aligned} \tag{14}$$

Each control variable $p_\kappa(\kappa_l)$ is associated to the constraint $\sum_{a \in A} \gamma_\kappa(a, \kappa_l) = \psi_\kappa(\kappa_l)$ of the primal problem. Also, each control variable $\mu_\kappa(a_j, a)$ corresponds to the incentive-compatibility condition $(IC)$ of problem $\mathcal{P}$, which provides $R$ with the informational incentives to choose action $a_j$ over the alternative $a$. As usual in any duality approach, the variables $p_\kappa(\kappa_l)$ have the interpretation of "shadow prices" for $S$'s decision. In the dual problem $\mathcal{D}$, the Sender wishes to choose the lowest values of each $p_\kappa(\kappa_l)$ so as to satisfy the constraint in $(D)$ of Eq. (14) above. Importantly, we can resort to the saddle point theorem of linear programming and make use of the complementary slackness conditions to characterize the optimal solutions to both problems $\mathcal{P}$ and $\mathcal{D}$ by the two following conditions:

$$\begin{aligned}&(c1)\;\; \mu^*_\kappa(a_j, a)\sum_{\kappa_l}\sigma^*_\kappa(a_j \mid \kappa_l)\delta^{a_j,a}_{\kappa_l} = 0 \quad \forall a_j, a \in A;\\ &(c2)\;\; \gamma^*_\kappa(a_j, \kappa_l)\Big[\psi_\kappa(\kappa_l)p^*_\kappa(\kappa_l) - \beta^S_\kappa(a_j \mid \kappa_l) - \sum_{a\neq a_j}\delta^{a_j,a}_{\kappa_l}\mu^*_\kappa(a_j, a)\Big] = 0 \quad \forall \kappa_l \in \mathcal{K}.\end{aligned} \tag{15}$$

To fix ideas about the logic behind the dual problem $\mathcal{D}$, consider first a hypothetical situation with no conflict of interests whatsoever between $S$ and $R$, conditional on experimentation over a given aspect $\kappa$. In particular, consider a situation where, for each action $a^*_S \in \overline{A}^S_\kappa(\kappa_l)$, it is the case that $\delta^{a^*_S,a}_{\kappa_l} \geq 0$. In other words, conditional on knowing only that $\kappa = \kappa_l$, $R$ prefers one of $S$'s most preferred actions, $a^*_S$, over any alternative. Then, from the constraint $(D)$ of the dual problem in Eq. (14), we observe that the $S$ would optimally choose $\mu^*_\kappa(a^*_S, a) = 0$ for each action $a^*_S \in \overline{A}^S_\kappa(\kappa_l)$ and each alternative $a \neq a^*_S$. Then, the optimal expected utility of $S$ given experimentation over aspect $\kappa$ would be equal to $\sum_{\kappa_l}\beta^S_\kappa(a^*_S \mid \kappa_l) = \sum_{\kappa_l}\psi_\kappa(\kappa_l)p_\kappa(\kappa_l)$, where $a^*_S \in \max_{a\in A}\sum_{-\kappa_l}\psi(\theta)v(a, \theta)$ for $\theta = (\kappa_l, -\kappa_l)$. In this (uninteresting) situation, the Sender would receive the highest possible expected utility conditional on each realization $\kappa_l$.

The model, though, aims at capturing situations where there is indeed a conflict of interests between $S$ and $R$. Suppose then that, for a given action $a^*_S \in \overline{A}^S_\kappa(\kappa_l)$, there is some alternative action $a$ such that $\delta^{a^*_S,a}_{\kappa_l} < 0$ so that, conditional on knowing only that $\kappa_l$ is the actual realization of aspect $\kappa$, the Receiver strictly prefers action $a$ over action $a^*_S$. In this case, we observe from the constraint $(D)$ of the problem Eq. (14) that the Sender finds optimal to choose $\mu^*_\kappa(a^*_S, a) > 0$. By doing so, $S$ would receive an expected utility $\beta^S_\kappa(a^*_S \mid \kappa_l) = \sum_{-\kappa_l}\psi(\theta)v(a^*_S, \theta)$ for $\theta = (\kappa_l, -\kappa_l)$. Such an optimal expected utility of $S$ satisfies $\beta^S_\kappa(a^*_S \mid \kappa_l) > \psi_\kappa(\kappa_l)p_\kappa(\kappa_l)$. This implication is in fact a central message of the classical information design approach under complete commitment (Kamenica and Gentzkow, 2011; Bergemann and Morris, 2016; Taneva, 2018). In particular, when Sender and Receiver have conflicting interests, it is beneficial for $S$ to choose experiments that hide some information—"obfuscation." Hiding information leads $R$ to make "mistakes" and thus choose an action such as $a_S^*$, which she would certainly dislike conditional on learning that $\kappa = \kappa_l$. Conditional on hiding some (restricted) information, it is also optimal for $S$ that the selected information structures do in fact reveal (partially) some information. The dual problem $\mathcal{D}$ then captures that $S$ "pays a price" $p_\kappa(\kappa_l)$ in order to induce $R$ to choose an action that is harmful for herself, conditional on knowing only the realization $\kappa_l$ of the aspect. Because of this cost, there is then a limit to the extent to which $S$ can induce $R$ to make these mistakes. This restriction is captured by the condition $(c1)$ in Eq. (15). We observe from $(c1)$ that the $S$ can choose $\mu_\kappa^*(a_S^*, a) > 0$ if and only if $\sum_{\kappa_l} \sigma_\kappa^*(a_S^* \mid \kappa_l)\delta_{\kappa_l}^{a_S^*,a} = 0$. In other words, provided that $S$ is restricted to choose experiments over an aspect $\kappa$, $S$ optimally provides $R$ with "obfuscation" by designing an experiment $\sigma_\kappa^*$ that discloses new information up to the point where $R$ is made indifferent between choosing $a_S^*$ and $a$.

For the two action-model, the costs captured by $p_\kappa(\kappa_l)$ in the dual problem play an important role in optimal experimentation that recommends an action which is harmful for $R$ conditional on some aspect realizations. In particular, for one of two available actions, say $a_0$, and a set $\mathcal{K}_R^-$ of aspect realizations $\kappa_l$ conditional on which $R$ dislikes action $a_0$, $S$ will have to pay the "shadow prices" $p_\kappa(\kappa_l)$ to select experiments that recommend $a_0$ conditional on such realizations $\kappa_l \in \mathcal{K}_R^-$. Because of such costs $p_\kappa(\kappa_l)$, $S$ must optimally pick the aspects $\tilde{\kappa} \in \mathcal{K}_R^-$ that recommend action $a_0$ by ensuring that such realizations $\tilde{\kappa}$ maximize a ratio that captures the net gains for $S$ from $a_0$—conditional on the realizations $\kappa_l \in \mathcal{K}_R^-$—, relative to the net loses induced on $R$. Intuitively, since inducing $R$ to choose the action that she dislikes is costly, optimal experimentation needs to suitably gauge what $S$ gains relative to what $R$ looses. This particular insight will be provided by Theorem 3 and Corollary 3.

The two necessary and sufficient conditions conditions in Eq. (15) allow us to derive Corollary 2 below. Loosely, the implication of Corollary 2 can be viewed as a counterpart of the result in Corollary 1.

**Corollary 2.** *Consider a given realization $\kappa_l \in \mathcal{K}$ of some aspect $\kappa \in \{x, y\}$ and a given action $a_j \in A$. If there exists an alternative action $a_s \neq a_j$ such that*

$$\beta_\kappa^S(a_s \mid \kappa_l) + \sum_{a \neq a_s} \mu_\kappa(a_s, a)\delta_{\kappa_l}^{a_s,a} > \beta_\kappa^S(a_j \mid \kappa_l) + \sum_{a \neq a_j} \mu_\kappa(a_j, a)\delta_{\kappa_l}^{a_j,a} \tag{16}$$

*for each $\mu_\kappa : A \times A \to \mathbb{R}_+$, then each optimal experiment $\sigma_\kappa^* \in \Sigma_\kappa$ satisfies $\sigma_\kappa^*(a_j \mid \kappa_l) = 0$.*

In turn, by using the insight of Corollary 2, we can propose a sufficient condition, which has the flavor of a welfare consideration, under which a given action *is not recommended* under *any* (aspect-restricted) experiment conditional on a given realization of the aspect. More precisely, a given action $a_j$ will not be recommended by any experiment conditional on a given realization $\kappa_l$ of aspect $\kappa$ if there exists another action $a_s$ such that knowing that $\kappa = \kappa_l$ leads to that choosing $a_s$ is a particular welfare improvement relative to $a_j$. To propose the particular form of welfare improvement, note that the result of Corollary 2 enables us to restrict attention to functions $\mu_\kappa$ such that $\mu_\kappa(a, a') = 1/n$ for each $a, a' \in A$. Then, for each action $a \in A$ and each aspect realization $\kappa_l \in \mathcal{K}$, we can specify the function

$$W(a, \kappa_l) \equiv \beta_\kappa^S(a \mid \kappa_l) + \beta_\kappa^R(a \mid \kappa_l) - \frac{1}{n} \sum_{a' \neq a} \beta_\kappa^R(a' \mid \kappa_l).$$

Intuitively, function $W(a, \kappa_l)$ gives us a "social welfare measure" at the interim stage of the information disclosure game, conditional on $S$ and $R$ knowing only that the realization of aspect $\kappa$ is $\kappa_l$. Notice that the term $\beta_\kappa^S(a \mid \kappa_l) + \beta_\kappa^R(a \mid \kappa_l)$ is simply the sum of the conditional expected utilities of both players—i.e., $\sum_{-\kappa_l} \psi(\theta)[v(a, \theta) + u(a, \theta)]$ for $\theta = (\kappa_l, -\kappa_l)$. On the other hand, the term $(1/n) \sum_{a' \neq a} \beta_\kappa^R(a' \mid \kappa_l)$ captures the (average) conditional expected utility that $R$ obtains by not choosing action $a$—i.e., $(1/n) \sum_{a' \neq a} \sum_{-\kappa_l} \psi(\theta) u(a', \theta)$ for $\theta = (\kappa_l, -\kappa_l)$. Then, an interesting message of Corollary 2 is that a given action $a_j$ will not be recommended by optimal experimentation over aspect $\kappa$, conditional on $\kappa = \kappa_l$, if the pair $(a, \kappa_l)$ does not maximize the proposed social welfare function $W(a, \kappa_l)$.

