Diverse Opinions and Obfuscation through Hard Evidence in Voting Environments*

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Abstract

Can leaders in voting environments obfuscate voters using only hard evidence? We study a model in which it is mandatory for leaders to make research efforts to obtain evidence. After that, leaders choose how much of the obtained evidence they disclose to voters. Research efforts maybe unsuccessful, which allows leaders to strategically conceal pieces of evidence, thus obfuscating voters. We study how obfuscation strategies, and their welfare implications, depend on whether leaders are moderate or radical. A key insight is that, through obfuscation, leaders want to persuade those voters who are closer to them within the spectrum of opinions. Radical leaders have stronger incentives than moderate leaders to conceal evidence. We also ask which majority rules and distributions of voters' external sources of information would the leaders prefer. Radical leaders prefer that voters with opinions similar to them do not have externals sources of information. Moderate leaders do not care about which voters possess external sources. In general, radical leaders and voters disagree about which voters should have external sources. When leaders are moderate, voters prefer that external sources be in the hands of those voters who are closer to the leaders in their opinions.

Keywords: Hard Evidence; Strategic Obfuscation; Persuasion; Voting

JEL Classification: C72; D72; D83; D84

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1 Introduction

In voting environments, such as political elections or committees, information provision is central to gauge the available alternatives. By listening to informed leaders, voters may modify their initial opinions and, in consequence, their preferred alternatives. Economists and political scientists have abundantly used the classical models of strategic information disclosure to explore how leaders can influence voters. In this paper, we are concerned with the disclosure of "hard evidence" to a group of voters who have diverse opinions about the best alternative. Political and committee leaders are often enforced (by institutional mandates) to make investigation efforts to collect verifiable evidence. Government agencies must seek for information on public health, extreme weather, or economic conditions. Leaders in hiring committees must obtain additional information about job candidates. Campaign leaders are enforced to disclose information, which is subject to fact-checking by the media and by state agencies. In some legal systems, board leaders must conduct research about prospective mergers prior to submitting such proposals for the approval of the shareholders.

In most practical situations, nonetheless, leaders are themselves interested in the outcome of the voting processes, and their opinions about the best course of action do not always concur with those of the (required majority of) voters. Another feature usually present in voting environments is that voters have also access to other means of information, external to the leader's provision. In this respect, the prevalent view among political scientists (Carmines and Stimson, 1980; Sniderman et al., 1991; Carpini and Keeter, 1996) is that having access to external means of information, such as education, make voters rely less on initial opinions attached to ideological positions. In practice, what voters learn from external means shift their initial opinions and interacts with the leaders' disclosures, affecting plausible obfuscation strategies and the well-beings of leaders and voters.

In these scenarios, some fundamental questions remain open. Can a leader that is enforced to investigate and disclose *only hard evidence* strategically obfuscate voters? If so, in which ways? How does the existing majority rule influence the investigation efforts of the leader? Which type of majority rule would the leader prefer? Which voters would the leader prefer to have external sources of information, voters with opinions more similar or dissimilar to his own's? Which majority rules and which distributions of external sources of information would the group of voters prefer?

To shed light on those questions, we investigate a model where a leader must acquire evidence about a variable of interest before choosing how to publicly disclose it to a group of voters. We formalize the way in which the leader can obfuscate voters through evidence by adapting key features of the model proposed by Che and Kartik (2009). A central consideration is that the leader's investigation effort may be unsuccessful, which enables him to act strategically when he obtains evidence that would harm him if disclosed. The possibility of not obtaining completely accurate information is thus employed as a resource to obfuscate voters. In addition to their information acquisition and disclosure machinery, we also incorporate Che and Kartik (2009)'s approach of "diverse opinions" about the relevant variable.

We restrict attention to voting environments with a binary choice, either accept a new initiative or remain in the status quo, such as it is the case in referendums (e.g., to hire a certain job candidate or to leave the post open, to remain or to leave the EU, to issue or not company shares). Acceptance of the new initiative requires that at least a certain number of voters prefer such an alternative, which specifies the *majority rule* in our model. We allow majority rules to vary generally, ranging from dictatorial to unanimous. Also, we distinguish between moderate and radical leaders. Radical leaders are strongly biased for either one alternative or the other, and they remain biased even if they receive immense amounts of information in favor of the other alternative. Moderate leaders hold a "centrist" view about the most suitable alternative and small changes in what they learn makes them switch his preferred alternative. As to the role played by possible external means of information, we consider that a fraction of voters may fully learn the relevant variable by themselves, according to a certain probability. Thus, following the insights of the aforementioned prevalent view in political science, external means of information may drastically change the initial opinions of voters.

Our insights can be summarized as follows.

1. Leaders have incentives (that may be either weak or strong) to conceal evidence. Through such a strategic obfuscation, leaders will target some voters with the goal of persuading them to switch to the leaders' preferred alternatives. In general, leaders want to persuade those voters with opinions more similar to their own opinions, while they do not care about voters whose opinions are very different. Radical leaders *always* want to conceal evidence. Unlike this, disclosing all successfully obtained evidence is always part of the obfuscation strategies of moderate leaders.

2. Radical leaders in favor of accepting the new initiative prefer more dictatorial majority rules. Such radical leaders wish to persuade larger sets of voters as the majority rule becomes more unanimous. Analogously, radical leaders in favor of remaining in the status quo prefer more unanimous majority rules. Such radical leaders want to persuade larger sets of voters as the majority rule becomes more dictatorial.

3. Moderate leaders are incentivized to *invest more in obtaining evidence and to obfuscate less when majority rules make voters with moderate opinions to be decisive for the election outcome*.

4. *Radical leaders prefer that external means of information not be in the hands of those voters more similar* to the leader within the opinion spectrum. In general, radical leaders and voters disagree about which voters should be educated. *Moderate leaders care little about whether or not external means of information are in the hands of voters that are similar in their opinions to the leader*. When leaders are

moderate, *the group of voters prefer that those voters whose opinions are closer to the leaders' opinions have external means of information.*

Recent history has provided anecdotal evidence that may help illustrate our model's implications. Consider, for instance, the 2016 Brexit (simple majority) referendum in the UK. For this voting environment, our model would suggest that, by concealing evidence, radical leaders in favor of any of the two options would seek to persuade those voters with similar views to their own's. According to media coverage, the campaign director of the Leave option, Dominic Cummings, spent months doing detailed evidence-based research into the relationships between the UK and the EU. However, in his campaign disclosure, he revealed a narrow set of pieces of evidence. It became famous the display of a figure (mostly on buses) that said "Let's give our NHS the £350 million the EU takes every week." Shortly after, the Office for National Statistics, concluded that such a £350 amount "did not take into account the rebate or other flows from the EU to the UK public sector (or flows to non-public sector bodies), alongside the suggestion that this could be spent elsewhere, without further explanation, was potentially misleading."[1] Thus, the available evidence that full research on the flows from the EU to the UK could have gathered should have included also rebates and other flows. Clearly, efforts to obtain all the available evidence on this point can be unsuccessful and, in particular, such pieces of evidence were not disclosed to the public by Mr. Cummings. This suggests a concealment strategy, under a logic of imperfect evidence acquisition, that is at the heart of our theoretical approach. In addition, at least to the extent that it was voiced out only through Leave campaign events and speeches,[2] not revealing all potentially available evidence was largely targeted to voters who were already followers of the Leave campaign and, thus, with opinions favorable to leaving the EU.

On the side of the Remain campaign, it became also famous the extensive reporting of the BBC on a statement by the Confederation of British Industry (CBI). "The CBI says that all the trade, investment, jobs and lower prices that come from our economic partnership with Europe is worth £3000 per year to every household." However, UK in a Changing Europe Fellow Jonathan Portes subsequently detailed that this was not a complete disclosure estimate.[3] Here again, full research could have gathered also key

[1] Also, in an interview with BBC's journalist Andrew Marr, NHS chief executive questioned the veracity of the £350 million figure.

[2] Since the UK government supported the Remain option, most prominent Remain campaigners, including David Cameron, could use official government channels—sometimes even echoed through international meetings or institutions such as the IMF—to disclose information. For instance, US president Barack Obama used government press conferences to campaign in favor of the Remain option. Leave campaigners, on the other hand, had to resort to non-official channels that, consequently, required a certain degree of involvement by attendees and followers. Presumably, such an audience was mainly composed of voters closer to the Leave option.

[3] According to Mr. Porter, such an estimate was "based on a selection of studies produced at different times (some date back well over a decade), with different methodologies, and designed to answer different questions. Some looked at the economic impact of EU membership to date, and some at the future impact of a vote to leave. Some are not even specific to the UK."

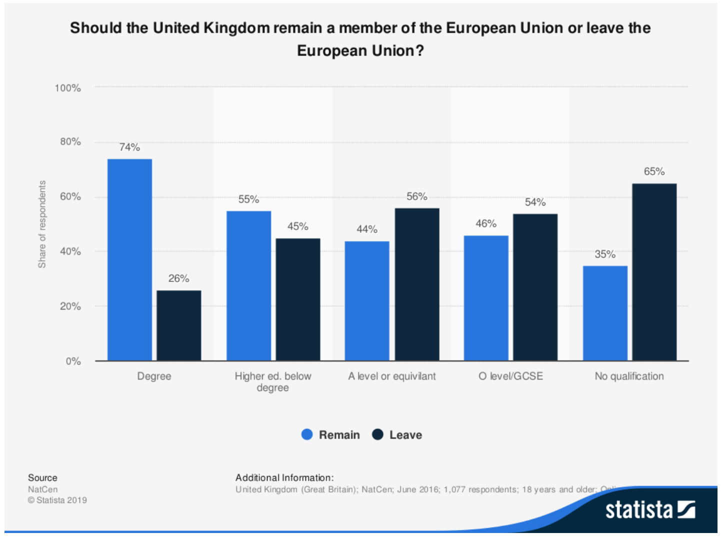

Figure 1 – Brexit Vote and Education Levels

qualifications to complete the potentially available evidence on the topic. Arguably, not disclosing all the potentially available evidence was aimed at persuading voters that paid attention to the BBC's reporting and whose options were already closer to the Remain option.[4]

The logic that our model provides for such obfuscation strategies is that trying to make voters who are far away in the opinion spectrum to change their preferred alternatives requires to conceal larger amounts of evidence (formally, according to larger subsets in the set inclusion order). This raises skepticism on the voters' side, which could be harmful for the leader. On the other hand, concealing smaller amounts of evidence can more easily change the minds up of those voters who are already very close to the leader's opinion.

On the role played by external sources of information, such as education, data on the Brexit referendum shows (Fig. 1) that the Remain option was strongly supported by highly educated voters. Inclination for the Remain option decreased dramatically among less educated voters (in particular, among voters with higher schooling or lower levels of education). One might be tempted to think that sociological or income factors—which are usually correlated with education levels—could account as well for the inclination of highly educated voters towards the Remain option. However, Fig. 2 shows that the relative inclination of voters from the upper social class in favor of the Remain option stood far below that of highly educated voters. While the margin of support for the Remain option was 14 points for voters that belong to the upper social class, the corresponding margin

[4] More broadly, our model would suggest that the incomplete disclosure of evidence by Leave leaders was aimed at persuading those voters who were already biased to thinking that leaving the EU would reduce immigration, and improve their social and income prospects. Similarly, the incomplete disclosure by Remain leaders would have been addressed to persuade voters who were already more inclined to considering that leaving the EU would damage the size and productivity of the UK's economy.