# 7. The Two-Action Case

The case $n = 1$ typically capures situations where the Receiver must decide whether to accept or reject a certain proposal. This is a benchmark case extensively studied by the Bayesian persuasion/information design models under complete commitment. Most applications and results in the literature have been developed for this two-action case.

Suppose, without loss of generality, that $a_1$ is a default action—a *status quo*—and that action $a_0$ means accepting a new proposal over the status quo. To capture interesting situations with conflict of interests between the players, consider (again, without loss of generality) that, according to the priors, $S$ prefers action $a_0$ whereas $R$ is more inclined towards action $a_1$. Specifically,

**Assumption 1.** *The prior about the state and the preferences of Sender and Receiver satisfy:*

*(i)* $\sum_\theta \psi(\theta)v(a_0,\theta) > \sum_\theta \psi(\theta)v(a_1,\theta)$;

*(ii)* $\sum_\theta \psi(\theta)u(a_0,\theta) < \sum_\theta \psi(\theta)u(a_1,\theta)$.

Note that requirement (ii) of Assumption 1 can be equivalently rewritten as $\sum_{\kappa_l} \delta_{\kappa_l}^{a_0,a_1} < 0$ for any given aspect $\kappa \in \{x,y\}$. Take a given aspect $\kappa \in \{x,y\}$. For each player $i = S, R$, we shall be interested in the following subsets of aspect realizations:

$$\mathcal{K}_i^+ \equiv \{\kappa_l \in \mathcal{K} \mid \beta_\kappa^i(a_0 \mid \kappa_l) \geq \beta_\kappa^i(a_1 \mid \kappa_l)\};$$
$$\mathcal{K}_i^- \equiv \{\kappa_l \in \mathcal{K} \mid \beta_\kappa^i(a_0 \mid \kappa_l) < \beta_\kappa^i(a_1 \mid \kappa_l)\}.$$

In addition, it will be useful to consider the ratio

$$\phi_\kappa(\kappa_l) \equiv \frac{\beta_\kappa^S(a_0 \mid \kappa_l) - \beta_\kappa^S(a_1 \mid \kappa_l)}{\beta_\kappa^R(a_1 \mid \kappa_l) - \beta_\kappa^R(a_0 \mid \kappa_l)}, \tag{17}$$

which captures the utility change for $S$ from choosing the "acceptance" action $a_0$ rather than $a_1$ over the utility change for $R$ from choosing the "status quo" $a_1$ rather than $a_0$. Then, the set $\tilde{\mathcal{K}} \equiv \arg\max_{\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-} \phi_\kappa(\kappa_l)$ consists of those aspect realizations $\kappa_l$ that satisfy the following two conditions: (1) conditional on any $\kappa_l \in \tilde{\mathcal{K}}$, $R$ prefers action $a_1$ over $a_0$ and (2) any $\kappa_l \in \tilde{\mathcal{K}}$ yields the highest ratio of utility increase for $S$ from choosing $a_0$, relative to the utility increase of $R$ from choosing the alternative $a_1$. Intuitively, suppose that we restrict attention to data from experimentation upon which $R$ wants to choose the status quo action $a_1$. Then, $\tilde{\mathcal{K}}$ captures the subset of such data attainable by experimentation over aspect $\kappa$ associated to the highest marginal increase in $S$'s utility relative to the induced decrease in $R$'s utility. Also, let $\Phi_\kappa \equiv \phi_\kappa(\kappa_l)$, for $\kappa_l \in \tilde{\mathcal{K}}$, be the value function of the problem $\max_{\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-} \phi_\kappa(\kappa_l)$.

Provided that experimentation over aspect $\kappa$ is actually able to persuade $R$ to pick $a_0$, it seems natural then that $S$ wants so select experiments that place positive probability on the realizations $\kappa_l \in \tilde{\mathcal{K}}$.

**Theorem 3.** *Consider the case where $A = \{a_0, a_1\}$ and assume Assumption 1.*

*(i) An optimal experiment over aspect $\kappa$ is able to recommend action $a_0$ with positive probability if and only if both sets $\mathcal{K}_S^+$ and $\mathcal{K}_R^+$ are nonempty.*

*(ii) If both sets $\mathcal{K}_S^+$ and $\mathcal{K}_R^+$ are nonempty, then any optimal experiment $\sigma_\kappa^*$ over aspect $\kappa$ is such that: (a) $\sigma_\kappa^*(a_0 \mid \kappa_l) = 0$ for each $\kappa_l \in \mathcal{K}_S^-$; (b) $\sigma_\kappa^*(a_0 \mid \kappa_l) = 1$ for each*

*$\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+$, provided that $\mathcal{K}_S^+ \cap \mathcal{K}_R^+ \neq \emptyset$; (c) for all aspect realizations $\tilde{\kappa} \notin \mathcal{K}_R^+$, it follows that $\sigma_\kappa^*(a_0 \mid \tilde{\kappa}) \in [0,1)$ if $\tilde{\kappa} \in \tilde{\mathcal{K}}$, with the restriction that $\sum_{\tilde{\kappa} \in \tilde{\mathcal{K}}} \sigma_\kappa^*(a_0 \mid \tilde{\kappa}) = \bar{\varrho}_\kappa$ with*

$$\bar{\varrho}_\kappa \equiv \frac{\sum_{\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+} [\beta_\kappa^R(a_0 \mid \kappa_l) - \beta_\kappa^R(a_1 \mid \kappa_l)]}{\beta_\kappa^R(a_1 \mid \bar{\kappa}) - \beta_\kappa^R(a_0 \mid \bar{\kappa})},$$

*where $\bar{\kappa}$ is some arbitrarily chosen $\bar{\kappa} \in \tilde{\mathcal{K}}$. Moreover, if there is a single aspect realization $\tilde{\kappa}$ such that $\tilde{\mathcal{K}} = \{\tilde{\kappa}\}$, then $\sigma_\kappa^*(a_0 \mid \tilde{\kappa}) = \bar{\varrho}_\kappa$; (d) for all $\kappa_l \notin \mathcal{K}_R^+$ such that $\kappa_l \notin \tilde{\mathcal{K}}$, it follows that $\sigma_\kappa^*(a_0 \mid \kappa_l) = 0$.*

Condition (i) of Theorem 3 states that a (restricted) experiment $\sigma_\kappa$ can recommend the "acceptance" action $a_0$ if and only if there is data available from experimentation over aspect $\kappa$ conditional on which both $S$ and $R$ agree on that action $a_0$ is the most suitable one. Interestingly, experimentation is able to make such incentive-compatible recommendation even if the pieces of data from experimentation that would make $S$ and $R$ prefer action $a_0$ do not coincide. This is a message already conveyed by the more general setup.

Condition (ii)-(b) ensures that $S$'s information design problem has solutions where the chosen experiments recommend action $a_0$ with probability one, conditional on some aspect realization(s). The requirement that the set $\mathcal{K}_S^+ \cap \mathcal{K}_R^+$ be nonempty means that there is always at least one realization of the chosen aspect $\kappa$ conditional on which the interests of the two players are perfectly aligned: both $S$ and $R$ prefer action $a_0$.

Condition (ii)-(c) of Theorem 3 establishes the criterion under which the experiment selected by $S$ recommends acceptance with non zero (yet, less than one) probability, conditional on aspect realizations other than the ones in the agreement set $\mathcal{K}_S^+ \cap \mathcal{K}_R^+$. As already mentioned, $S$ wants to pick experiments that disclose pieces of data $\kappa_l \in \tilde{\mathcal{K}}$ associated to the highest increase in $S$'s utility relative to the induced decrease in $R$'s utility.

Which aspect would the Sender optimally choose when he is constrained to selecting a single one for experimentation in an "accept-reject" action setup? For those cases where the primitives of the model enable the Sender to persuade the Receiver to pick action $a_0$ by designing information over any of the two aspects of uncertainty, Theorem 4 provides the criterion that determines the optimal aspect choice.

**Theorem 4.** *Consider the case where $A = \{a_0, a_1\}$ and assume Assumption 1. Suppose that for each aspect $\kappa \in \{x, y\}$, both sets $\mathcal{K}_S^+$ and $\mathcal{K}_R^+$ are nonempty so that optimal experimentation over any of the two aspects $\kappa$ is able to recommend action $a_0$ with positive* *probability. Then, the optimal aspect for experimentation is the aspect $\kappa^*$ that maximizes the corresponding value function $V_\kappa(\sigma^*_\kappa)$, which has the form*

$$V_\kappa(\sigma^*_\kappa) = \sum_{\kappa_l} \beta^S_\kappa(a_1 \mid \kappa_l) + \sum_{\kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^+_R} \left[\beta^S_\kappa(a_0 \mid \kappa_l) - \beta^S_\kappa(a_1 \mid \kappa_l)\right]$$
$$+ \sum_{\kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^+_R} \left[\beta^R_\kappa(a_0 \mid \kappa_l) - \beta^R_\kappa(a_1 \mid \kappa_l)\right] \times \max_{\kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^-_R} \left\{\frac{\beta^S_\kappa(a_0 \mid \kappa_l) - \beta^S_\kappa(a_1 \mid \kappa_l)}{\beta^R_\kappa(a_1 \mid \kappa_l) - \beta^R_\kappa(a_0 \mid \kappa_l)}\right\}.$$

A case of particular interest in many applications with two available actions is one where the set of possible states is partitioned into two sets, an "acceptace" set $\Theta_0$ and its complement $\Theta_1$. In this case, typically $S$ prefers action $a_0$ for all possible states $\theta \in \Theta$, whereas $R$ prefers $a_0$ if $\theta \in \Theta_0$ and $a_1$ if $\theta \in \Theta_1$. The following Assumption 2 captures these situations. In particular, the preferences of the players in our leading example of Section 3 satisfy both Assumption 1 and Assumption 2.