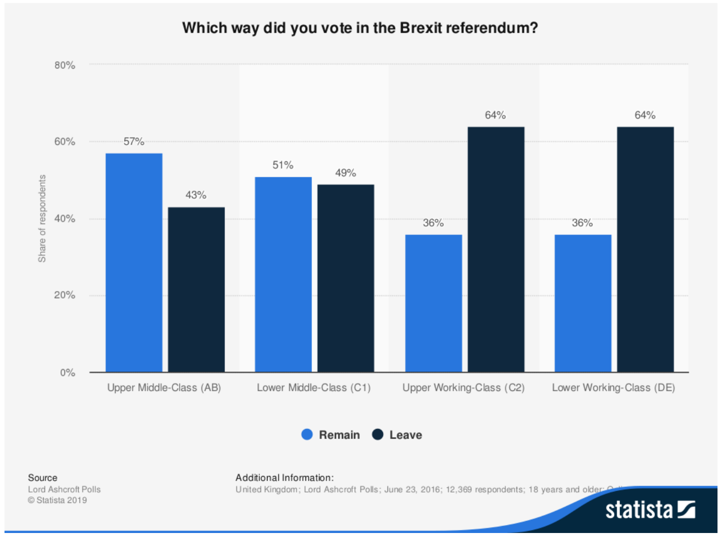

Figure 2 – Brexit Vote and Social Class

of support was 48 points for highly educated voters. This bit of evidence suggests that the informative content of education did play a role in voting in the referendum. Therefore, under the premise that education has informative content and provides analytical tools (independent of the investigation efforts and disclosure of either campaign) to gauge the convenience of leaving or staying, our model's insights would suggest the following. First, it would convey the message that it was in the interest of radical leaders in favor of the Leave option that voters inclined to leaving were relatively less educated than voters whose opinions were more aligned with remaining. Analogously, radical leaders in favor of the Remain option would prefer that education, or other means of external information about the Brexit, be in the hands of voters with opinions more aligned with the Leave option.

1.1 Literature Connections

Our paper contributes to a growing political economy literature of strategic information disclosure in voting environments. In particular, our approach critically considers a setup that allows for incomplete revelation in the presence of mandatory verifiable disclosure. The canonical model of verifiable disclosure (Grossman, 1981; Milgrom, 1981) features an "unravelling" mechanism that typically leads to full revelation. Some recent contributions, however, have proposed realistic twists to the classical model that are able to break down the unravelling mechanism. Our model builds upon one of such contributions, Che and Kartik (2009), wherein a single decision-maker receives information from an expert. The sort of questions we investigate, though, are quite different as we are interested in the features of the leader's obfuscation strategy, and its implications on welfare, in the presence of voting and of external sources of information in the hands of the voters. In an environment where voting is not considered, the primary interest of Che and Kartik (2009) is to study which level of discrepancy in opinions (between expert and receiver) would a single receiver prefer. Key ingredients of the two setups are also different.[5] Another paper where the unravelling mechanism of verifiable information fails is Dziuda (2011), wherein the expert may provide a number of bits of evidence in support of one alternative or the other. In this case, it is the assumption that receivers are uncertain about the total number of bits available for disclosure what breaks down the unravelling mechanism.

The provision of hard evidence in voting environments is also explored by Titova (2021) in a model that takes on the approach of Bayesian persuasion. While her approach does not consider the logic used in our paper of diverse opinions and imperfect evidence acquisition, leaders have in her model the ability to target voters individually so as follow fully discriminatory revelation policies. In our paper, leaders must disclose evidence publicly to all voters, thus lacking any discrimination power in their disclosure strategies.

The political economy literature has explored key questions of influential communication to voters using other traditional models of strategic information revelation. For instance, multi-dimension cheap-talk from leaders to voters has been by analyzed by Schnakenberg (2015). A model of signaling by political platforms in the presence of information acquisition on the voters' side has been studied by Bandyopadhyay et al. (2020) to propose a logic for radicalization in the information choices of electoral platforms. Bayesian persuasion in voting environments has been studied by Alonso and Camara (2016) and by Chan et al. (2019). Whereas Alonso and Camara (2016) consider a voting environment in which a sender uses a policy experiment (public signal) that aims to target different winning coalitions, Chan et al. (2019) consider a framework in which voting is costly and the sender may provide voters with private signals. The focus of Chan et al. (2019) is thus on the analysis of the benefits of private persuasion in voting environments. Also, in an environment where voters have private information of their own, vote-buying screening mechanisms have been studied by Eguia and Xefteris (2021). As to other models in which voters are not rational, a behavioral approach to information processing on the side of voters has recently been considered by Bonomi et al. (2021) to explore influential communication in voting environments.

Our interest in exploring a logic for obfuscation by leaders connects with Dewan and Myatt (2008). Using a model where individuals want to take actions that be both suitable to a variable of interest and to the actions of the entire group, Dewan and Myatt (2008) provide a rationale for obfuscation when leaders compete for audiences and are able to choose the clarity of their information disclosure. The idea here is that the incentives of the leaders to attract the attention of the audience for longer periods makes them lower strategically the clarity of their speeches, therefore, obfuscating their followers. The fundamental questions explored, as well as the main ingredients of the two models, are quite different. For instance, Dewan and Myatt (2008) do not consider voting or verifiable infor-

[5] Che and Kartik (2009) considers a continuum of possible actions which involves a totally different approach to explore equilibrium, relative to the one considered in the current paper.

mation, whereas we do not consider the implications of individuals acting in consonance with others.

Manipulative behavior from informed experts is also connected to media biased reporting. Using a bias confirmatory approach where listeners wish to see their own opinions confirmed by new information, Mullainathan and Shleifer (2005) investigate slanting in media reporting. Also, exploiting reputation concerns of media firms to signal high qualities, Gentzkow and Shapiro (2006) provide a rationale for such sources of information strategically adjusting their reporting to the listeners' opinions.

While our paper does not consider how competition among leaders affect their disclosure policies, there are also connections with our motivation to study the incentives of leaders to withhold information. A fast-developing literature on political science has investigated the effects of increased competition on the informative content of the leaders' disclosures. When voters are rational, an insight largely put forward by this strand of the literature is that competition forces leaders to align better their incentives with those of the voters, thereby enabling more precise information disclosure. This is the general message conveyed, among others, by Baron (2006); Chan and Suen (2009); Anderson and McLaren (2012); Duggan and Martinelli (2011). However, following recent empirical evidence on traditional and social media based polarization (Gentzkow and Shapiro, 2010; Allcott et al., 2020), the direction of this insight has been questioned recently by Perego and Yuksel (2021). In their model, information providers are non-partisans and compete for profits. Using the key consideration of different dimensions of interest in the voters' preferences, Perego and Yuksel (2021)'s main takeaway is that competition among information providers may boost disagreement across voters.

At a theoretical level, there are some connections with the literature on probabilistic voting in spatial models of elections, as reviewed by Banks and Duggan (2005). In the model that we study, obfuscation make voters assess their preferred alternatives in a probabilistic way when evidence is not disclosed by the leader. As a consequence, leaders must assess the outcome of voting in a probabilistic way. The sort of questions that we analyze, though, depart fundamentally from the existence and optimality issues addressed typically by this strand of literature.

Finally, at a more empirical level, our exploration of optimal obfuscation strategies has also some connections to the empirical research of Kono (2006) on the transparency of trade policies followed by leaders to obfuscate voters.

The paper is organized as follows. In Section 2 we present the model. Section 3 offers preliminaries to the equilibrium analysis, which we develop fully in Section 4. Section 5 and Section 6 provide insights about the well-beings of the leader and of the group of voters, respectively. Section 7 comments on further empirical evidence to illustrate the paper's insights. The formal arguments omitted in the main text are relegated to Appendices A,B.

2 Model Setup

A political, or committee, leader $i = l$ (he) and a group of voters $i \in N \equiv \{1, \ldots, n\}$ (each of them, she) are interested in some underlying *state of the world* ω, such as the state of the economy or the quality of a job candidate. The leader must (by institutional mandate) investigate to obtain "hard evidence" (e.g., data, scientific reports) about ω, which he can subsequently disclose (publicly) to the voters.

The state ω is distributed $N(\mu_i, 1)$ *from the perspective of player* $i \in \{l\} \cup N$. Thus, we assume that all players agree on the underlying distribution (and variance) of the relevant state, but they disagree on its mean. This approach crudely captures the idea that the players have different (prior) opinions about the unknown variable of interest. In some practical situations, we may regard such opinions μ_i as being attached to ideological positions. We assume that such diverse opinions are common knowledge though, so that players "agree to disagree."[6] Throughout the paper, we will use $\mathbb{P}_i[\cdot]$ and $\mathbb{E}_i[\cdot]$ to denote the probability and expectation operators, respectively, from the perspective of player i. In some cases where all players agree on the underlying probability space, we will switch to notation $\Pr[\cdot]$ to indicate the corresponding probability operator.[7]

Without loss of generality, we consider that there is an even number n of voters. Furthermore, we assume that opinions are heterogenous across players, with the particular form: $\mu_l \in \mathbb{R}$, and $\mu_n < \cdots < \mu_{(n/2)+1} < 0 < \mu_{n/2} < \cdots < \mu_1$.[8]

Each voter $i \in N$ must simultaneously announce her preferred alternative, or personal vote, $v_i \in \{A, R\}$, where A means accept a new initiative and R means reject it to remain in the "status quo." The new initiative is accepted by means of voting if at least a certain number $k \in N$ of voters announce a personal vote $v_i = A$. Otherwise, the proposal is rejected so that k parameterizes the *majority rule* required to approve the new initiative.

[6] Thus, such assumptions challenge the commonly accepted view in game-theoretic models—known as the *Harsanyi doctrine*—that heterogenous priors cannot persist if fully rational players have common knowledge either of such priors or of the learning processes of others. Nevertheless, we follow some recent efforts to understand the practical implications of individuals having different opinions—e.g., Che and Kartik (2009), Mullainathan and Shleifer (2005). An alternative, more complex, formulation to allow for different opinions could consider that the players have instead common priors but receive private uncorrelated signals about such priors. Then, if we assume that how and what the players learn from their signals remains their private information, these considerations would provide a setup with common priors that yet captures the idea proposed in this paper that the players begin with heterogeneous opinions before making any strategic choice. Acemoglu et al. (2016) have recently proposed an interesting approach to justify the persistence of different opinions in game-theoretical models by introducing uncertainty on learning processes.

[7] In particular, all players will agree on the probability of the outcome of the voting process even though they disagree about the state of the world. In short, the players are aware of the opinions of others and incorporate such different opinions to assess, in a common manner, which will be the outcome from voting.

[8] Thus, without any further information, half of the voters prefer acceptance of the new initiative and half of them prefer rejection. Based on the opinions of the voters, the proposal would be accepted under the simple majority rule, $k = n/2$, or under more dictatorial rules, $k < n/2$.

Thus, given a *profile of personal votes* $v = (v_1, \ldots, v_n)$, the outcome $o(v)$ of the voting process is given by $o(v) = A$ if $|\{i \in N \mid v_i = A\}| \geq k$, and $o(v) = R$ otherwise.[9] The two extreme majority rules $k = 1$ and $k = n$ correspond, respectively, to *dictatorship* and to *unanimity*. For values of $k \leq n/2$, we will say that the rule becomes "more dictatorial" as k lowers and, similarly, for values of $k > n/2$, we will say that the rule becomes "more unanimous" as k raises.

Players care about the suitability of the available alternatives to the state of the world according to a common utility function $u : \{A, R\} \times \mathbb{R} \to \{0, -1\}$. In particular, for each $i \in \{l\} \cup N$, u is specified as: for $\omega < 0$, we have $u(R, \omega) = 0$ and $u(A, \omega) = -1$, whereas for $\omega \geq 0$, we have $u(R, \omega) = -1$ and $u(A, \omega) = 0$. Thus, each player (strictly) prefers rejection if $\omega < 0$, and acceptance if $\omega \geq 0$. Notably, there is no conflict of interests regarding the most desirable course of action *conditional on the actual realization of the state*. The disagreement among the players takes place only at the level of opinions.