**Assumption 2.** *The preferences of the players satisfy:*

*(i)* $v(a_1, \theta) = u(a_1, \theta) = 0$ *for each* $\theta \in \Theta$*;*

*(ii)* $\overline{v} \equiv v(a_0, \theta) > 0$ *for each* $\theta \in \Theta$*;*

*(iii)* $\overline{u} \equiv u(a_0, \theta) > 0$ *if* $\theta \in \Theta_0$ *and* $\underline{u} \equiv u(a_0, \theta) < 0$ *if* $\theta \in \Theta_1$.

For an aspect $\kappa \in \{x, y\}$ and a given aspect realization $\kappa_l \in \mathcal{K}$, we will be interested in a particular subset of aspect realizations for the remaining aspect $-\kappa$, which will be denoted as $-\bar{\mathcal{K}}(\kappa_l) \subseteq -\mathcal{K}$. The set of realizations $-\bar{\mathcal{K}}(\kappa_l)$ crucially depends on the geometrical shape of the acceptance set $\Theta_0$ according to $-\bar{\mathcal{K}}(\kappa_l) \equiv \{-\kappa_l \in -\mathcal{K} \mid \theta = (\kappa_l, -\kappa_l) \in \Theta_0\}$. Observe that $\Theta_0 = \cup_{\kappa_l \in \mathcal{K}} - \bar{\mathcal{K}}(\kappa_l)$.

Under Assumption 2, the set of aspect realizations $\mathcal{K}^+_S \cap \mathcal{K}^+_R$ coincides with the set $\hat{\mathcal{K}}(a_0)$ which was specified earlier in Eq. (9). In particular, we can now write the set of aspect realizations $\mathcal{K}^+_S \cap \mathcal{K}^+_R = \hat{\mathcal{K}}(a_0)$ as

$$\hat{\mathcal{K}} \equiv \left\{ \kappa_l \in \mathcal{K} \;\middle|\; \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l) \geq -\underline{u}/(\overline{u} - \underline{u}) \right\}, \tag{18}$$

where the reference to action $a_0$ has been removed for simplicity in this two-action case. The condition that specifies the set $\hat{\mathcal{K}}$ in Eq. (18) above provides an intuitive interpretation of what makes aspect realizations, or data from experimentation, capable of persuading the Receiver to choose the "acceptance" action $a_0$. The expression $\sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l)$ in the left-hand side of the inequality in Eq. (18) measures the likelihood $P(\theta \in \Theta_0 \mid \kappa = \kappa_l)$ according to which the aspect realization $\kappa_l$ is associated state realizations $\theta = (\kappa_l, -\kappa_l) \in \Theta_0$. In other words, it measures the probability of the event that knowing only that the realization of aspect $\kappa$ is $\kappa_l$ lead $R$ to prefer $a_0$ over $a_1$. The expression $-\underline{u}/(\overline{u} - \underline{u})$ in the right-hand side is a measure of the difference between what $R$ receives by choosing $a_0$ in states in which it is beneficial for her to do so, relative to what she loses in states in which she gets harmed by choosing $a_0$. Note that both lower rewards $\overline{u}$ from choosing $a_0$ when $\theta \in \Theta_0$ and higher penalties $-\underline{u}$ from choosing $a_0$ when $\theta \in \Theta_1$ lead to higher values of the ratio $-\underline{u}/(\overline{u} - \underline{u})$. Therefore, for $\kappa_l \in \hat{\mathcal{K}}$, knowing only that the coordinate $\kappa$ of the state of the world $\theta = (\kappa, -\kappa)$ is $\kappa_l$ makes picking $a_0$ beneficial for $R$. In short, $\hat{\mathcal{K}}$ gives us the realizations of aspect $\kappa$ upon which $R$ is inclined to choose action $a_0$ rather than $a_1$.

In addition, under Assumption 2, it can be verified that the set of aspect realizations $\tilde{\mathcal{K}}$ specified earlier in Eq. (17), can now take the simpler form

$$\tilde{\mathcal{K}} = \arg \max_{\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}} P(\theta \in \Theta_0 \mid \kappa = \kappa_l) = \arg \max_{\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}} \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l).$$

Recall that the set $\tilde{\mathcal{K}}$ consists of data from experimentation over aspect $\kappa$, conditional on which $R$ prefers action $a_1$ over $a_0$, which lead $S$ to the highest marginal increase in his payoffs relative to what $R$ loses from choosing $a_0$. As we can observe from the expression above, Assumption 2 on the players' preferences gives an intuitive form to such a key ratio, which optimal experimentation over aspect $\kappa$ seeks to maximize.

Given the expressions derived above and the coefficients specified earlier in Eq. (6), direct application of the results of Theorem 3 and Theorem 4 to an environment under the preference specification imposed by Assumption 2 yields the following two corollaries.

**Corollary 3.** *Consider the case where $A = \{a_0, a_1\}$ and assume Assumption 1 and Assumption 2.*

*(i) An optimal experiment over aspect $\kappa$ is able to recommend action $a_0$ with positive probability if and only if the set of aspect realizations $\hat{\mathcal{K}} \subseteq \mathcal{K}$ specified in Eq. (18) is nonempty.*

*(ii) Provided that the set $\hat{\mathcal{K}}$ specified in Eq. (18) is nonempty, then (a) $\sigma^*_\kappa(a_0 \mid \tilde{\kappa}) = 1$ for each $\tilde{\kappa} \in \hat{\mathcal{K}}$; (b) for all aspect realizations $\tilde{\kappa} \notin \hat{\mathcal{K}}$, it follows that $\sigma^*_\kappa(a_0 \mid \tilde{\kappa}) \in [0, 1)$ if*

$\tilde{\kappa} \in \tilde{\mathcal{K}}$, *with the restriction that* $\sum_{\tilde{\kappa}\in\tilde{\mathcal{K}}} \sigma^*_\kappa(a_0 \mid \tilde{\kappa}) = \bar{\varrho}_\kappa$ *with*

$$\bar{\varrho}_\kappa \equiv \frac{\sum_{\kappa_l\in\hat{\mathcal{K}}} \left[\underline{u}\,\psi_\kappa(\kappa_l) + (\overline{u}-\underline{u})\sum_{-\kappa_l\in-\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l,-\kappa_l))\right]}{(\underline{u}-\overline{u})\sum_{-\kappa_l\in-\bar{\mathcal{K}}(\bar{\kappa})} \psi((\bar{\kappa},-\kappa_l)) - \underline{u}\,\psi_\kappa(\bar{\kappa})},$$

*where* $\bar{\kappa}$ *is some arbitrarily chosen* $\bar{\kappa} \in \tilde{\mathcal{K}}$. *Moreover, if there is a single aspect realization* $\tilde{\kappa}$ *such that* $\tilde{\mathcal{K}} = \{\tilde{\kappa}\}$, *then optimal experimentation requires* $\sigma^*_\kappa(a_0 \mid \tilde{\kappa}) = \bar{\varrho}_\kappa$; *(c) for all* $\kappa_l \notin \hat{\mathcal{K}}$, *it follows that* $\sigma^*_\kappa(a_0 \mid \kappa_l) = 0$ *if* $\kappa_l \notin \tilde{\mathcal{K}}$.

Corollary 4 allows us to determine how the Sender would optimally choose an aspect for experimentation by studying the marginal priors over each aspect and the joint priors about the state.

**Corollary 4.** *Consider the case where* $A = \{a_0, a_1\}$ *and assume Assumption 1 and Assumption 2. Suppose that both sets* $\hat{X}$ *and* $\hat{Y}$ *are nonempty so that optimal experimentation over any of the two aspects* $x$ *or* $y$ *is able to recommend action* $a_0$ *with positive probability. Then, the optimal aspect for experimentation is the aspect(s)* $\kappa^*$ *that that solve(s) the problem* $\max_{\kappa\in\{x,y\}} \Gamma(\kappa)$, *where the function* $\Gamma : \{x,y\} \to \mathbb{R}$ *is specified as*

$$\Gamma(\kappa) \equiv \sum_{\kappa_l\in\hat{\mathcal{K}}} \psi_\kappa(\kappa_l)\left(\overline{v} + \Phi_\kappa\left[\underline{u} + (\overline{u}-\underline{u})\sum_{-\kappa_l\in-\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l)\right]\right),$$

*where*

$$\Phi_\kappa \equiv \max_{\kappa_l\in\mathcal{K}\setminus\hat{\mathcal{K}}} \frac{\overline{v}}{(\underline{u}-\overline{u})\sum_{-\kappa_l\in-\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l) - \underline{u}}.$$

The functions specified in Corollary 4 can be useful to describe the optimal strategy that $S$ follows to choose an aspect for experimentation. Function $\Gamma(\kappa)$ simply gives us a convenient expression of the value function $V_\kappa(\sigma^*_\kappa)$ under Assumption 2. As already pointed out, $\Phi_\kappa$ gives us the maximal increase in $S$'s utility, relative to the induced decrease in $R$'s utility, which can be attained by all those pieces of data from experimentation over aspect $\kappa$ conditional on which $R$ wishes to choose $a_1$ instead of $a_0$. From the specification of the set of aspect realizations $\hat{\mathcal{K}}$ it follows that $\left[\underline{u} + (\overline{u}-\underline{u})\sum_{-\kappa_l\in-\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l)\right] > 0$ for each $\kappa_l \in \hat{\mathcal{K}}$. In addition, the definition of the value function $\Phi_\kappa$, together with the specification of the set $\hat{\mathcal{K}}$, directly implies that $\Phi_\kappa > 0$. Therefore, for each $\kappa_l \in \hat{\mathcal{K}}$, we observe that

$$\left(\overline{v} + \Phi_\kappa\left[\underline{u} + (\overline{u}-\underline{u})\sum_{-\kappa_l\in-\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l)\right]\right) > 0.$$

Given this, the first message of Corollary 4 is that $S$ wants to choose the aspect such that the corresponding marginal priors put relatively high weights on aspect realizations $\kappa_l \in \hat{\mathcal{K}}$. These are aspect realizations such that the marginal probabilities $P(\kappa_l \in \hat{\mathcal{K}}) = \sum_{\kappa_l \in \hat{\mathcal{K}}} \psi_\kappa(\kappa_l)$ are relatively high. In other words, $S$ wishes to select the aspect(s) that make it likely for experimentation to release data conditional on which $R$ prefers action $a_0$ over $a_1$. Not surprisingly, the role of the marginal priors $\psi_\kappa$ of the two aspects $\kappa \in \{x, y\}$ is essential to derive the required likelihoods according to which (restricted) experimentation recommends the acceptance action $a_0$. Also, from the specification of the functions $\Phi_\kappa$ and $\Gamma(\kappa)$, we observe that the correlations described by the associated family $\tau$ of conditional distributions $\psi(-\kappa_l \mid \kappa_l)$ are also key to determine which aspect is optimally selected. In particular, in order to maximize $\phi_\kappa(\kappa_l)$ for aspect realizations $\kappa_l \notin \hat{\mathcal{K}}$, $S$ wants to choose experiments such that the conditional probability $P(\theta \in \Theta_0 \mid \kappa = \kappa_l) = \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l)$ is relatively high.

The substance of the message conveyed by Corollary 4 is that optimal (restricted) experimentation favors the aspect $\kappa$ such that both the marginal distribution $\psi_\kappa$ and the family of conditional distributions $\psi(-\kappa_l \mid \kappa_l)$ make it likely for the state $\theta = (\kappa_l, -\kappa_l)$ to belong to the acceptance set $\Theta_0$. In this particular sense, the optimal choice of the aspect seeks to adjust to the preferences of the Receiver.

Although the expressions derived by Corollary 4 are lengthy and seem complex, the insights of Corollary 3 and Corollary 4 readily allow us to use the characterization results provided by Theorem 3 and Theorem 4 in applications. Our leading examples are helpful to illustrate how the analysis of optimal aspect-restricted experimentation can be conveniently simplified.

## 7.1. Leading Examples: Simplified Analysis

By resorting to Corollary 3 and Corollary 4, this subsection applies the characterization provided by Theorem 3 and Theorem 4 to our main examples of Section 3. Recall that in those examples we had $\overline{u} = 1$, $\underline{u} = -1$, and $\overline{u} = 1/3$, so that $-\underline{u}/(\overline{u} - \underline{u}) = 3/4$. Also, notice that the details of those examples lead directly to $\bar{X}(y_1) = \emptyset$, $\bar{X}(y_2) = \{x_3\}$, and $\bar{X}(y_3) = \{x_1, x_2, x_3\}$, whereas $\bar{Y}(x_1) = \{y_3\}$, $\bar{Y}(x_2) = \{y_3\}$, and $\bar{Y}(x_3) = \{y_2, y_3\}$. Let us first use Corollary 3 to describe optimal experimentation for the three examples considered earlier in Section 3.

*Example I. The Two Aspects Are Independent.*—Consider first the situation $\tau = \tau_x$ described by the matrix of conditional probabilities

$$\begin{pmatrix} \psi(y_1 \mid x_1) & \psi(y_1 \mid x_2) & \psi(y_1 \mid x_3) \\ \psi(y_2 \mid x_1) & \psi(y_2 \mid x_2) & \psi(y_2 \mid x_3) \\ \psi(y_3 \mid x_1) & \psi(y_3 \mid x_2) & \psi(y_3 \mid x_3) \end{pmatrix} = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix},$$

wherein income growth and environmental concerns are unrelated with each other. Then, for experimentation over aspect $\kappa = x$, we can readily compute:

$$\sum_{y \in \bar{Y}(x_1)} \psi(y \mid x_1) = 1/3 < 3/4, \quad \sum_{y \in \bar{Y}(x_2)} \psi(y \mid x_2) = 1/3 < 3/4,$$
$$\text{and} \quad \sum_{y \in \bar{Y}(x_3)} \psi(y \mid x_3) = 2/3 < 3/4 \quad \Rightarrow \quad \hat{X} = \emptyset.$$

On the other hand, for experimentation over aspect $\kappa = y$, we obtain

$$\sum_{x \in \bar{X}(y_1)} \psi(x \mid y_1) = 0 < 3/4, \quad \sum_{x \in \bar{X}(y_2)} \psi(x \mid y_2) = 1/3 < 3/4,$$
$$\text{and} \quad \sum_{x \in \bar{X}(y_3)} \psi(x \mid y_3) = 1 > 3/4 \quad \Rightarrow \quad \hat{Y} = \{y_3\}.$$

Using the insights from Corollary 4, we observe that experimentation over aspect $x$ is useless to persuade the Board, whereas experimentation over aspect $y$ is capable of persuading it by recommending acceptance with probability one when the realization of aspect $y$ is $y_3$. In addition, for the case of experimentation over aspect $y$, we easily observe that $\tilde{Y} = \{y_2\}$. Thus, the CEO's experimentation decision can optimally recommend acceptance with positive probability for realization $y_2$ as well. Also, can use Corollary 4 to determine which aspect the CEO optimally selects for information design. We easily observe that the CEO optimally chooses aspect $\kappa = y$, since $\hat{X} = \emptyset$ whereas $\hat{Y} = \{y_3\}$.

*Example II. The Two Aspects Depend on Each Other.*— Now consider the family $\tau = \tau_x'$ of conditional distributions described by the matrix of conditional probabilities

$$\begin{pmatrix} \psi(y_1 \mid x_1) & \psi(y_1 \mid x_2) & \psi(y_1 \mid x_3) \\ \psi(y_2 \mid x_1) & \psi(y_2 \mid x_2) & \psi(y_2 \mid x_3) \\ \psi(y_3 \mid x_1) & \psi(y_3 \mid x_2) & \psi(y_3 \mid x_3) \end{pmatrix} = \begin{pmatrix} 2/3 & 0 & 0 \\ 1/3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

wherein there is some correlation between income growth and environmental concerns. Then, for experimentation over aspect $\kappa = x$, we can now compute:

$$\sum_{y \in \bar{Y}(x_1)} \psi(y \mid x_1) = 0 < 3/4, \quad \sum_{y \in \bar{Y}(x_2)} \psi(y \mid x_2) = 0 < 3/4,$$
$$\text{and} \quad \sum_{y \in \bar{Y}(x_3)} \psi(y \mid x_3) = 1 > 3/4 \quad \Rightarrow \quad \hat{X} = \{x_3\}.$$

On the other hand, for experimentation over aspect $\kappa = y$, we obtain

$$\sum_{x \in \bar{X}(y_1)} \psi(x \mid y_1) = 0 < 3/4, \quad \sum_{x \in \bar{X}(y_2)} \psi(x \mid y_2) = 0 < 3/4,$$
$$\text{and} \quad \sum_{x \in \bar{X}(y_3)} \psi(x \mid y_3) = 1 > 3/4 \quad \Rightarrow \quad \hat{Y} = \{y_3\}.$$

Using the insights from Corollary 3, we observe that experimentation over aspect $x$ is capable of persuading the Board by recommending acceptance with probability one when the realization of aspect $x$ is $x_3$. Also, experimentation over aspect $y$ is capable of persuading the Board by recommending acceptance with probability one when the realization of aspect $y$ is $y_3$. For the case of experimentation over aspect $x$, we trivially obtain that $\tilde{X} = \{x_1, x_2\}$. Therefore, acceptance can optimally be recommended with positive probability for any of the realizations $x_1$ and/or $x_2$, with the restriction imposed the incentive-compatibility constraint (with equality). Similarly, for the case of information design over aspect $y$, we easily observe that $\tilde{Y} = \{y_1, y_2\}$.

We can use Corollary 4 to determine which aspect the CEO optimally selects for information design. Notice that For the case where $\tau = \tau_x'$, we can easily derive, given any $\bar{x} \in X \setminus \hat{X} = \{x_1, x_2\}$,

$$\frac{\overline{v}}{(\underline{u} - \overline{u}) \sum_{y_l \in \bar{X}(\bar{x})} \psi(y_l \mid \bar{x}) - \underline{u}} = \frac{1}{-(4/3)(0) + 1} = 1 = \Phi_x.$$

Similarly, given any $\bar{y} \in Y \setminus \hat{Y} = \{y_1, y_2\}$, we can compute

$$\frac{\overline{v}}{(\underline{u} - \overline{u}) \sum_{x_l \in \bar{Y}(\bar{y})} \psi(x_l \mid \bar{y}) - \underline{u}} = \frac{1}{-(4/3)(0) + 1} = 1 = \Phi_y.$$

Given such values for $\Gamma(x)$ and $\Gamma(y)$, we can then compute

$$\begin{aligned}\Gamma(x) &= \psi_x(x_3)\Big(\overline{v} + \Phi_x\Big[\underline{u} + (\overline{u} - \underline{u}) \sum_{y_l \in \bar{X}(x_3)} \psi(y_l \mid x_3)\Big]\Big) \\ &= (2/6)\Big(1 + (1)[-1 + 4/3 \cdot 1]\Big) = 4/9,\end{aligned}$$

and, analogously,

$$\begin{aligned}\Gamma(y) &= \psi_y(y_3)\Big(\overline{v} + \Phi_y\Big[\underline{u} + (\overline{u} - \underline{u}) \sum_{x_l \in \bar{Y}(y_3)} \psi(x_l \mid y_3)\Big]\Big) \\ &= (1/3)\Big(1 + (1)[-1 + 4/3 \cdot 1]\Big) = 4/9.\end{aligned}$$

Therefore, since $\Gamma(x) = \Gamma(y)$, we know that, for the Example II, the CEO is optimally indifferent between choosing any aspect, either $x$ or $y$, for experimentation.