We will consider three (qualitatively different) categories for the leader, based on his own position relative to the spectrum of opinions. A *(centrist) moderate* leader will be captured by considering that $\mu_l = 0$, a radical leader biased towards approving the new initiative will be described by considering that $\mu_l = \overline{\mu} > \mu_1$, and a *radical* leader biased towards remaining in the status quo will be captured by considering that $\mu_l = \underline{\mu} < \mu_n$. Furthermore, merely for technical reasons relative to the Gaussian distribution, in most of the analysis we will additionally consider that $\overline{\mu} \to +\infty$ and $\underline{\mu} \to -\infty$.

Given the preferences of the leaders, note that in some circumstances it could be the case they prefer not to acquire any information at all about the state if they had such an option. A key assumption of the model, however, is that the leader *must* make a positive investment in acquiring additional information about the state. Accordingly, our model considers that there will be additional information available about the relevant state in the hands of the leaders.

2.1 Voters' Private Information

Voters may possess some private information by themselves about the underlying state. We interpreted this as information that voters can obtain from any source external to the leader's information disclosure. For simplicity, we encompass all possible forms of private information that a voter may have under the label *education*.[10] In particular, each voter $i \in N$ may be either uneducated, $x_i = ne$, or educated, $x_i = e$. Let $x = (x_1, \ldots, x_n) \in \{ne, e\}^n$ be a *profile of education levels*. Educated voters are endowed with a (common) probability $\varepsilon \in (0, 1)$ of learning the true state of the world. Conditional on obtaining

[9] Therefore, without loss of generality, we consider that ties are broken by having the proposal accepted in the case of a tie.

[10] In modern democracies or voting systems, our "education" label thus captures as well any other sources external to the leader's efforts, such as independent media, social networks, information accessible through the Internet, and so on.

education, the distribution according to which voters learn the true state is independent across voters. The outcome of an education level $x_i \in \{ne, e\}$ remains voter i's private information. Thus, we consider that there are no *educational spillovers* across voters, neither from the voters to the leader.

2.2 *Leader's Information Acquisition and Disclosure*

Although he cares about the outcome of the election as well, the leader has an institutional mandate to acquire further information about the relevant state, and to communicate publicly with the voters about his findings. The analysis of the leader's information acquisition and disclosure follows closely the approach that Che and Kartik (2009) propose for the case of a single decision-maker that receives information from an expert. In particular, the leader acquires information by choosing the probability $\lambda \in (0, \bar{\lambda}]$, with $\bar{\lambda} < 1$, of obtaining a noisy signal s about ω, at a cost $c(\lambda)$. By doing so, the leader chooses the likelihood of his investigation being successful. The cost function $c(\cdot)$ is smooth, increasing, convex, and it satisfies the typical Inada conditions $\lim_{\lambda \to 0} c'(\lambda) = 0$ and $\lim_{\lambda \to \bar{\lambda}} c'(\lambda) = +\infty$. Notably, because of his institutional role, the leader does not have the option of not acquiring any information about the state. Therefore, the leader must choose positive efforts $\lambda > 0$ to learn about the variable of interest. Voters can verify the investigation effort λ exerted by the leader. While we consider that the maximum investigation effort that the leader can exert is bounded (by $\bar{\lambda}$), we impose no specific minimum required level of investigation effort.[11]

Then, with probability λ, the leader obtains a signal $s \equiv \omega + \eta$, where η is a "noise term" distributed $N(0,1)$, uncorrelated with the state. Under our normality assumptions, the information structure that relates the state with the signal is captured by a normally distributed random pair (ω, s) such that, for each player $i \in \{l\} \cup N$, it follows that $\mathbb{E}_i[s] = \mathbb{E}_i[\omega] = \mu_i$, $var[\omega] = 1$, $var[s] = 2$, and $cov[\omega, s] = 1$. We shall use $f(\cdot; \mu_i)$ and $F(\cdot; \mu_i)$ to indicate the density and the (cumulative) distribution function of the signal s from player i's perspective, which distributes $N(\mu_i, 2)$. With the complementary probability $1 - \lambda$, the leader's investigation is unsuccessful and he obtains nothing from his investigation efforts, which we denote as obtaining signal $s = \emptyset$.

After learning privately the outcome of his investigation, the leader chooses whether or not to disclose such findings *publicly to all* voters. The information contained in signal s is (verifiable) "hard evidence" and it cannot be modified or falsified. Thus, if the leader obtains the signal and chooses to disclose it, he is constrained to transmitting true information to the voters. In addition to the intensity of his investigation effort, the only other strategic choice of the leader, therefore, is whether to disclose or to conceal the signal when his investigation is successful. Since the leader can choose to conceal signals, in the event that the voters are reported signal $s = \emptyset$, they update their beliefs (in a Bayesian

[11] The ideas behind these requirements are intuitive, yet we choose the above stated forms for such restrictions for technical reasons.

way) to assess whether the leader's investigation has indeed been unsuccessful, or he is instead hiding evidence. Although the leader is restricted to making positive investigation efforts, notice that the suggested mechanism enables him to act strategically about how he communicates with the voters.

Given an investigation effort λ and a set of signals $C \subseteq \mathbb{R}$ that the leader conceals from all voters, let $v_i(\cdot \mid C, \lambda) : \mathbb{R} \cup \{\emptyset\} \to \{A, R\}$ be a *personal voting rule* for voter i. In particular, $v_i(s \mid C, \lambda)$ gives us the preferred alternative of voter i upon observing signal $s \in \mathbb{R} \cup \{\emptyset\}$. Also, let $v(s \mid C, \lambda) \equiv (v_1(s \mid C, \lambda), \ldots, v_n(s \mid C, \lambda))$ be a *profile of personal voting rules* conditional on the observed signal $s \in \mathbb{R} \cup \{\emptyset\}$.

2.3 Time Line

The timing of the game played by the leader and the voters is as follows. First, nature chooses the state of the world ω and the profile of education levels x. While ω remains unknown to everyone, the profile x becomes publicly known. In a second stage, without any further information about the underlying state ω, the leader chooses his investigation effort λ. The leader's investigation effort (or, alternatively, the investigation cost $c(\lambda)$ incurred) becomes commonly known to all voters. The leader observes the outcome of his investigation, which is unobservable to the voters. Then, the leader chooses whether to disclose or to conceal the successful outcome of his investigation. In a third stage, each voter i announces her preferred alternative v_i. Based on the considered majority rule k, an outcome $o(v)$ is then obtained from the preferred alternatives $v = (v_1, \ldots, v_n)$ announced.

The equilibrium notion that we use is that of perfect Bayes equilibrium—to which we will simply refer as *equilibrium*. To avoid uninteresting equilibria, we will restrict attention to equilibria in which each voter announces her personal vote according to her preferred alternative, regardless of whether her vote would be inconsequential to the voting outcome.[12]

2.4 Interim Information

Given our normality assumptions, if a player $i \in \{l\} \cup N$ observes a signal s and has no further information about the underlying state, then i considers that $\omega \mid s$ follows a normal distribution with posterior mean $\mathbb{E}_i[\omega \mid s] = [\mu_i + s]/2$.[13] Similarly, when voter $i \in N$ is educated, she considers that $\omega \mid s, e$ follows a normal distribution with posterior mean $\mathbb{E}_i[\omega \mid s, e] = \varepsilon\omega + (1-\varepsilon)[\mu_i + s]/2$.

[12] Specifically, we focus on equilibria such that for any two given $v_i \neq v_i'$, if $u(o(v_i, v_{-i}), \omega) = u(o(v_i', v_{-i}), \omega)$, then voter i announces v_i if and only if $u(v_i, \omega) > u(v_i', \omega)$. Although our model does not consider an abstention alternative, we wish to guarantee that the analysis focuses on meaningful equilibria that avoid the sort of "swing voter's curse" implications (see, e.g., the seminal paper by Feddersen and Pesendorfer (1996), and a subsequent body of literature both in economics and in political science).

[13] That is the case when voter $i \in N$ is not educated. Then, $\mathbb{E}_i[\omega \mid s, ne] = \mathbb{E}_i[\omega \mid s] = [\mu_i + s]/2$.

3 Preliminaries: Persuading a Single Voter

Before exploring the logic behind the (optimal) behavior of the leader in the proposed voting setup, let us study a situation with *a single voter i* whom the leader wishes to persuade. Suppose that the investigation effort of the leader is successful so that he obtains a certain signal $s \in \mathbb{R}$.

Consider first the situation where voter i is uneducated ($x_i = ne$). This voter can improve her information about the state only by observing the signal s obtained by the leader. Then, the expected utility that such a voter receives satisfies $\mathbb{E}_i[u(A,\omega) \mid ne, s] > \mathbb{E}_i[u(R,\omega) \mid ne, s]$ if and only if $\mathbb{E}_i[\omega \mid s] \geq 0$. Clearly, such a voter i strictly prefers acceptance over rejection if and only if $s \geq -\mu_i$. Secondly, consider the situation where voter i is educated ($x_i = e$). Then, note that

$$\begin{aligned} \mathbb{E}_i[u(A,\omega) \,| e, s] > \mathbb{E}_i[u(R,\omega) \mid e, s] \;&\Leftrightarrow\; \mathbb{E}_i[\omega \mid e, s] \geq 0 \\ &\Leftrightarrow\; s \geq -\mu_i - \left(\frac{2\varepsilon}{1-\varepsilon}\right)\omega. \end{aligned}$$

Education gives the voter the opportunity of learning the true realization ω of the state. Unlike this, the leader does not learn ω and can only compute $\mathbb{E}_l[\omega \mid s]$. Accordingly, the leader considers that voter i prefers acceptance if and only if

$$s \geq -\mu_i - \left(\frac{2\varepsilon}{1-\varepsilon}\right)\mathbb{E}_l[\omega \mid s] \;\Leftrightarrow\; s \geq -\big[(1-\varepsilon)\mu_i + \varepsilon\mu_l\big].$$

Notice that the leader anticipates that, with probability ε, the voter will be better informed than himself.

The conditions derived above for both cases, those of an uneducated and of an educated voter, allow us to define the *critical signal realization* $\bar{s}_i(x;\varepsilon,\mu_l)$ (for any voter $i \in N$) as

$$\bar{s}_i(x;\varepsilon,\mu_l) \equiv \begin{cases} -\mu_i \;\text{ if } x_i = ne; \\ -\big[(1-\varepsilon)\mu_i + \varepsilon\mu_l\big] \;\text{ if } x_i = e. \end{cases} \tag{1}$$

Given the profile of education levels x, the probability ε of education being fruitful, and the opinion μ_l of the leader, the critical signal realization $\bar{s}_i(x;\varepsilon,\mu_l)$ determines then a cutoff value for observed signals such that voter i prefers rejection whenever she observes $s < \bar{s}_i(x;\varepsilon,\mu_l)$ and acceptance whenever she observes $s \geq \bar{s}_i(x;\varepsilon,\mu_l)$. On the other hand, the leader himself can only obtain additional information about the underlying state through his investigation. Therefore, he prefers acceptance if and only if he observes a signal $s \geq -\mu_l$. Thus, we set the critical signal realization for the leader as $\bar{s}_l \equiv -\mu_l$.

Noting the discrepancies between the critical signal realizations $\bar{s}_l$ and $\bar{s}_i(x;\varepsilon,\mu_l)$, it will be convenient throughout the analysis to pay attention to subsets of signals $C_i =$

$C_i(\mu_l;x)$ with the following forms: $C_i(\mu_l;x) \equiv [\bar{s}_l, \bar{s}_i(x;\varepsilon,\mu_l))$ if $\bar{s}_l < \bar{s}_i(x;\varepsilon,\mu_l)$, or $C_i(\mu_l;x) \equiv [\bar{s}_i(x;\varepsilon,\mu_l), \bar{s}_l)$ if $\bar{s}_l > \bar{s}_i(x;\varepsilon,\mu_l)$. For those signals $s \notin C_i$, the leader's preferred alternative coincides with that of voter i. Conditional on $s \notin C_i$, the preferred alternative of voter i is

$$v_i^*(s \mid C_i, \lambda) = \begin{cases} A & \text{if } s \geq \bar{s}_i(x;\varepsilon,\mu_l); \\ R & \text{if } s < \bar{s}_i(x;\varepsilon,\mu_l). \end{cases} \tag{2}$$

However, for signals $s \in C_i$, the leader and the voter disagree on the best course of action.[14] Then, the leader will have certain incentives to conceal signals $s \in C_i$. By doing so the leader *obfuscates* voter i in an attempt of persuading her to change her preferred alternative.