*Example III. The Two Aspects Depend on Each Other.*— Finally, consider the family $\tau = \tau''_x$ of conditional distributions described by the matrix of conditional probabilities

$$\begin{pmatrix} \psi(y_1 \mid x_1) & \psi(y_1 \mid x_2) & \psi(y_1 \mid x_3) \\ \psi(y_2 \mid x_1) & \psi(y_2 \mid x_2) & \psi(y_2 \mid x_3) \\ \psi(y_3 \mid x_1) & \psi(y_3 \mid x_2) & \psi(y_3 \mid x_3) \end{pmatrix} = \begin{pmatrix} 3/6 & 1/6 & 1/6 \\ 2/6 & 4/6 & 1/6 \\ 1/6 & 1/6 & 4/6 \end{pmatrix},$$

where again there is some correlation between income growth and environmental concerns. Then, for experimentation over aspect $\kappa = x$, we can now compute:

$$\begin{aligned}&\sum_{y \in \bar{Y}(x_1)} \psi(y \mid x_1) = 1/6 < 3/4, \quad \sum_{y \in \bar{Y}(x_2)} \psi(y \mid x_2) = 1/6 < 3/4, \\ &\text{and} \quad \sum_{y \in \bar{Y}(x_3)} \psi(y \mid x_3) = 5/6 > 3/4 \quad \Rightarrow \quad \hat{X} = \{x_3\}.\end{aligned}$$

On the other hand, for experimentation over aspect $\kappa = y$, we obtain

$$\begin{aligned}&\sum_{x \in \bar{X}(y_1)} \psi(x \mid y_1) = 0 < 3/4, \quad \sum_{x \in \bar{X}(y_2)} \psi(x \mid y_2) = 1/6 < 3/4, \\ &\text{and} \quad \sum_{x \in \bar{X}(y_3)} \psi(x \mid y_3) = 1 > 3/4 \quad \Rightarrow \quad \hat{Y} = \{y_3\}.\end{aligned}$$

Using the insights from Corollary 3, we observe that experimentation over aspect $x$ is capable of persuading the Board upon recommending acceptance with probability one when the realization of aspect $x$ is $x_3$. Also, experimentation over aspect $y$ is capable of persuading the Board by recommending acceptance with probability one when the realization of aspect $y$ is $y_3$. For the case of experimentation over aspect $x$, we trivially obtain that $\tilde{X} = \{x_1, x_2\}$. Therefore, acceptance can optimally be recommended with positive probability for any of the realizations $x_1$ and/or $x_2$, with the restriction imposed the incentive-compatibility constraint (with equality). Similarly, for the case of information design over aspect $y$, we easily observe that $\tilde{Y} = \{y_2\}$.

We can also use Corollary 4 to determine which aspect the CEO optimally selects for information design. For the case where $\tau = \tau''_x$, we can easily derive, given any $\bar{x} \in X \setminus \hat{X} = \{x_1, x_2\}$,

$$\frac{\overline{v}}{(\underline{u} - \overline{u}) \sum_{y_l \in \bar{X}(\bar{x})} \psi(y_l \mid \bar{x}) - \underline{u}} = \frac{1}{-(4/3)(1/6) + 1} = 9/7 = \Phi_x.$$

Similarly, given any $\bar{y} = y_2$, we can compute

$$\frac{\overline{v}}{(\underline{u} - \overline{u}) \sum_{x_l \in \bar{Y}(\bar{y})} \psi(x_l \mid \bar{y}) - \underline{u}} = \frac{1}{-(4/3)(1/6) + 1} = 9/7 = \Phi_y.$$

Given such values for $\Gamma(x)$ and $\Gamma(y)$, we can then compute

$$\begin{aligned}\Gamma(x) &= \psi_x(x_3)\Big(\overline{v} + \Phi_x\Big[\underline{u} + (\overline{u} - \underline{u}) \sum_{y_l \in \bar{X}(x_3)} \psi(y_l \mid x_3)\Big]\Big) \\ &= (2/6)\Big(1 + (9/7)[-1 + 4/3 \cdot (5/6)]\Big) = 8/21,\end{aligned}$$

and, analogously,

$$\begin{aligned}\Gamma(y) &= \psi_y(y_3)\Big(\overline{v} + \Phi_y\Big[\underline{u} + (\overline{u} - \underline{u}) \sum_{x_l \in \bar{Y}(y_3)} \psi(x_l \mid y_3)\Big]\Big) \\ &= (1/3)\Big(1 + (9/7)[-1 + 4/3 \cdot (1)]\Big) = 10/21.\end{aligned}$$

Therefore, since $\Gamma(x) > \Gamma(y)$, we know that, for the Example III, the CEO optimally wants to choose aspect $y$ for experimentation.

Overall, we observe that the entire description provided now for the CEO's optimal disclosure behavior in our three examples coincides exactly with the one presented earlier in Section 3.

## 8. Application: Media *Slant*

I am particularly motivated by applications that allow us to understand better media tactics of persuasion. This paper main's assumption translates into media outlets being exogenously restricted—due to technological, cognitive, or, simply, time constraints—to choosing a subset of all plausible aspects in their reporting. For the two-action case, the framework proposed here offers a logic for how media outlets can *slant*[20] by selecting deliberately the aspects over which they provide data from experimentation.[21]

To illustrate how the current model could be useful to rationalize media slant, let us consider an example used by Mullainathan and Shleifer (2005). Suppose that the Bureau of Labor Statistics (BLS) discloses that the number of unemployed increased from 6.1 percent to a 6.3 unemployment rate. We can interpret this piece of data as the outcome of experimentation about a particular aspect of interest to Receivers. We can regard a Receiver as an undecided voter that must choose whether or not to support the current governing party for the next term. In the example used by Mullainathan and Shleifer (2005), in addition to the outcome of the experiment over employment, a media outlet may offer one of two alternative pieces of additional information. One piece, under the headline "Recession Fears Grow," would suggest an imminent recession. This headline would release information by Harvard economist John K. Galbraith, who would elaborate on that the implemented economic policies are ill-suited. Another piece, under the headline "Turnaround in Sight," would point instead towards an imminent expansion. This second headline would provide information by the chief stock market analyst of Goldman and Sachs, Abbie J. Cohen, who would argue that there are clear signs of profitable investment opportunities.

Using the benchmark proposed here, one could formalize this situation by considering three relevant aspects of uncertainty.[22] A state of the world in this case would be $\theta = (x, y, z)$, where $x$ describes job market conditions, $y$ accounts for the quality of economic

[20] The idea of *slant* was introduced by Hayakawa (1940). Slant roughly refers to the act of selecting verifiable details that are favorable or unfavorable to a certain view for an action choice under uncertainty.

[21] While some contemporary efforts to explain media slant consider that either Receivers (Mullainathan and Shleifer, 2005), Senders (Baron, 2006), or public institutions (Besley and Prat, 2006) are biased in ways that make them benefit from slanted information, other models assume that reputation concerns incentivize Senders to distort their reporting (Gentzkow and Shapiro, 2006). Neither exogenous preferences for distorted news nor reputation concerns are required to obtain a rationale for slant under the setup proposed in this paper.

[22] Although the model has been developed in terms of two dimensions of uncertainty for simplicity, its functioning and implications go through for a general multi-dimensional state with a finite number of coordinates.

policies, and $z$ measures the position of economic fundamentals relevant for the stock market. We would consider that, before election day, voters are able to pay attention only to data from experimentation about job market conditions and either about the quality of economic policies or the stock market fundamentals. As experts in their respective fields, we can regard J. K. Galbraith and A. J. Cohen as experimentation choices by the media outlets, respectively, on the quality of economic policies and on the stock market fundamentals. Both experts in fact support their arguments with pieces of evidence. We need to assume that, once the media outlet selects the remaining aspect ($y$ or $z$) and the experiment (exemplified by the choices of J. K. Galbraith or A. J. Cohen), it "ties its hand out" regarding to what such experiments can release (e.g., to what experts such as J. K. Galbraith or A. J. Cohen can argue with the support of hard evidence). In other words, information design rules out cases in which the media outlet can influence, or manipulate, the points made by the chosen experiments/experts.

Suppose that, independently of the state of the world, a particular media outlet may either want voters to support the current governing party or to not reelect the governing party. If we wish to interpret this story according to the analysis developed in this paper, then the crucial point lies in the preferences that the media outlets perceive as the predominant ones among the voters (e.g., the median voter or a type of voter that has a certain weight in the population). In a stylized way, suppose that there are only two types of voters, type $A$ and $B$. Type $A$ voter is relatively more interested in job market conditions and in the quality of the economic policies implemented. Instead, type $B$ voter is relatively more interested in job market conditions and in investment opportunities. We could capture these considerations by considering two different acceptance sets, $\Theta_0^A$ and $\Theta_0^B$, such that type $A$ voter cares less about the $z$-aspect, whereas type $B$ gives relatively less importance to the $y$-aspect.

Then, the first point highlighted by the model (Theorem 4 and Corollary 4) is that the media outlet needs to assess what is the type of the predominant voter, in order to decide between economic policies quality or stock market opportunities. In particular, the proposed model predicts that the media outlet would commit on economic policy quality (by, e.g., offering the views of J. K. Galbraith) if there is at least one possible piece of data on economic policy quality that would make the media outlet and the voter's type (which is perceived as the predominant one by the media outlet) to agree on the best course of action. Likewise, the media outlet would commit on stock market opportunities (by, e.g., offering the arguments of A. J. Cohen) if there is one possible piece of data on stock market opportunities upon which the preferences of the voter's type (which is perceived as the predominant one) and the media outlet about the outcome of the election are perfectly aligned. Thirdly, if there are pieces of data over both aspects such that the predominant voter the media outlet agree on the best course of action, then the media outlet would choose the aspect with the highest likelihood of agreement across all possible realizations of the respective aspect.

The logic that the proposed model provides for rationalizing media slant is consistent with other theoretical proposals and with empirical findings as well. Although Mullainathan and Shleifer (2005)'s behavioral assumption that Receivers want to see their initial beliefs confirmed makes the analyses quite different, there are also similarities in the ways in which the implications about slant work. In particular, they find that Senders slant by disclosing information in order to adjust to the tastes of the Receivers, which shares the basic rationale of the result that the Sender chooses a single aspect for experimentation in a way crucially driven by the preferences of the Receiver. At the empirical level, Gentzkow and Shapiro (2010) estimate that roughly a 20 of the variation on their recent US sample on media slant obeys to media outlets having incentives to respond to the preferences of their consumers. Interestingly, in their estimations, the identities, or preferences, of the media outlets play no role in explaining slant.

# 9. Further Literature Connections

The study of influential communication goes back to the literature on strategic advise. Since its seminal works (Green and Stokey, 1980; Crawford and Sobel, 1982), the cheap talk framework has established that influential communication is critically bounded when the conflict of interests is high.[23] Credible communication is enhanced when the Sender can commit to design information.[24] Following the assumptions of Bayesian persuasion (Rayo and Segal, 2010; Kamenica and Gentzkow, 2011), or information design (Bergemann and Morris, 2013, 2016), the current paper has considered that the Sender has the ability to make (ex ante) commitments to design information. In contrast to the classical approach, however, the setup proposed here assumes that the Sender lacks the ability to commit over

[23] Although fully revealing cheap talk communication can be obtained when there are multiple dimensions of uncertainty (Battaglini, 2002; Chakraborty and Harbaugh, 2007, 2010), the message that costless−uncommitted−unverifiable disclosure is severely restricted continues to hold even for such environments. In particular, multi-dimensional cheap talk communication is severely restricted if either the state space is bounded (Ambrus and Takahashi, 2008) or if the dimensions of uncertainty are strongly correlated between them according to the prior (Levy and Razin, 2007).

[24] As already suggested by Levy and Razin (2007)'s analysis even before the Bayesian persuasion literature comprehensively explored these topics.

all dimensions of uncertainty simultaneously and must instead resort to full, but "isolated," commitment over any single one of the separate dimensions.

In relation with the verifiable disclosure literature (Grossman, 1981; Milgrom, 1981; Milgrom and Roberts, 1986; Krishna and Morgan, 2001; Che and Kartik, 2009; Kartik et al., 2017), there are clear connections, mainly in motivation and raised questions, to models that deal with how selective reporting in multi-dimensional environments can affect security price dynamics (Shin, 1994, 2003).

Since one natural motivation for the restriction that the Sender faces is to consider that the Receiver is able to process information from experimentation about any single one of the aspects, this paper is also related to the study of persuasion rules, explored by Glazer and Rubinstein (2004), under restrictions on the amount of (verifiable) information that can be accumulated or processed.

The pursued approach of weakening the Sender's commitment power bears also similarities with the setup proposed by Nguyen and Tan (2019). They consider a Bayesian persuasion model where the Sender first receives privately the signals disclosed by the selected information structure and then sends a costly message to the Receiver about the original signals. The differences, though, are of substance. Nguyen and Tan (2019) do not consider information design restricted to particular aspects of uncertainty and they model the communication that complements the information structure as costly signaling. Sharing motivations and interests with Nguyen and Tan (2019), other papers have also weaken the commitment power of the Sender in information design problems. In Min (2017) and Frechette et al. (2019), the Sender has the ability to distort the information structure that he selects before releasing it to the Receiver. Lipnowski and Ravid (2019) relaxes the Sender's commitment power to formalize cheap talk environments where the payoffs of the Sender do not depend on the state of the world, while Lipnowski et al. (2019) analyses credibility issues regarding the quality of the chosen information structures. At a more abstract level, Bester and Strausz (2001) have explored how incentive-compatibility conditions and the Revelation Principle need to be modified in contracting situations with a single agent where the principal has limited commitment power. Importantly, all these papers are part of a recent literature on persuasion where the key consideration is *limited or partial commitment*: there is an exogenous positive probability that the chosen information structure is not binding for the information-designer. Unlike this assumption, in the current paper the Sender can commit fully, without any possibility of subsequent manipulation of the chosen information structure.

For environments where the relevant uncertainty is multi-dimensional, Frankel and Kartik (2019) consider a model of separate costly signaling over two aspects of uncertainty and obtain that information provision about one aspect can diminish ("muddle") disclosure over the other aspect. Unlike their paper, though, the current setup does not consider costly signaling.

Similar to the assumption that the Sender chooses to design information over one aspect or another of uncertainty, Deimen and Szalay (2019) allow the Sender to acquire information selectively about one aspect (the Sender's ideal action) or another (the Receiver's ideal action). Theirs is a setup of delegated expertise enriched with the possibility of costless information acquisition about two separate aspects of uncertainty and cheap talk communication. Other than the shared interest in strategic aspect choice for information provision, the setup and questions explored are quite distinct.

Lastly, the questions investigated in the current paper are reminiscent of those addressed by Che et al. (2013), who explore how verifiable ("hard") information in the hands of the Receiver combines with comparative cheap talk ("soft information") by the Sender. They obtain that, in equilibrium, the Sender biases his cheap talk towards recommendations favorable to the Receiver ("pandering"), provided that there is a mild conflict of interests only over an outside option.

# 10. Concluding Comments

This paper has explored persuasion by information design for environments with two-aspect relevant uncertainty. The novelty of the approach lies in the assumption that the Sender is exogenously constrained to picking any single one of the aspects for experimentation. For the two-action case, the Sender commits to disclose data from experimentation for the aspect that allows the highest (ex ante) likelihood of mitigation for the initial conflict of interests. For persuasive aspect-restricted experimentation to take place, it is necessary (and sufficient) to have at least one piece of data about the chosen aspect over which the incentives of the two parties are aligned.

This paper restricted attention to two-aspect settings where commitment for experimentation is feasible for only one of the two aspects. While this is convenient for tractability and expositional reasons, the mechanisms that drive the model's qualitative implications go entirely through for settings with general (finite) multi-dimensional uncertainty and commitment restricted to (strict) subsets of all the relevant aspects. For such cases, by following the arguments behind the paper's main results, it is intuitive to see that each optimally selected experiment would similarly recommend higher acceptance probabilities the lower the conflict of interests associated with the corresponding aspect realization. In addition, the Sender would analogously select the subset of aspects for experimentation associated to the highest (ex ante) likelihood of reducing the most the original source of conflict of interests.

# 11. Appendix

## Omitted Proofs

**Proof of Theorem 1.**

Consider a given aspect $\kappa \in \{x, y\}$. Using the terms introduced in Eq. (6) and Eq. (7), together with the short-hand notation $z^{\kappa}_{hl} \equiv \sigma_{\kappa}(a_h \mid \kappa_l)$ for $h = 0, \ldots, n$ and $l = 1, \ldots, m$, the optimal experimentation choice of $S$ over aspect $\kappa$ expressed in Eq. (4) can be rewritten as the linear programming problem

$$\begin{aligned} &\max_{\{z^{\kappa}_{hl} \in [0,1]\}} \quad \sum_{h=0}^{n} \sum_{l=1}^{m} \beta^{S}_{\kappa}(a \mid \kappa_l)\, z^{\kappa}_{hl} \\ &\text{s.t.:} \quad \forall l = 1 \ldots, m : \quad \sum_{h=0}^{n} z^{\kappa}_{hl} = 1; \\ &(IC) \quad \forall h = 0, \ldots, n : \quad \sum_{l=1}^{m} \delta^{a_h, a}_{\kappa_l}\, z^{\kappa}_{hl} \geq 0 \quad \forall a \in A. \end{aligned} \tag{19}$$

Consider some given action $a_j \in A$.

(i) First, suppose that $\beta^{S}_{\kappa}(a_j \mid \kappa_l) > 0$ so that $S$ wishes to choose $z^{\kappa}_{jl} > 0$ in his optimal experiment decision. This is a necessary condition for action $a_j$ to be recommended with (strictly) positive probability in equilibrium. In addition, the following additional conditions, combined together, are necessary and sufficient for the problem in Eq. (19) to have a solution such that $z^{\kappa}_{jl} > 0$.

(ii) Secondly, it follows from the set of incentive-compatible conditions $(IC)$ in Eq. (19), for $h = j$, that $z^{\kappa}_{jl} > 0$ is incentive-compatible only if, for each action $a \neq a_j$, there is some aspect realization $\kappa(a)$ such that $\delta^{a_j, a}_{\kappa(a)} > 0$. Otherwise, incentive-compatibility would lead to that $z^{\kappa}_{jl} = 0$ for each realization $\kappa_l \in \mathcal{K}$. Furthermore, if $\delta^{a_j, a}_{\kappa(a)} > 0$ for some aspect realization

$\kappa(a)$, for each alternative action $a \neq a_j$, then $z^{\kappa}_{jl} > 0$ does solve $S$'s optimization problem in Eq. (19).

(iii) Thirdly, provided that for each action $a \neq a_j$, there is some aspect realization $\kappa(a)$ such that $\delta^{a_j,a}_{\kappa(a)} > 0$, we crucially need to verify that the system of inequalities

$$\sum_{l=1}^{m} \delta^{a_j,a}_{\kappa_l} \; z^{\kappa}_{jl} \geq 0 \qquad (a \in A), \tag{20}$$

is consistent. Theorem 2 of Dines (1936) shows that the system in Eq. (20) is consistent if and only if the convex hull of the set $\mathcal{B}_{\kappa}(a_j) = \{(\delta^{a_j,a}_{\kappa_l})_{\kappa_l \in \mathcal{K}} \in \mathbb{R}^m \mid a \neq a_j\}$ does not contain the $n$-dimensional vector of zeroes.