We turn now to study how the single voter i processes (and best responds to) the information disclosed when the leader reports that his investigation efforts have been unsuccessful, so that the voter observes $s = \emptyset$. From the previous arguments, it will be convenient to restrict attention to the family $\mathscr{C}_i$ of all subsets of the interval $C_i = C_i(\mu_l;x)$. Of course, the set $C_i(\mu_l;x)$ will give us the largest set (according to the set inclusion order) within the family $\mathscr{C}_i$ of plausible sets of concealed signals.

Suppose then that the leader chooses a set C of concealed signals from the suggested family $\mathscr{C}_i$ of subsets. Then, voter i assigns probability λ to the leader's investigation having been successful or, equivalently, to the event that $s \in C$. In this case, the voter places herself in the leader's position and uses the signals $s \in C$, yet combined with her own prior information about the state, in order to determine her preferred alternative v_i.[15] In addition, voter i assigns probability $1-\lambda$ to the leader's investigation having indeed been unsuccessful. In this case, the voter is left only with her own prior opinion μ_i about the state to determine her preferred alternative v_i.

Given a subset of signals $C \in \mathscr{C}_i$ that the leader may conceal and an investigation effort λ, notice that the optimal personal voting rule $v_i^*(\emptyset \mid C, \lambda)$ of voter i, when the leader discloses signal $s = \emptyset$, takes the form of a *mixed strategy*. In particular, with probability

$$\pi_i(C;\lambda) \equiv \frac{\lambda \mathbb{P}_i[s \in C]}{\lambda \mathbb{P}_i[s \in C] + (1-\lambda)}, \tag{3}$$

voter i assesses her preferred alternative according to

$$v_i^*(\emptyset \mid C, \lambda) = \begin{cases} A & \text{if } \mathbb{E}_i[s \mid s \in C] \geq \bar{s}_i(x;\varepsilon,\mu_l); \\ R & \text{if } \mathbb{E}_i[s \mid s \in C] < \bar{s}_i(x;\varepsilon,\mu_l), \end{cases} \tag{4}$$

where $\mathbb{E}_i[s \mid s \in C] = \int_C s f(s;\mu_i) ds$. With the complementary probability $1 - \pi_i(C;\lambda)$

[14] In particular, conditional on observing any signal $s \in C_i$, the disagreement takes the following particular form: (a) if $\bar{s}_l < \bar{s}_i$, then the leader prefers acceptance whereas the voter prefers rejection and (b) if $\bar{s}_l > \bar{s}_i$, then the leader prefers rejection whereas the voter prefers acceptance.

[15] Intuitively, the voter would in this case use the leader's "technology" but according to her own opinion.

such a voter i assesses her preferred alternative according to

$$v_i^*(\emptyset \mid C, \lambda) = \begin{cases} A & \text{if } \mu_i \geq 0; \\ R & \text{if } \mu_i < 0. \end{cases} \tag{5}$$

We can now comment on the skeptical reaction of the voter to the leader's disclosure. Note that, for any pair of subsets $C, C' \in \mathscr{C}_i$, we have that $C \subset C'$ implies $\pi_i(C'; \lambda) > \pi_i(C; \lambda)$. That is, the probability $\pi_i(C; \lambda)$, according to which voter i uses the concealed signals is strictly increasing in the set inclusion order—with the restriction to the class of subsets of interest $\mathscr{C}_i$. Therefore, enlarging the set of concealed signals beyond $C_i(\mu_l; x)$ raises the voter's "skepticism" and leads to a personal voting decision clearly unfavorable to the leader. This idea is nicely captured under the term *prejudicial effect* by Che and Kartik (2009). Because of this skeptical reaction, the leader has no incentives to conceal signals $s \notin C_i(\mu_l; x)$ to the single voter i. Therefore, *the real interval* $C_i(\mu_l; x)$ gives us the largest set that a leader with opinion μ_l optimally wishes to conceal from voter i.

For the case of interest in our model, in which the leader faces instead a set of voters under a given majority rule, we will consider the class $\mathscr{C} \equiv \cup_{i \in N} \mathscr{C}_i$ of possible largest *concealment sets* $C_k(\mu_l; x)$ *under a majority rule* k. Then, we will resort to the logic presented in this Section 3 to investigate how a leader with opinion μ_l will be interested in concealing all signals $s \in C_k^*(\mu_l; x) \subseteq C_k(\mu_l; x)$, for some largest concealment set $C_k(\mu_l; x) \in \mathscr{C}$, and disclosing the rest of signals that he obtains. In addition to the leader's opinion μ_l, the optimal concealment set $C_k^*(\mu_l; x)$ will naturally depend on the education profile x and on the existing majority rule k.

4 Equilibrium Analysis

Equilibrium requires that each player best responds to the choices of the rest of players. In particular, the leader must choose the probability $\lambda \in (0, \bar{\lambda}]$ of his investigation being successful and a subset of signals $C_k^*(\mu_l; x) \in \mathscr{C}$ that he conceals to *all* voters, whereas each voter i must decide her preferred alternative, or personal vote, $v_i \in \{A, R\}$.

4.1 Leader's (Optimal) Obfuscation Strategy

We turn to study how the leader designs his optimal concealment set $C_k^*(\mu_l; x)$ by taking into account the role played by voting. Though similar in spirit to the case where the leader persuades a single voter (Section 3), the analysis of information disclosure to several voters (that may possess information of their own as well) requires additional considerations.

Fix a given majority rule $k \in N$. Recall that the voters' opinions are ordered in a way that entails $-\mu_1 < \cdots < -\mu_{n/2} < 0 < -\mu_{(n/2)+1} < \cdots < -\mu_n$. However, given that some voters may be educated while others uneducated, the relevant ordering is the one induced over the set of critical signal realizations $\bar{s}_i(x;\varepsilon,\mu_l)$ defined in Eq. (1). Therefore, we simply reorder the critical signal realizations of all the voters by considering

$$\bar{s}_{i_1}(x;\varepsilon,\mu_l) < \cdots < \bar{s}_{i_j}(x;\varepsilon,\mu_l) < \bar{s}_{i_j+1}(x;\varepsilon,\mu_l) < \ldots \bar{s}_{i_n}(x;\varepsilon,\mu_l). \tag{6}$$

Notice that we may need to relabel indexes as expressed above (say, from i to i_j). Let us then use $\sigma(x) = (\bar{s}_{i_1},\ldots,\bar{s}_{i_j},\ldots,\bar{s}_{i_n})$ to refer to such an ordering of the critical signal realization under a given education profile x.[16]

Suppose first that $C = \emptyset$ so that the leader does not conceal any signal that he obtains. Since approval of the new initiative requires that at least k voters prefer acceptance, it follows that the particular voter i_k, whose name is associated to the reordering $\sigma(x)$ described in Eq. (6), would be *pivotal* in the election process. Rejection would be the outcome of the election conditional on the voters observing any signal $s < \bar{s}_{i_k}$, whereas acceptance would be the outcome of the election conditional on the voters observing any signal $s \geq \bar{s}_{i_k}$. In short, using the derivation of a voter's personal vote in Eq. (2), the probability that the outcome of the election be acceptance, conditional on the leader not conceling any signal and on the voters observing observing a signal $s \in \mathbb{R}$, is simply given by $\Pr[o(v^*(s \mid C,\lambda)) = A] = 0$ if $s < \bar{s}_{i_k}$ and $\Pr[o(v^*(s \mid C,\lambda)) = A] = 1$ if $s \geq \bar{s}_{i_k}$.

Now, consider a majority rule k and a concealment set $C \in \mathscr{C}$ such that $C \neq \emptyset$. Then, the probability that any voter i prefers acceptance when she receives a signal $s = \emptyset$ is

$$\Pr[v_i^*(\emptyset \mid C,\lambda) = A] = \pi_i(C,\lambda)\mathscr{I}(i,C) + [1-\pi_i(C,\lambda)]\mathscr{J}(i), \tag{7}$$

where $\mathscr{I}(i,C)$ and $\mathscr{J}(i)$ are indicator functions specified, respectively, by $\mathscr{I}(i,C) = 1$ if $\mathbb{E}_i[s \mid s \in C] \geq \bar{s}_i$, and $\mathscr{I}(i,C) = 0$ otherwise, and by $\mathscr{J}(i) = 1$ if $\mu_i \geq 0$, and $\mathscr{J}(i) = 0$ otherwise. The expression given in Eq. (7) follows from our earlier analysis (in Section 3, where optimal voting behavior was described by Eq. (4) and Eq. (5)) for the case where the leader faces a a single voter.

The expression in Eq. (7) is derived by putting together the following considerations. On the one hand, voter i considers that the leader obtained and concealed a signal with probability $\pi_i(C,\lambda)$. Conditional on this event, it follows from Eq. (4) that voter i prefers acceptance if and only if $\mathbb{E}_i[s \mid s \in C] \geq \bar{s}_i$. This leads directly to i preferring acceptance with probability $\pi_i(C,\lambda)$ if $\mathbb{E}_i[s \mid s \in C] \geq \bar{s}_i$, and with probability zero if $\mathbb{E}_i[s \mid s \in C] < \bar{s}_i$. On the other hand, voter i considers that the leader obtained no signal with probability $1 - \pi_i(C,\lambda)$. From Eq. (5), we know that, conditional on this event, voter i prefers acceptance if and only if $\mu_i \geq 0$. Since the two described events (that the leader is successful in his efforts and conceals the signal, and that he is unsuccessful) are disjoint, the probability derived in Eq. (7) follows in an additive manner by applying the total probability rule.

[16] As mentioned in fn. 7, all players incorporate the different opinions of everyone and are able to assess in a common manner the probability that the outcome of the election be either acceptance or rejection.

We turn now to obtaining key features of the voting outcome when voters do not observe signals in the presence of a concealment interval $\emptyset \neq C \in \mathscr{C}$. For a majority rule k, an investigation effort λ, and a set of signals $C \in \mathscr{C}$ that the leader may conceal, let

$$\phi_k(C,\lambda) \equiv \Pr[o(v^*(\emptyset \mid C,\lambda)) = A] \tag{8}$$

denote the probability that the outcome of the election be acceptance conditional on the voters receiving $s = \emptyset$. Consider the voter i_k that results from the ordering $\sigma(x)$ induced by the existing profile of education levels x in the previously described situation where all successfully obtained signals are disclosed. Recall, however, that we are now considering that $C \neq \emptyset$ so that some obtained signals are concealed. If the associated critical signal realization $\bar{s}_{i_k}$ satisfies $\bar{s}_{i_k} \notin C$, then it follows that

(a) $\bar{s}_{i_k} < \inf C \Rightarrow \phi_k(C,\lambda) = 1$ because signals $s \in C$ such that $s \geq \bar{s}_{i_k}$ make at least k voters prefer acceptance.

(b) $\bar{s}_{i_k} \geq \sup C \Rightarrow \phi_k(C,\lambda) = 0$ because signals $s \in C$ such that $s < \bar{s}_{i_k}$ make less than k voters prefer acceptance.

On the other hand, for the case where $\bar{s}_{i_k} \in C$, the players want to assess whether or not the concealed signals $s \in C$ could induce at least a number k of voters to prefer acceptance. To this end, we use the expression in Eq. (7) above to obtain insights, in Lemma 1 below, about the probability of attaining the voting outcome $o(v) = A$ when $\bar{s}_{i_k} \in C$. It is useful to distinguish between the three different categories of the leader.