This completes the required arguments. ∎

**Proof of Theorem 2.**

Consider a given aspect $\kappa \in \{x, y\}$ and fix some action $a_j \in A$. The arguments required for conditions in (i) and (ii) of Theorem 2 coincide exactly with the ones provided for (i) and (ii) of Theorem 1. As for the condition in (iii), note we can construct the dual to the system of inequalities in Eq. (20) by considering a set of variables $q_{\kappa}(a_j, a)$ for $a \in A$. The dual to the system in Eq. (20) is then the system of equalities

$$\sum_{a \in A \setminus \{a_j\}} \delta^{a_j,a}_{\kappa_l} \, q_{\kappa}(a_j, a) = 0 \qquad (l = 1, \ldots, m). \tag{21}$$

Theorem 9 of Dines (1936) shows that the system in Eq. (21) is consistent if and only if it does not have solutions $q^*(a_j) \equiv (q^*_{\kappa}(a_j, a))_{a \neq a_j}$ such that $q^*_{\kappa}(a_j, a) \geq 0$ for each $a \neq a_j$ and $q^*_{\kappa}(a_j, a) \neq 0$ for some $a \neq a_j$. Therefore, it must be the case that at least some solution $q^*(a_j)$ to Eq. (21) satisfies $q^*_{\kappa}(a_j, a) < 0$ for some $a \neq a_j$. This completes the required arguments. ∎

**Proof of Corollary 1.**

Consider a given aspect $\kappa \in \{x, y\}$. Fix some action $a_j$ such that $\beta^S_{\kappa}(a_j \mid \kappa_l) > 0$. Theorem 2 has shown that $\sigma_{\kappa}(a_j \mid \kappa_l) > 0$ in equilibrium if and only if the system of linear equations

$$\sum_{a \in A \setminus \{a_j\}} \delta^{a_j,a}_{\kappa_l} \, q_{\kappa}(a_j, a) = 0 \qquad (l = 1, \ldots, m)$$

has at least one negative solution $q^*_\kappa(a_j, a)$, for some $a \neq a_j$. If $a_j \in \overline{A}^R_\kappa(\kappa_s)$ for some aspect realization $\kappa_s$, then it must be the case that $\delta^{a_j,a}_{\kappa_s} \geq 0$ for each $a \neq a_j$. Furthermore, since the model assumes that $\overline{A}^R_\kappa(\kappa_s) \neq A$, then it must be the case that $\delta^{a_j,a}_{\kappa_s} > 0$ for some $a \neq a_j$. Therefore, for the equality

$$\sum_{a \in A \setminus \{a_j\}} \delta^{a_j,a}_{\kappa_s} \, q^*_\kappa(a_j, a) = 0$$

to be satisfied, it is a necessary condition that $q^*_\kappa(a_j, a) < 0$ for at least some $a \neq a_j$. The result of Theorem 2 then leads to that action $a_j$ is recommended in equilibrium conditional on aspect realization $\kappa_l$ in any optimal choice of experiments over aspect $\kappa$. ∎

**Proof of Theorem 3.**

Consider a given aspect $\kappa \in \{x, y\}$. Using the terms introduced in Eq. (6) and Eq. (7), together with the short-hand notation $z^\kappa_{hl} \equiv \sigma_\kappa(a_h \mid \kappa_l)$ for $h = 0, 1$ and $l = 1, \ldots, m$, the optimal experimentation choice of $S$ over aspect $\kappa$ expressed in Eq. (4) can be expressed as

$$\begin{aligned}
&\max_{\{z^\kappa_{0l}, z^\kappa_{1l} \in [0,1]\}^m_{l=1}} && \sum_{h=0}^{1} \sum_{l=1}^{m} \beta^S_\kappa(a_h \mid \kappa_l) \, z^\kappa_{hl} \\
&\text{s.t.:} \quad (IC) && \sum_{l=1}^{m} \delta^{a_0,a_1}_{\kappa_l} z^\kappa_{0l} \ \geq 0; \\
& && \sum_{l=1}^{m} \delta^{a_1,a_0}_{\kappa_l} z^\kappa_{1l} \ \geq 0.
\end{aligned}$$

Furthermore, the problem above can be rewritten in terms of the choice variables $z^\kappa_{0l}$ alone by noting that $z^\kappa_{1l} = 1 - z^\kappa_{0l}$ and $\delta^{a_1,a_0}_{\kappa_l} = -\delta^{a_0,a_1}_{\kappa_l}$. We then obtain

$$\begin{aligned}
&\max_{\{z^\kappa_{0l} \in [0,1]\}^m_{l=1}} && \sum_{l=1}^{m} \beta^S_\kappa(a_1 \mid \kappa_l) + \sum_{l=1}^{m} [\beta^S_\kappa(a_0 \mid \kappa_l) - \beta^S_\kappa(a_1 \mid \kappa_l)] \, z^\kappa_{0l} \\
&\text{s.t.:} \quad (IC) && \sum_{l=1}^{m} \delta^{a_0,a_1}_{\kappa_l} z^\kappa_{0l} \ \geq \max\{0, \sum_{l=1}^{m} \delta^{a_0,a_1}_{\kappa_l}\}.
\end{aligned}$$

In addition, by considering Assumption 1, and by setting the short-hand notations $\rho_l \equiv \beta^S_\kappa(a_0 \mid \kappa_l) - \beta^S_\kappa(a_1 \mid \kappa_l)$ and $\delta_l \equiv \delta^{a_0,a_1}_{\kappa_l}$ for simplicity, $S$'s aspect-restricted information design problem can be conveniently written as the simple linear programming problem:

$$\begin{aligned} \max_{\{z^{\kappa}_{0l}\in[0,1]\}_{l=1}^{m}} \quad & \sum_{l=1}^{m} \rho_l \, z^{\kappa}_{0l} \\ \text{s.t.:} \quad (IC) \quad & \sum_{l=1}^{m} \delta_l z^{\kappa}_{0l} \ \geq 0, \end{aligned} \tag{22}$$

where Assumption 1 translates into $\sum_{l=1}^{m} \rho_l > 0$ and $\sum_{l=1}^{m} \delta_l < 0$.

(i) Using the version in Eq. (22) of $S$'s problem, we observe that maximizing the expression $\sum_{l=1}^{m} \rho_l \, z^{\kappa}_{0l}$ subject to the $(IC)$ condition yields optimal choices where $z^{\kappa *}_{0l} > 0$ for some $\kappa_l \in \mathcal{K}$ if and only if $\kappa_l \in \mathcal{K}^+_S$ and there is some $\kappa_{l'} \in \mathcal{K}^+_R$. Thus, the required necessary and sufficient condition is that both sets $\mathcal{K}^+_S$ and $\mathcal{K}^+_R$ be nonempty.

(ii) Suppose that both sets $\mathcal{K}^+_S$ and $\mathcal{K}^+_R$ are nonempty. From the form of the problem in Eq. (22), under the conditions imposed by Assumption 1, we observe that $S$'s optimal experiments must satisfy:

(a) $z^{\kappa *}_{0l} = 0$ for each $\kappa_l \in \mathcal{K}^-_S$;

(b) provided that $\mathcal{K}^+_S \cap \mathcal{K}^+_R$ is nonempty, then $z^{\kappa *}_{0l} = 1$ for each $\kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^+_R$.

After incorporating the features (a) and (b) above of any optimal choice, it follows that solving the problem in Eq. (22) is equivalent to solve the problem

$$\begin{aligned} \max_{\{z^{\kappa}_{0l}\in[0,1] \,|\, \kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^-_R\}} \quad & \sum_{\{l \,|\, \kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^-_R\}} \rho_l \, z^{\kappa}_{0l} \\ \text{s.t.:} \quad & \sum_{\{l \,|\, \kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^-_R\}} (-\delta_l) z^{\kappa}_{0l} \ \leq \sum_{\{l \,|\, \kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^+_R\}} \delta_l. \end{aligned} \tag{23}$$

Now, to solve the linear programming problem in Eq. (24), we need to compare each directional slope $\rho_l/\rho_{l'}$ (for $\kappa_l, \kappa_{l'} \in \mathcal{K}^+_S \cap \mathcal{K}^-_R$ such that $\kappa_l \neq \kappa_{l'}$) according to which $S$'s expected utility increases with the respective directional slope $\delta_l/\delta_{l'}$ of the incentive-compatible set for $R$ to follow the recommendations from restricted experimentation. Any solution to Eq. (24) satisfies $z^{\kappa}_{0l} > 0$ for those aspect realizations $\kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^-_R$ which are associated with the highest positive differences of directional slopes $\rho_l/\rho_{l'} - \delta_l/\delta_{l'} > 0$ for each aspect realization $\kappa_{l'} \in \mathcal{K}^+_S \cap \mathcal{K}^-_R \setminus \{\kappa_l\}$. This translates directly into finding aspects realizations $\kappa_l$ associated to the highest ratios $(-\rho_l/\delta_l)$ among the aspect realizations $\kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^-_R$ such that $(-\rho_l/\delta_l) > 1$.

Therefore, to solve the problem in Eq. (24), we first need to obtain the set of aspect realizations $\tilde{\mathcal{K}} \equiv \arg\max_{\{l \,|\, \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-\}} (-\rho_l/\delta_l)$. Given this, the optimal experiment then satisfies (c) $z_{0l}^{\kappa\,*} \in [0,1)$ for all aspect realizations $\kappa_l \in \tilde{\mathcal{K}}$, with the requirement that $\sum_{\{l \,|\, \kappa_l \in \tilde{\mathcal{K}}\}} z_{0l}^{\kappa\,*} = \bar{\varrho}_\kappa \equiv \sum_{\{l \,|\, \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+\}} \delta_l/(-\delta_{\bar{l}})$ for any arbitrarily chosen aspect realization $\kappa_{\bar{l}} \equiv \bar{\kappa} \in \tilde{\mathcal{K}}$. Of course, using the description of the optimal choice $\bar{\varrho}_\kappa$, it directly follows that if there is a single aspect realization $\bar{\kappa}$ such that $\{\bar{\kappa}\} = \tilde{\mathcal{K}}$, then optimal experimentation requires $z_{0\bar{l}}^{\kappa\,*} = \bar{\varrho}_\kappa \equiv \sum_{\{l \,|\, \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+\}} \delta_l/(-\delta_{\bar{l}})$. Finally, (d) if $\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-$ but $\kappa_l \notin \tilde{\mathcal{K}}$, then it follows directly from the description of $S$'s problem in Eq. (24) that $z_{0l}^{\kappa\,*} = 0$. ∎

**Proof of Theorem 4.**

The dual problem of the problem in Eq. (22) above is

$$\begin{aligned} &\min_{\{q \geq 0\}} \ \Big( \sum_{\{l \,|\, \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+\}} \delta_l \Big)\, q \\ &\text{s.t.:} \ \ (-\delta_l) q \geq \rho_l \qquad \forall l \ \ \text{s.t.} \ \ \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-. \end{aligned} \tag{24}$$