LEMMA 1. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader designs a concealment set $C \in \mathscr{C}$ such that $\bar{s}_{i_k} \in C$. Further, suppose that voters observe signal $s = \emptyset$. Then, the following implications about probability $\phi_k(C,\lambda)$ hold.

1. Moderate leader ($\mu_l = 0$).

(a) Suppose that $k \leq n/2$ and consider the largest concealment set $C_k = [\bar{s}_{i_k}, 0)$. Then, $\phi_k(C,\lambda) = 1$ for any subset $C \subseteq C_k$.

(b) Suppose that $k > n/2$ and consider the largest concealment set $C_k = [0, \bar{s}_{i_k})$. Then, $\phi_k(C,\lambda) = 0$ for any subset $C \subseteq C_k$.

2. Radical leader in favor of the new initiative ($\mu_l \to +\infty$). Consider the largest concealment set $C_k = (-\infty, \bar{s}_{i_k})$. Then, for any subset $C \subseteq C_k$, (a) $\phi_k(C,\lambda) > 0$ for $k \leq n/2$ and (b) $\phi_k(C,\lambda) = 0$ for $k > n/2$.

3. Radical leader in favor of the status quo ($\mu_l \to -\infty$). Consider the largest concealment set $C_k = [\bar{s}_{i_k}, +\infty)$. Then, for any subset $C \subseteq C_k$, (a) $\phi_k(C,\lambda) = 1$ for $k \leq n/2$, and (b) $\phi_k(C,\lambda) < 1$ for $k > n/2$.

The result in 1. of Lemma 1 leads directly to the implication that the leader would be indifferent between disclosing all obtained signals or concealing all signals $s \in C$. The result in 2. (a) of Lemma 1 follows because, upon observing $s = \emptyset$, the set of voters $\{i_1, \dots, i_k\}$ prefer the new initiative with positive probability. This is the case since each of those voters will place a positive probability on the event that the leader investigation efforts were unsuccessful. In consequence, assessing their preferred alternative will be based (to some extent) on their priors. This leads to that the outcome $o(v) = A$ happens with positive probability (i.e., $\phi_k(C, \lambda) > 0$) when all signals $s \in C = (-\infty, \bar{s}_{i_k})$ are concealed. Crucially, if voters received instead signals $s < \bar{s}_{i_k}$, then the voting outcome $o(v) = A$ would have probability zero, as only less than k voters would prefer for the new initiative. The result in 3. (b) of Lemma 1 follows because, upon observing $s = \emptyset$, the set of voters $\{i_k, \dots, i_n\}$ prefer the new initiative with probability less than one. Similarly to the situation described in 2. (a), each of those voters will place a positive probability on the event that the leader investigation efforts were unsuccessful. In consequence, assessing their preferred alternative will be based (to some extent) on their priors. This leads to that the outcome $o(v) = A$ happens with probability less that one (i.e., $\phi_k(C, \lambda) < 1$) when all signals $s \in C = [\bar{s}_{i_k}, +\infty)$ are concealed. If the voters instead received signals $s \geq \bar{s}_{i_k}$, then the voting outcome $o(v) = A$ would happen with probability one, as no less than k voters would prefer the new initiative.

The main implications of this analysis are that, conditional on his investigation effort being successful, the leader will have (sometimes, only weak) incentives to design the optimal concealment set $C_k^*(\mu_l; x) \subseteq C_k(\mu_l; x)$ so as to conceal either (a) all signals $s \in [\bar{s}_l, \bar{s}_{i_k})$ if $\bar{s}_l < \bar{s}_{i_k}$, or (b) all signals $s \in [\bar{s}_{i_k}, \bar{s}_l)$ if $\bar{s}_l > \bar{s}_{i_k}$.[17] The key point is that the leader would obtain the negative payoff -1 with probability one regardless of whether he conceals such signals, as it is the case in point 1. of Lemma 1. On the other hand, concealment of such signals allows him to obtain the expected payoff of either $-[1 - \phi_k(C_k^*, \lambda)]$, in the sort of situations described by point 2. (a) of Lemma 1, or of $-\phi_k(C_k^*, \lambda)$, in situations as the ones described by point 3. (b) of Lemma 1. This gives us the formal description of the logic behind obfuscation suggested by our model. In addition, the earlier arguments lead to that the leader only wants to conceal those signals. This is the case because disclosing signals $s \notin C_k^*(\mu_l; x)$ already makes the outcome of the election coincide with the alternative preferred by the leader. Concealing any of those signals raises the voters' skepticism, which could be harmful for the leader (as argued in Section 3).

In general, it is not obvious whether or not to regard the voter with label i_k (that stems from the induced ordering $\sigma(x)$) as decisive, or pivotal, to switch the outcome, conditional on signals being concealed ($C \neq \emptyset$) and on the voters receiving no signal ($s = \emptyset$). In particular, the possibility of concealed signals poses difficulties to the intuitive idea of a pivotal voter in cases where voters different from i_k could change their preferred alternative upon receiving $s = \emptyset$. In turn, these difficulties affect the determination of the probability $\phi_k(C, \lambda)$ according to which the outcome of voting is acceptance. Observa-

[17] Recall that conditional on the leader observing a signal $s \in [\bar{s}_l, \bar{s}_{i_k})$, he prefers acceptance whereas the voting outcome would be rejection. Similarly, for those signals $s \in [\bar{s}_{i_k}, \bar{s}_l)$, the leader prefers rejection, yet the outcome of the election would be acceptance.

tion 1 below develops some analytical arguments on this point.

OBSERVATION 1. In our setup, voter i_k stands formally as *the voter who would be pivotal in a (hypothetical) situation where all signals were disclosed*. When all signals are disclosed, we can straightforwardly consider such a notion of pivotal voter in quite a natural manner. This is the case because the preferred alternative of each voter is assessed in a deterministic manner. However, a notion of pivotal voter for situations in which signals are concealed is less clear, and needs further considerations. Crucially, in these cases, the preferred alternative of some voters can only be assessed in probabilistic terms—as such preferred alternatives are given by mixed strategies. Therefore, we need to propose a notion of pivotal voters in the presence of concealed signals. Our particular notion for such cases seeks to (i) rely, as much as possible, on deterministic optimal choices by the voters, and to (ii) select a single voter as being pivotal. The practical goal of our notion is to verify whether or not voters who would be clearly pivotal when all signals are disclosed continue to be pivotal (under such a notion) when signals are concealed and they receive no signal. Following the criteria described in (i) and (ii) above, and taking into account the order $\sigma(x)$ of the voters' critical signal realizations, we propose the following notion.

DEFINITION 1. Consider the order $\sigma(x)$ of critical signal realizations induced by the profile of education levels x. We say that voter i_k continues to be *pivotal to the voting process in the presence of concealed signals*, if given a concealment set $C \neq \emptyset$, and conditional on the voters receiving signal $s = \emptyset$, then either (a) all voters $\{i_1, \dots, i_{k-1}\}$ prefer acceptance of the new initiative with probability one, or (b) all voters $\{i_{k+1}, \dots, i_n\}$ prefer rejection of the new initiative with probability one.

As a consequence, in case (a) above, if voter i_k votes for acceptance with probability one as well, then the outcome of voting would be $o(v) = A$ with probability one. Even further, if voter i_k votes for acceptance with probability $\phi_k(C, \lambda)$, then the outcome of voting would be $o(v) = A$ with such a probability $\phi_k(C, \lambda)$. Analogously, in case (b), if voter i_k votes for rejection with probability $1 - \phi_k(C, \lambda)$, then the outcome of voting would be $o(v) = R$ with such a probability $1 - \phi_k(C, \lambda)$.

However, the conditions (a) and (b) provided by our notion in Definition 1 do not cover all possible cases that could follow in our model. To fix ideas about the difficulties that concealed signals pose to the problems of (a) proposing a notion of pivotal voter and of (b) identifying pivotal voters in particular situations, consider the following example. Suppose that the leader is a radical in favor of the new initiative (so that $\bar{s}_l \to -\infty$) and consider the set $C = (\bar{s}_l, \bar{s}_{i_k}]$ of concealed signals. In addition, suppose that the voting rule satisfies $k < n/2$ so that $i_k < j$ for some voter j with $\mu_j > 0$. Note then that, conditional on C and on the voters receiving signal $s = \emptyset$, we know from the specification in Eq. (7) that all voters $i_{(n/2)+1}, \dots, i_n$ prefer rejection with probability one. Such a number $(n/2)$ of voters, though, would be not sufficient to reject the proposal in this example. In this case, our notion of pivotal voter in the presence of concealed signals (given by Definition 1) is not useful to conclude whether voter i_k is the pivotal voter, neither to identify a pivotal voter. The critical point in this example is that (even when we invoke such a notion in Definition 1) whether voter i_k ends up being pivotal or not depends crucially on the preferred alternative (conditional on C and on the voters receiving signal $s = \emptyset$) of those voters j with $i_k < j \leq n/2$, so that $\mu_j > 0$. Obviously, further structure and assumptions would be necessary to obtain general messages about whether or not voter i_k continue to be pivotal (when moving from a situation where no signals are concealed to another with concealed signals) in this sort of particular situations. In particular, further key considerations on the differences between all the players opinions would be necessary. A general analysis to explore all possible situations would be ill-suited to overcome such difficulties. We opt for not introducing further structure to the model and, instead, in Section 5 we will impose a reduced-form assumption to focus on interesting situations for our investigation of the well-beings of the leader (for each of his possible categories) and of the group of voters.

Following the previous arguments, and the results in Lemma 1, Proposition 1–Proposition 3 below characterize the optimal design of concealment sets by the leader.

PROPOSITION 1. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Then, the moderate leader ($\mu_l = 0$) designs the concealment set $C^*_k(0;x)$ as follows:

(a) for $k \leq n/2$, the leader selects any subset $C^*_k(0;x) \subseteq C_k(0;x) = [\bar{s}_{i_k}(x;\varepsilon,0), 0)$;

(b) for $k > n/2$, the leader selects any subset $C^*_k(0;x) \subseteq C_k(0;x) = [0, \bar{s}_{i_k}(x;\varepsilon,0))$.

The moderate leader has *weak* incentives to conceal signals that would critically influence the voters whose opinions are closer to his own opinions. Nevertheless, a profound multiplicity of optimal concealment sets arises in this case. In particular $C^*_k(0;x) = \emptyset$ is included in the description given by Proposition 1. Thus, *disclosing all signals obtained through his investigation efforts is also part of the optimal behavior of the moderate leader*. We make no formal claims regarding equilibrium selection.[18]

In sharp contrast with the case of the moderate leader, *radical leaders have strict incentives to conceal all signals that would critically influence those voters whose opinions are closer to his own's*. Depending on the majority rule, there could be a unique equilibrium in which the radical leader seeks to persuade those voters that are closer in opinions by concealing evidence. In particular, the radical leader in favor of the new initiative *always* obfuscates for majority rules $k \leq n/2$. He also obfuscates more (according to the set inclusion order) as the majority rule becomes more unanimous. On the other hand, the radical leader in favor of the status quo *always* obfuscates for majority rules $k > n/2$, and he obfuscates more as the majority rule becomes more dictatorial.

[18] For environments where we could naturally consider that disclosing the obtained signals involves any sort of cost for the leader, our setup would deliver the message that concealing $C^*_k(0;x) = C_k(0;x)$ appears as the most reasonable behavior.

PROPOSITION 2. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Then, the radical leader biased in favor of the new initiative ($\mu_l = \overline{\mu} > \mu_1$) designs the concealment set $C_k^*(\overline{\mu};x)$ as follows:

(a) for $k \leq n/2$, the leader selects the interval $C_k^*(\overline{\mu};x) = C_k(\overline{\mu};x) = [\bar{s}_l, \bar{s}_{i_k}(x;\varepsilon,\overline{\mu}))$ with $C_k^*(\overline{\mu};x) = C_k(\overline{\mu};x) = (-\infty, \bar{s}_{i_k}(x;\varepsilon,\overline{\mu}))$ for $\overline{\mu} \to +\infty$;

(b) for $k > n/2$, the leader selects any subset $C_k^*(\overline{\mu};x) \subseteq C_k(\overline{\mu};x)$ with the form $C_k^*(\overline{\mu};x) = [\bar{s}_l, \bar{s}_{i_{n/2}}) \cup B$, for any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k}(x;\varepsilon,\overline{\mu}))$. Moreover, $\bar{s}_l \to -\infty$ for $\overline{\mu} \to +\infty$.

PROPOSITION 3. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Then, the radical leader biased in favor of the status quo ($\mu_l = \underline{\mu} < \mu_n$) designs the concealment set $C_k^*(\underline{\mu};x)$ as follows:

(a) for $k \leq n/2$, the leader selects any subset $C_k^*(\underline{\mu};x) \subseteq C_k(\underline{\mu};x)$ with the form $B \cup C_k^*(\underline{\mu};x) = (\bar{s}_{i_{(n/2)+1}}, \bar{s}_l]$, for any subset $B \subseteq (\bar{s}_{i_k}(x;\varepsilon,\underline{\mu}), \bar{s}_{i_{(n/2)+1}}]$. Moreover, $\bar{s}_l \to +\infty$ for $\underline{\mu} \to -\infty$

(b) for $k > n/2$, the leader selects the interval $C_k^*(\underline{\mu};x) = C_k(\underline{\mu};x) = [\bar{s}_{i_k}(x;\varepsilon,\underline{\mu}), \bar{s}_l)$ with $C_k^*(\underline{\mu};x) = C_k(\underline{\mu};x) = [\bar{s}_{i_k}(x;\varepsilon,\underline{\mu}), +\infty)$ for $\underline{\mu} \to -\infty$.

The following example illustrates the construction of the (optimal) concealment sets by a moderate leader, described by Proposition 1.

EXAMPLE 1. Suppose that $n = 4$ and consider the moderate leader ($\mu_l = 0$). Suppose that there are exactly two voters located on each side of the leader across the opinion spectrum, that is, $-\mu_1 < -\mu_2 < 0 < -\mu_3 < -\mu_4$. Consider a situation where $x = (ne, ne, ne, ne)$ so that no voter can obtain information from an external source. It then follows that the induced ordering $\sigma(x)$ of critical signal realizations is simply given by $\bar{s}_1 < \bar{s}_2 < \bar{s}_3 < \bar{s}_4$ so that $i_k = k$.

On one extreme of the possible majority rules, consider first that $k = 1$. Then, conditional on observing a signal $s < 0$, the leader has weak incentives to conceal all signals $s \in [-\mu_i, 0)$ from each of the two voters i such that $\mu_i > 0$. Notice that, conditional on such negative signals, the leader strictly prefers rejection but observing them would make any of such two voters $i = 1,2$ (with opinions $\mu_i > 0$) to prefer acceptance instead. Under the dictatorship majority rule $k = 1$, the leader clearly wants to avoid this. The optimal strategy of the leader is then to try to persuade these two voters $i = 1,2$. The incentives are weak though. Given the obfuscation that the leader can create by concealing evidence, these two voters would continue to prefer acceptance with probability one as well. The moderate leader is then indifferent between concealing signals from any subset $C \subseteq [-\mu_i, 0)$. On the other hand, if the leader observes a signal $s \geq 0$, then he does not have incentives to conceal such a nonnegative signal. This is so because, upon observing such signals, already two voters prefer acceptance (those two voters $i = 1,2$ with $\mu_i > 0$). In this case, the new initiative is approved regardless of the preferred alternative of those two voters with $\mu_i < 0$. It follows that the optimal concealment set takes the form of any set $C_1(0;x)$ such that $C_1(0;x) \subseteq [\bar{s}_1, 0) = [-\mu_1, 0)$.

On the other extreme, consider now a majority rule $k = 4$ so that unanimity in favor of acceptance is required to approve the new initiative. Then, conditional on observing a signal $s < 0$, the leader does not care about whether the two voters $i = 1,2$ with $\mu_i > 0$ prefer or not acceptance since the remaining two voters (i.e., those two voters $i = 3,4$ with $\mu_i < 0$) prefer rejection upon disclosing such negative signals. The leader would then disclose all negative signals because four personal votes in favor of acceptance are now required for the new initiative to be approved. On the other hand, conditional on observing a signal $s \geq 0$, the leader wishes that all voters prefer acceptance. In this case, he would be indifferent between concealing all signals $s \in C$ for any subset $C \subseteq [0, -\mu_4)$. Thus the optimal concealment set takes the form of any set $C_4(0;x)$ such that $C_4(0;x) \subseteq [0, \bar{s}_4) = [0, -\mu_4)$.

Finally, consider $k = 2$ so that simple majority is sufficient to approve the new initiative. Then, conditional on observing a signal $s < 0$, the leader has weak incentives to try to persuade only one voter with positive priors, in particular, the voter with the smallest $|\bar{s}_i|$ among those two voters $i = 1,2$ with $\mu_i > 0$. If the leader did not try to persuade such a voter, then two voters would prefer acceptance for signals under which the leader strictly prefers rejection. Thus, the leader would be indifferent between concealing all signals $s \in C$ for any set $C \subseteq [-\mu_2, 0)$. On the other hand, conditional on observing a signal $s \geq 0$, the leader already can count on the two voters with positive priors preferring acceptance. Therefore, we obtain that the optimal concealment set takes the form of any $C_2(0;x)$ such that $C_2(0;x) \subseteq [\bar{s}_2, 0) = [-\mu_2, 0)$.

How do education levels influence the leader's optimal design of the concealment set? In situations where no voter had education ($x = (ne, \dots, ne)$), we could simply resort to the ordering $-\mu_1 < \cdots < -\mu_{n/2} < 0 < -\mu_{(n/2)+1} < \cdots < -\mu_n$ to determine directly the critical signal realization $\bar{s}_k = -\mu_k$ associated to voter k. However, in situations where some voters are educated, we observe from the specification in Eq. (1) that such an ordering needs further qualification. In particular, the relevant ordering is given by the condition in Eq. (6). Leaving aside the corresponding analytical expressions, such comparative implications convey an intuitive message. The basic idea is that varying degrees of access to external sources of information across voters make them learn differently. As a consequence, such discrepancies in the access to external means of information affect the distribution of cutoff signals (for the voters to prefer one alternative over the other).

If we restrict attention to the (largest) selection $C^*_k(0;x) = C_k(0;x)$ of optimal concealment sets, then the general message in the presence of education is that optimal concealment sets shrink, compared to the situation where no voter possesses education. Furthermore, the new optimal concealment sets shrink relative more as the probability ε of the education efforts being fruitful increases. The message conveyed is that the leader is aware that his obfuscation strategy becomes less effective when voters are able to obtain larger amounts of information by themselves about ω.

The most interesting situations arise when some voters are educated and others are not so that the information disclosed by the leader stands as their unique source of additional information about ω. The following example illustrates how the moderate leader would optimally design the concealment set $C_k^*(0;x)$ when some voters are educated. Example 2 highlights differences about the leader's optimal behavior, relative to the case where voters are not educated (as it was the case in Example 1).

EXAMPLE 2. Suppose that $n = 4$ and consider a moderate leader ($\mu_l = 0$). As in Example 1, there are two voters located on each side of the leader across the opinion spectrum, that is, $-\mu_1 < -\mu_2 < 0 < -\mu_3 < -\mu_4$. Let us focus on the dictatorship majority rule $k = 1$. For situations where all signals were disclosed, voter 1 would be the pivotal voter in the absence of education. As argued above, this might change in the presence of education.

Suppose that voters 3 and 4 are educated. Since the vote of a single voter in favor of the initiative is sufficient to achieve the outcome preferred by the leader when he receives signals $s \geq 0$, he does not care about the education levels of such voters 3 and 4. In particular, the leader finds optimal to disclose all nonnegative signals. He does care, however, about whether or not to conceal negative signals, depending on the education levels of voters 1 and 2. We thus consider plausible education levels for such voters.

First, suppose that $x_1 = x_2 = e$. Then, we obtain the following induced ordering $\sigma(x)$ of the critical signal realizations $\bar{s}_i$.

$$-(1-\varepsilon)\mu_1 < -(1-\varepsilon)\mu_2 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

Thus, conditional on observing a signal $s < 0$, the leader wants to conceal signals so as to try to persuade the two voters with positive priors. The optimal concealment strategy of the leader takes the form now of any subset $C_1(0;x) \subseteq [-(1-\varepsilon)\mu_1, 0)$. We observe that the largest set within the family of optimal concealment sets shrinks relative to the largest set within the family of subsets that we derived in Example 1 for the case in which voters were not educated.

Secondly, suppose that $x_1 = ne$ and $x_2 = e$. Then, we obtain the following induced ordering $\sigma(x)$ of the critical signal realizations $\bar{s}_i$.

$$-\mu_1 < -(1-\varepsilon)\mu_2 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

Thus, conditional on observing a signal $s < 0$, the leader wants to conceal signals so as to try to persuade the two voters with positive priors. The optimal concealment strategy of the leader takes the form now of any subset $C_1(0;x) \subseteq [-\mu_1, 0)$. This family of subsets indeed coincides with the family of optimal concealment sets that we derived in Example 1 for the case in which voters were not educated.

Lastly, suppose that $x_1 = e$ and $x_2 = ne$. Then, there are two different orderings of the critical signal realizations, depending on the relation between μ_1 and μ_2, and the probability ε. In particular, if $\varepsilon < (\mu_1 - \mu_2)/\mu_1$, then it follows that

$$-(1-\varepsilon)\mu_1 < -\mu_2 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

The optimal concealment set in this case is any subset $C_1(0;x) \subseteq [-(1-\varepsilon)\mu_1, 0)$. The largest possible optimal concealment set shrinks relative to the largest possible set within the family of subsets that was optimally chosen when voters were not educated. On the other hand, if $\varepsilon > (\mu_1 - \mu_2)/\mu_1$, then

$$-\mu_2 < -(1-\varepsilon)\mu_1 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

In this case, the optimal concealment set takes the form of any subset $C_1(0;x) \subseteq [-\mu_2, 0)$. Again, the largest possible optimal concealment set shrinks relative to the largest set within the family of subsets that was optimally chosen when voters were not educated.

5 Leader's Utility in Equilibrium

We turn now to investigate key features of the (ex ante) utility that the leader receives from his investigation effort and obfuscation behavior in equilibrium. Notably, even though there is a profound multiplicity of equilibria in terms of optimal concealment sets, equilibrium payoffs are unique.

Following our previous comments on Observation 1, we now make the reduced form assumption of focusing on situations in which voter i_k continues to be the pivotal voter (according to Definition 1) when the leader moves from a hypothetical situation of not concealing signals to doing so (so that $C_k \neq \emptyset$), and voters receive no signal (i.e., they receive $s = \emptyset$). Unless we impose further (restrictive) assumptions on the differences among the players' opinions, focusing in such situations is crucial to obtain insights about all possible categories of the leader.[19]

Suppose that the leader has an opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$ and that the profile of education levels is x. Conditional on the selection of the concealment set $C_k^*(\mu_l;x) \in \mathscr{C}$, let $U_{\mu_l}(\lambda;x,k)$ be the leader's ex ante expected utility for an investigation effort λ, given the profile of education levels x and the majority rule k. Then, Lemma 2–Lemma 4 derive useful expressions for the (ex ante) expected utility of the leader, provided that he designs optimally a concealment set $C_k^*(\mu_l;x)$ at the interim stage of the game.

[19] Given our notion of pivotal voter in the presence of concealed signals in Definition 1, this assumption holds always when the leader is moderate, when the leader is a radical in favor of the new initiative and the majority rule is not below simple majority, or when the leader is a radical in favor of the status quo and the majority rule is not above simple majority.