Clearly, the solution to the dual problem in Eq. (24) is $q^* = \max_{\{l \,|\, \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-\}} \rho_l/(-\delta_l)$. Then, using the saddle point Theorem of linear programing, we can derive the value function of the problems in Eq. (22) and Eq. (24) as

$$\Big( \sum_{\{l \,|\, \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+\}} \delta_l \Big) \times \max_{\{l \,|\, \kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-\}} \rho_l/(-\delta_l).$$

Therefore, by undoing the notational simplifications, it follows that the value function of $S$'s optimal experimentation problem over aspect $\kappa$ is given by

$$\begin{aligned} V_\kappa(\sigma_\kappa^*) = &\sum_{\kappa_l \in \mathcal{K}} \beta_\kappa^S(a_1 \mid \kappa_l) + \sum_{\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+} \big[\beta_\kappa^S(a_0 \mid \kappa_l) - \beta_\kappa^S(a_1 \mid \kappa_l)\big] \\ &+ \sum_{\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+} \big[\beta_\kappa^R(a_0 \mid \kappa_l) - \beta_\kappa^R(a_1 \mid \kappa_l)\big] \times \max_{\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-} \left\{ \frac{\beta_\kappa^S(a_0 \mid \kappa_l) - \beta_\kappa^S(a_1 \mid \kappa_l)}{\beta_\kappa^R(a_1 \mid \kappa_l) - \beta_\kappa^R(a_0 \mid \kappa_l)} \right\}. \end{aligned} \tag{25}$$

Then, $S$ optimally chooses for experimentation the aspect $\kappa$ that yields a higher value for $V_\kappa(\sigma_\kappa^*)$ according to the expression in Eq. (25) above. ∎

**Proof of Corollary 3.**

Consider the case where $A = \{a_0, a_1\}$ and assume Assumption 1 and Assumption 2. Fix an aspect realization $\kappa \in \{x, y\}$. First, it follows directly from (i) and (ii) of Assumption 2 that

$\mathcal{K}_S^+ = \mathcal{K}$. Secondly, (iii) of Assumption 2 directly implies that

$$\begin{aligned}
\beta_\kappa^R(a_0 \mid \kappa_l) - \beta_\kappa^R(a_1 \mid \kappa_l) &= \sum_{-\kappa_l} \psi((\kappa_l, -\kappa_l))\big[u(a_0, (\kappa_l, -\kappa_l)) - u(a_1, (\kappa_l, -\kappa_l))\big] \\
&= \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l))\overline{u} + \Big[\psi_\kappa(\kappa_l) - \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l))\Big]\underline{u} \\
&= \underline{u}\,\psi_\kappa(\kappa_l) + (\overline{u} - \underline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l)),
\end{aligned}$$

where recall that $-\bar{\mathcal{K}}(\kappa_l) \equiv \{-\kappa_l \in -\mathcal{K} \mid (\kappa_l, -\kappa_l) \in \Theta_0\}$. Since $(\overline{u} - \underline{u}) > 0$, it then follows that $\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^+$ if and only if $\sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l))/\psi_\kappa(\kappa_l) \geq -\underline{u}/(\overline{u} - \underline{u})$. Therefore, upon noting that $\psi((\kappa_l, -\kappa_l))/\psi_\kappa(\kappa_l) = \psi(-\kappa_l \mid \kappa_l)$, we observe that optimal investigation over aspect $\kappa$ is able to recommend action $a_0$ with positive probability if and only if (i) there exist some aspect realization $\kappa_l$ that belongs to the set of aspect realizations $\hat{\mathcal{K}} \subseteq \mathcal{K}$ specified in Eq. (18). Furthermore, it follows from (ii)-(b) of Theorem 3 that $\sigma_\kappa^*(a_0 \mid \kappa_l) = 1$ for each $\kappa_l \in \hat{\mathcal{K}}$. On the other hand, for aspect realizations $\tilde{\kappa} \notin \mathcal{K}_R^+$—equivalently, aspect realizations $\tilde{\kappa} \notin \hat{\mathcal{K}}$—, it follows from (ii)-(c) of Theorem 3 that $\sigma_\kappa^*(a_0 \mid \tilde{\kappa}) \in [0, 1)$ if $\tilde{\kappa} \in \tilde{\mathcal{K}}$. Again, by using the specification of the set of aspect realizations $-\bar{\mathcal{K}}(\kappa_l) \subseteq -\mathcal{K}$, it follows from Assumption 2 that

$$\begin{aligned}
\tilde{\mathcal{K}} &\equiv \arg \max_{\kappa_l \in \mathcal{K}_S^+ \cap \mathcal{K}_R^-} \frac{\beta_\kappa^S(a_0 \mid \kappa_l) - \beta_\kappa^S(a_1 \mid \kappa_l)}{\beta_\kappa^R(a_1 \mid \kappa_l) - \beta_\kappa^R(a_0 \mid \kappa_l)} \\
&= \arg \max_{\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}} \frac{\overline{v}\,\psi_\kappa(\kappa_l)}{(\underline{u} - \overline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l)) - \underline{u}\,\psi_\kappa(\kappa_l)} \\
&= \arg \max_{\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}} \frac{\overline{v}}{(\underline{u} - \overline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l))/\psi_\kappa(\kappa_l) - \underline{u}} \\
&= \arg \max_{\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}} \frac{\overline{v}}{(\underline{u} - \overline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l) - \underline{u}}.
\end{aligned}$$

By noting that $(\underline{u} - \overline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l) - \underline{u} > 0$ for each $\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}$, together with $(\underline{u} - \overline{u}) < 0$, it then follows that solving the problem $\max_{\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}} \phi_\kappa(\kappa_l)$ is equivalent, under Assumption 1 and Assumption 2, to solving $\max_{\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}} \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi(-\kappa_l \mid \kappa_l)$. Then, using again the specification of $-\bar{\mathcal{K}}(\kappa_l) \subseteq -\mathcal{K}$ together with Assumption 2, (ii)-(c) of Theorem 3 establishes that $\sigma_\kappa^*(a_0 \mid \tilde{\kappa}) \in [0, 1)$ for all aspect realizations $\tilde{\kappa} \in \tilde{\mathcal{K}}$, with the requirement that $\sum_{\tilde{\kappa}\in\tilde{\mathcal{K}}} \sigma^*_\kappa(a_0 \mid \tilde{\kappa}) = \bar{\varrho}_\kappa$ where, upon fixing some arbitrarily chosen $\bar{\kappa} \in \tilde{\mathcal{K}}$, we have

$$\bar{\varrho}_\kappa \equiv \frac{\sum_{\kappa_l \in \mathcal{K}^+_S \cap \mathcal{K}^+_R} [\beta^R_\kappa(a_1 \mid \kappa_l) - \beta^R_\kappa(a_0 \mid \kappa_l)]}{\beta^R_\kappa(a_1 \mid \tilde{\kappa}) - \beta^R_\kappa(a_0 \mid \tilde{\kappa})}, \\ = \frac{\sum_{\kappa_l \in \hat{\mathcal{K}}} \left[ \underline{u}\, \psi_\kappa(\kappa_l) + (\overline{u} - \underline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l)) \right]}{\underline{u}\, \psi_\kappa(\bar{\kappa}) + (\overline{u} - \underline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\bar{\kappa})} \psi((\bar{\kappa}, -\kappa_l))}.$$

Moreover, (ii)-(c) of Theorem 3 also establishes that if there is a single aspect realization $\tilde{\kappa}$ such that $\tilde{\mathcal{K}} = \{\tilde{\kappa}\}$, then $\sigma^*_\kappa(a_0 \mid \tilde{\kappa}) = \bar{\varrho}_\kappa$. Finally, (ii)-(d) of Theorem 3 shows that $\sigma^*_\kappa(a_0 \mid \kappa_l) = 0$ for all $\kappa_l \in \mathcal{K} \setminus \hat{\mathcal{K}}$ such that $\kappa_l \notin \tilde{\mathcal{K}}$. ∎

**Proof of Corollary 4.**

Consider the case where $A = \{a_0, a_1\}$ and assume Assumption 1 and Assumption 2. Suppose that both sets of aspect realizations $\hat{X}$ and $\hat{Y}$ are nonempty. Consider an arbitrary aspect $\kappa \in \{x, y\}$. It follows from Assumption 2 and from the specification of the set of aspect realizations $-\bar{\mathcal{K}}(\kappa_l) \subseteq -\mathcal{K}$ that the value function $V_\kappa(\sigma^*_\kappa)$ provided by Theorem 4 can be rewritten as

$$V_\kappa(\sigma^*_\kappa) = \overline{v} \sum_{\kappa_l \in \hat{\mathcal{K}}} \psi_\kappa(\kappa_l) \\ + \sum_{\kappa_l \in \hat{\mathcal{K}}} \left[ \underline{u}\, \psi_\kappa(\kappa_l) + (\overline{u} - \underline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\kappa_l)} \psi((\kappa_l, -\kappa_l)) \right] \left\{ \frac{\overline{v}\, \psi_\kappa(\bar{\kappa})}{(\underline{u} - \overline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\bar{\kappa})} \psi((\bar{\kappa}, -\kappa_l)) - \underline{u}\, \psi_\kappa(\bar{\kappa})} \right\},$$

for some arbitrarily chosen $\bar{\kappa} \in \tilde{\mathcal{K}}$. Then, the stated result follows by setting

$$\Phi_\kappa \equiv \frac{\overline{v}}{(\underline{u} - \overline{u}) \sum_{-\kappa_l \in -\bar{\mathcal{K}}(\bar{\kappa})} \psi(-\kappa_l \mid \kappa_l) - \underline{u}},$$

for $\bar{\kappa} \in \tilde{\mathcal{K}}$, and, then, by setting $\Gamma(\kappa) \equiv V_\kappa(\sigma^*_\kappa)$ for any optimal experimentation choice $\sigma^*_\kappa \in \Sigma_\kappa$. ∎

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