LEMMA 2. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a moderate opinion ($\mu_l = 0$). Then, the leader's (ex ante) expected utility for an investigation effort $\lambda \in (0, \bar{\lambda}]$, conditional on the optimal selection of a concealment set $C_k^*(0;x) \subseteq C_k(0;x)$, can be expressed as

$$U_0(\lambda;x,k) = -(1/2)\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda),$$

where $C_k^* = C_k^*(0;x) \subseteq [\bar{s}_{i_k}, 0)$ for $k \leq n/2$ and $C_k^* = C_k^*(0;x) \subseteq [0, \bar{s}_{i_k})$ for $k > n/2$.

LEMMA 3. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a radical opinion in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Then, the leader's (ex ante) expected utility for an investigation effort $\lambda \in (0, \bar{\lambda}]$, conditional on the optimal selection of a concealment set $C_k^* = C_k^*(\overline{\mu};x) \subseteq C_k(\overline{\mu};x)$, can be expressed as

(a) for $k \leq n/2$, $U_{\overline{\mu}}(\lambda;x,k) = -\pi_k(C_k^*, \lambda)\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda)$;

(b) for $k > n/2$, $U_{\overline{\mu}}(\lambda;x,k) = -\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda)$.

LEMMA 4. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a radical opinion in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). Then, the leader's (ex ante) expected utility for an investigation effort $\lambda \in (0, \bar{\lambda}]$, conditional on the optimal selection of a concealment set $C_k^* = C_k^*(\underline{\mu};x) \subseteq C_k(\underline{\mu};x)$, can be expressed as

(a) for $k \leq n/2$, $U_{\underline{\mu}}(\lambda;x,k) = -\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda)$;

(b) for $k > n/2$, $U_{\underline{\mu}}(\lambda;x,k) = -\pi_k(C_k^*, \lambda)\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda)$.

The descriptions of the families of optimal concealment sets $C_k^*(\mu_l;x)$ that appear in Lemma 2–Lemma 4 correspond to those derived, respectively, in Proposition 1–Proposition 3. The tractable expressions for the leader's utility derived in Lemma 2–Lemma 4 allow us to study the optimal investigation effort of each category of the leader.

PROPOSITION 4. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is moderate ($\mu_l = 0$). Then, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$\mathbb{P}_l[s \notin C_k^*(0;x)] = 2c'(\lambda^*), \tag{9}$$

where we have $C_k^*(0;x) \subseteq [\bar{s}_{i_k}, 0)$ for $k \leq n/2$ and $C_k^*(0;x) \subseteq [0, \bar{s}_{i_k})$ for $k > n/2$.

PROPOSITION 5. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical biased in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Then,

(a) for $k \leq n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$\pi_k \mathbb{P}_l[s \notin C_k^*] - (\partial \pi_k / \partial \lambda)\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] = c'(\lambda^*), \tag{10}$$

where $C_k^* = C_k^*(\overline{\mu}; x)$ and $\pi_k = \pi_k(C_k^*, \lambda^*)$;

(b) for $k > n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$\mathbb{P}_l[s \notin C_k^*] = c'(\lambda^*), \tag{11}$$

where $C_k^* = C_k^*(\overline{\mu}; x)$.

PROPOSITION 6. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical biased in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). Then,

(a) for $k \leq n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$\mathbb{P}_l[s \notin C_k^*] = c'(\lambda^*), \tag{12}$$

where $C_k^* = C_k^*(\underline{\mu}; x)$.

(b) for $k > n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$\pi_k \mathbb{P}_l[s \notin C_k^*] - (\partial \pi_k / \partial \lambda)\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] = c'(\lambda^*), \tag{13}$$

where $C_k^* = C_k^*(\underline{\mu}; x)$ and $\pi_k = \pi_k(C_k^*, \lambda^*)$.

Two qualitatively different insights emerge from Proposition 4–Proposition 6. On the one hand, the characterizations of the leader's optimal effort provided by Eq. (9), Eq. (10), and Eq. (13) describe the neat requirement that the marginal benefit from the investigation effort must be equal to its marginal cost. In this case, the marginal benefit is directly given by the probability that the received signal be actually disclosed by the leader. Unlike this, the conditions provided by Eq. (11) and Eq. (12) include in the expressions of the marginal benefit both the probability that voter i_k uses the concealed signals (according to her own opinions) and the rate of change of such probability with respect to the investigation effort. Such discrepancies in the characterization of the optimal investigation effort are driven by the following forces.

On the one hand, for those situations in which (1) the leader is moderate, (2) the leader is a radical in favor of the new initiative and the majority rule is more unanimous, or (3) the leader is a radical in favor of the status quo and the majority rule is more dictatorial, it follows that, conditional on concealed signals $s \in C_k^*$, the voting outcome obtained is not affected by whether, in order to determine their preferred alternatives, voters use only their opinions or combine them with the concealed signals. The idea is that the outcome of the election does not change regardless of whether concealed signals are taken into account.[20]

On the other hand, for the situations in which (4) the leader is a radical in favor of the new initiative and the majority rule is more dictatorial, or (5) the leader is a radical in favor of the status quo and the majority rule is more unanimous, it follows that, conditional on concealed signals $s \in C_k^*$, the outcome of the election process depends crucially on whether voters consider only their opinions or their opinions together with the concealed signals. In particular, for the case described in (4), it follows that the radical leader who prefers the new initiative benefits from the requirement that only a relatively small number of personal votes—which correspond to voters relatively close to her own opinions—be sufficient to approve the new initiative. Conditional on concealed signals, the preferred alternative of such voters coincides with the leader's one when they use only their priors. This benefits the leader when the majority rule is more dictatorial. Similarly, for the case described in (5), we have that the radical leader who prefers to remain in the status quo benefits from the requirement that a high number of personal votes in favor of the new initiative be required in order to change the status quo. Conditional on concealed signals, voter i_k—who is relatively close to the leader's own opinions—prefers rejection based solely on her own opinions. This again benefits the leader. Such mechanisms explain why, in the situations described in (4) and (5), the leader incorporates explicitly—as part of the marginal benefit of his efforts—the probability π_k that voter i_k uses the undisclosed signals, as well as the rate $\partial \pi_k / \partial \lambda$ according to which such a probability changes with his investigation efforts.

5.1 The Role of the Majority Rule

We can use now our previous insights for the case of the moderate leader to comment (in Observation 2 below) about (i) how would the investigation efforts of the leader change as a function of the majority rule? and (ii) which would be the preferred majority rules of the leader?

To develop some arguments, we will find useful to consider explicitly the function $\varphi_{\mu_l}(\lambda; x, k) \equiv \partial U_{\mu_l}(\lambda; x, k)/\partial \lambda$, which gives us the marginal change in the leader's (ex

[20] For instance, suppose that the leader is strongly biased in favor of the new initiative and that approval of the new initiative requires a relatively large number of votes in favor of it. Then, the leader needs to persuade a relatively large number of voters. The priors of some of those voters will be necessarily far away from the opinion of the leader. Furthermore, upon concealed signals, such voters will give importance to their own priors (using either their own priors solely or their own priors combined with the leader's "technology"). As a result, their personal vote will be against the new initiative, regardless of the leader's obfuscation efforts.

ante) utility due to changes in his investigation effort, for a leader with opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$, a profile of education levels x and a majority rule k.

OBSERVATION 2. Consider the moderate leader ($\mu_l = 0$) and suppose that he faces a profile of education levels x and a majority rule $k \in N$. Consider the induced ordering $\sigma(x)$ of critical signal realizations described in Eq. (6) and the associated voter i_k. We would like then to consider a one-unit increase ($\Delta k = 1$) in the number of personal votes required to approve the new initiative so that we move from the initial required majority k to the (slightly modified) rule $k+1$.

As to our first question, (i) above, note that the implicit value theorem (adapted to the discrete change in k) can then be used on the condition $\varphi_0(\lambda^*; x, k) = 0$—which guarantees that $\lambda^* \in (0, \overline{\lambda})$ is part of the leader's best response in equilibrium—to derive a reasonable approximation of the induced change $\Delta\lambda^*$ in the leader's optimal investigation effort. In particular, given $\Delta k = 1$, it follows that

$$\frac{\Delta\lambda^*}{\Delta k} \approx -\frac{\Delta\varphi_0/\Delta k}{\partial\varphi_0/\partial\lambda} = (-1/2)^r \Big(\mathbb{P}_l[s \geq \bar{s}_{i_k}] - \mathbb{P}_l[s \geq \bar{s}_{i_{k+1}}]\Big) \Big/ c''(\lambda^*), \tag{14}$$

where $r = 2$ for majority rules $k, k+1 \leq n/2$ and $r = 1$ for rules $k, k+1 > n/2$. Thus, since $c''(\lambda^*) > 0$, we observe that an increase in the required number of votes in favor of acceptance incentivizes the moderate leader (a) to invest more in acquiring information (as a result, λ^* raises) when the majority rule is more dictatorial than simple majority, and (b) to invest less (as a result, λ^* lowers) when the majority rule is more unanimous than simple majority. By combining those insights, it follows that the moderate leader wants to invest more in acquiring information as the majority rule becomes closer to the simple majority. The critical insight on this point is that the leader invests more and, therefore, provides more accurate information, as the majority rule narrows the discrepancy between the opinion of voter i_k and the leader's opinion.

As to our second question of interest, (ii) above, observe that, by plugging the requirement given by Proposition 4 into the expression for the (ex ante) utility of the leader provided in Lemma 2, the expression

$$U_0(\lambda^*; x, k) = -(1/2) + \lambda^* c'(\lambda^*) - c(\lambda^*) \tag{15}$$

gives us the optimal (ex ante) utility of the leader in equilibrium. Now, for the considered one-unit increase in the number of personal votes required to approve the new initiative, we can use the approximation given in Eq. (14), together with the expression in Eq. (15), to derive the induced change

$$\begin{aligned} \Delta U_0(\lambda^*; x, k) &= \lambda^* c''(\lambda^*) \Delta\lambda^* \\ &\approx (-1/2)^r \lambda^* \Big(\mathbb{P}_l[s \geq \bar{s}_{i_k}] - \mathbb{P}_l[s \geq \bar{s}_{i_{k+1}}]\Big), \end{aligned}$$

where $r = 2$ for $k \leq n/2$ and $r = 1$ for $k > n/2$.[21] Then, $\Delta U_0(\lambda^*; x, k) > 0$ as k increases for majority rules $k \leq n/2$ and $\Delta U_0(\lambda^*; x, k) < 0$ as k increases for majority rules $k > n/2$. Thus, it follows that the moderate leader prefers rules either $k = n/2$ or $k = (n/2) + 1$ over the rest of majority rules.[22]

Pushing further the sort of insights offered by Observation 2, we now investigate where (each category of) the leader would wish to place the opinion of voter i_k, if he had the opportunity to do so.

PROPOSITION 7. Consider a given majority rule k and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader had the possibility of choosing the location in the real line of the critical signal realization $\bar{s}_{i_k} = \bar{s}_{i_k}(x; \varepsilon, \mu_l)$—conditional on the model's restriction that $\bar{s}_{i_k} \in (-\infty, 0)$ for $k \leq n/2$ and $\bar{s}_{i_k} \in (0, +\infty)$ for $k > n/2$. Then,

(a) the moderate leader ($\mu_l = 0$), would choose $\bar{s}_{i_k} \to 0$ for the largest selection of optimal concealment subsets ($C_k^*(0; x) = C_k(0; x)$), whereas he would not care about $\bar{s}_{i_k}$ for the selection in which he discloses all signals ($C_k^*(0; x) = \emptyset$);

(b) the radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$) would always prefer $\bar{s}_{i_k} \to -\infty$;

(c) the radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$) would always prefer $\bar{s}_{i_k} \to +\infty$.

The general message is that the leader would prefer that voter i_k be as much aligned as possible with his own opinions. For the equilibrium selection in which the leader conceals all signals within the disagreement set $C_k(0; x)$, the moderate leader prefers a rule as close as possible to the simple majority $k = n/2$. Importantly, for the equilibrium selection in which he discloses all obtained signals, the leader is indifferent between majority rules. In sharp contrast with these insights, we observe that a radical leader in favor of the new initiative always prefers the dictatorial majority rule, $k = 1$, and a radical leader in favor of the status quo always prefers the unanimous rule, $k = n$.

5.2 *The Role of Education*

We turn now to investigate (i) how the information obtained by voters from external sources interacts with the leader's obfuscation strategy and (ii) which profiles of education levels are preferred by each category of the leader.

[21] The expression for the change $\Delta U_0(\lambda^*; x, k)$ induced by a change $\Delta k = 1$ follows from (a discrete-perturbation adaptation of) the envelope theorem: $\Delta U_0(\lambda^*; x, k)/\Delta k = U_0(\lambda^*; x, k+1) - U_0(\lambda^*; x, k)$.

[22] From the expression derived in Eq. (15), we observe that determining which of the two rules, either $k = n/2$ or $k = (n/2) + 1$, is preferred by the leader depends largely on (1) the particular shape of the cost function c and (2) the magnitudes of the respective biases $\mu_{n/2}$ and $\mu_{(n/2)+1}$. In short, though, our model delivers the message that the moderate leader's (ex ante) utility in equilibrium is harmed by both very dictatorial and very unanimous majority rules.

For a profile of education levels x, Let $N_e(x) \equiv \{i \in N \mid x_i = e\}$ be the set of educated voters and $n_e(x) \equiv |N_e(x)|$ the associated number of educated voters. The most interesting situations arise when there are certain (exogenous) restrictions such that a number less than the required majority k (to accept the new initiative) can be educated. In such situations, a certain number of personal votes required to leave the status quo will then necessarily correspond to voters that do not have access to external sources of information. By focusing on this class of situations we want to avoid (less interesting) situations where all the voters who support the new initiative could in principle obtain information from an external source, and not from the leader's disclosure of information. Therefore, as we do in Example 3 and Example 4 below, in the sequel we will restrict attention to situations where $0 < n_e(x) < k$ for the existing majority rule $k \in N$. As a consequence, we restrict attention to majority rules that are not fully dictatorial, i.e., $1 < k \leq n$.[23]

EXAMPLE 3. Consider a leader in favor of the new initiative ($\mu_l = \overline{\mu} \to \infty$). Suppose that there are 8 voters and that the existing majority rule is simple majority, $k = 4$. Consider a situation where 3 voters receive education. Let us then consider two alternative profiles of education levels to illustrate certain answers to the questions asked in (i) and (ii) above.

First, consider an education profile x such that voters 1, 2, and 3 are educated, $x_1 = x_2 = x_3 = e$. Then, it follows that $\bar{s}_i(x; \varepsilon, \overline{\mu}) = -[(1-\varepsilon)\mu_i + \varepsilon\overline{\mu}] \to -\infty$ for the three educated voters $i = 1, 2, 3$. The induced ordering $\sigma(x)$ is given by

$$\bar{s}_1(x; \varepsilon, \overline{\mu}) \leq \bar{s}_2(x; \varepsilon, \overline{\mu}) \leq \bar{s}_3(x; \varepsilon, \overline{\mu}) < \bar{s}_4(x; \varepsilon, \overline{\mu}) < \cdots < \bar{s}_8(x; \varepsilon, \overline{\mu}),$$

so that the pivotal voter i_4 (for a situation where all signals were disclosed) is voter 4.

Secondly, consider another education profile x' such that voters 6, 7, and 8 are educated, $x'_6 = x'_7 = x'_8 = e$. Then, it follows that $\bar{s}_i(x'; \varepsilon, \overline{\mu}) = -[(1-\varepsilon)\mu_i + \varepsilon\overline{\mu}] \to -\infty$ for the three educated voters $i = 6, 7, 8$. The induced ordering $\sigma(x)$ is then given by

$$\bar{s}_6(x'; \varepsilon, \overline{\mu}) \leq \bar{s}_7(x'; \varepsilon, \overline{\mu}) \leq \bar{s}_8(x'; \varepsilon, \overline{\mu}) < \bar{s}_1(x'; \varepsilon, \overline{\mu}) < \cdots < \bar{s}_5(x'; \varepsilon, \overline{\mu}),$$

so that voter i_4 is now voter 1.

Notably, we learned from Proposition 2 that the respective optimal concealment sets are uniquely given by $C_4^*(\overline{\mu}; x) = (-\infty, -\mu_4)$ and by $C_4^*(\overline{\mu}; x') = (-\infty, -\mu_1)$. The optimal concealment set chosen by the leader diminishes (in the set inclusion order) when we move from the profile x to the profile x'. In addition, from the insights in Proposition 7, we observe that, in equilibrium, the leader prefers the profile x' over the profile x. In particular, the preferred concealment set by the leader is necessarily $(-\infty, -\mu_1)$.

Finally, it can be checked that other education profiles x'' can be similarly proposed in a way such that they lead to $C_4^*(\overline{\mu}; x'') = (-\infty, -\mu_1)$ as well. For example, any profile of education levels such that a subset of exactly three voters $j \in \{2, \ldots, 8\}$ get educated will

[23] Example 2 was not restricted by these considerations as the sort of questions illustrated there were substantially different, and not affected by the restrictions that we consider from this point onwards.

make their critical signal realizations $\bar{s}_{i_j} \to -\infty$. As a consequence, under the majority rule $k = 4$, we would have $i_4 = 1$. In short, to maximize his (ex ante) utility in equilibrium, all that this radical leader wants is that voter 1 be not one of the three educated voters. This message that the radical leader prefers that those voters who are closer to his own opinion do not get education is a general one (as we will see in Proposition 8 below).

EXAMPLE 4. Consider the moderate leader ($\mu_l = 0$). Suppose that there are 10 voters and that the existing majority rule is now $k = 3$. Consider a situation where exactly 2 voters receive education. We observe that the size of the optimal concealment set $C_3^*(0;x)$ according to the (largest) selection $C_3^*(0;x) = [\bar{s}_{i_3}(x;\varepsilon,0),0)$ is minimized when the corresponding critical signal realization $\bar{s}_{i_3}(x;\varepsilon,0)$ is as close as possible to zero. Since only two voters can receive education, this goal is achieved if voters 3 and 4 are chosen to get education. In this case, some voter $i_3 \in \{3,4,5\}$ will ultimately be the pivotal voter (in a situation were all signals were disclosed) and $\bar{s}_{i_3}(x;\varepsilon,0) = \min\{-\mu_5, -(1-\varepsilon)\mu_3\}$. This gives us a plausible optimal concealment set with minimal size that can be induced by x in the described situation. Of course, note that other optimal concealment sets $C_3^*(0;x) \subseteq [\bar{s}_{i_3}(x;\varepsilon,0),0)$ arise as part of equilibrium, regardless of the existing profile of education levels.

Interesting qualitative implications can be derived from Example 3 above, for the radical leader in favor of the new initiative. First, (i) the incentives of the leader to obfuscate voters lower when the voters that become educated are *not* those with opinions closer to the leader's opinion. In addition, (ii) the leader prefers such situations where the educated voters are not those whose opinions are more similar to his own's. The message conveyed by this Example 3, though, is very particular to any of the two radical leaders. A quite interesting mechanism lies behind such implications. In particular, the radical leader assesses what educated voters can learn about the underlying state *by using his own radical view about the state*. Thus, given his own "extreme" perspective, the radical leader's anticipation of the reorder of the voters' critical signal realizations upon education makes him regard as of no value the education of voters whose opinions are more similar to his own's.

That message, however, does not follow for the case of the moderate leader, as illustrated in Example 4. The moderate leader's anticipation of the change in the order to the voters' critical signal realizations (when moving from a hypothetical situation of absence of education to another where some voters may get education) is less "extreme," compared to the case of a radical leader. In particular, under the equilibrium selection $C_k^*(0;x) = C_k(0;x)$, the moderate leader only wants to make sure that the voter that would be pivotal (in a hypothetical situation where all signals were disclosed and no education were available) indeed gets education if she were allowed to. Given his own moderate perspective about the state of the world, this would reduce his incentives to obfuscate voters and, in turn, maximize his (ex ante) utility in equilibrium. Furthermore, in other equilibrium selections the moderate leader would care even less about who gets educated. In the extreme case given by the equilibrium selection $C_k^*(0;x) = \emptyset$, in which the leader discloses all obtained signals, he is indifferent in equilibrium among the profiles of education levels.

The insights provided by Proposition 8 below for radical leaders follow closely the arguments laid out in Example 3. As in the example, the proposition benefits from the results obtained earlier in Proposition 2 and Proposition 3, together with Proposition 7.

PROPOSITION 8. Consider a given majority rule $1 < k \leq n$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that a number $0 < n_e(x) < k$ of voters are educated under profile x. Suppose that the leader had the possibility of choosing which voters are educated and which voters are not (i.e., the possibility of choosing the composition of the set $N_e(x)$) under the restriction that exactly $n_e = n_e(x)$ voters are educated. Then,

(a) the radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$) would choose $N_e(x)$ in any way such that $N_e(x) \subset N \setminus \{1, \ldots, k - n_e\}$;

(b) the radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$) would choose $N_e(x)$ in any way such that $N_e(x) \subset N \setminus \{k - n_e, \ldots, n\}$.

In natural situations where there are constraints that restrict the amount of education that can be provided to the set of voters, radical leaders would prefer that the external means of information not be provided to voters that have opinions relatively close to their own's. Of course, this insight would have strong political economy implications if one were to consider environments where leaders had some power to choose who gets educated or has access to external means of information, such as independent media or the internet. Questions such as the provision of public education or media censorship could be addressed in the light of our model's insights on this point.

6 Voters' Welfare in Equilibrium

In this section, we investigate certain features of (i) majority rules and of (ii) profiles of education levels that are preferred by the group of voters. We continue to invoke the reduced-form assumption considered in Section 5 to focus on situations in which voter k continues to be the pivotal voter when the leader conceals signals and voters receive $s = \emptyset$.

Suppose that the leader has an opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$ and that the profile of education levels is x. Conditional on the optimal selection of the concealment set $C_k^*(\mu_l; x)$, let $V_j^{\mu_l}(\lambda; x, k)$ be voter j's (ex ante) expected utility for an investigation effort λ, given the profile of education levels x and the majority rule k. Consider the critical signal realization $\bar{s}_j = \bar{s}_j(x; \varepsilon, \mu_l)$ for voter j.

Suppose first that the leader does obtain a signal (an event which happens with probability λ). Then, we can express the ex ante expected utility of voter j as

$$V_j^{\mu_l, s\neq\emptyset} = -\big\{P[o(v) = A \mid s < \bar{s}_j]\mathbb{P}_j[s < \bar{s}_j] \\ + \Pr[o(v) = R \mid s \geq \bar{s}_j]\mathbb{P}_j[s \geq \bar{s}_j]\big\}. \tag{16}$$

Secondly, suppose that the leader does not receive a signal (an event which happens with probability $1-\lambda$). In this case, we can express the ex-ante expected utility of voter j as

$$V_j^{\mu_l, s=\emptyset} = -\big\{\Pr[o(v) = A \mid v_j = R]\mathbb{P}_j[v_j = R] \\ + \Pr[o(v) = R \mid v_j = A]\mathbb{P}_j[v_j = A]\big\}. \tag{17}$$

Thus, note that voter j suffers a loss whenever the outcome of voting is different from what she prefers. The expected utility of voter j is then

$$V_j^{\mu_l} = \lambda V_j^{\mu_l, s\neq\emptyset} + (1-\lambda)V_j^{\mu_l, s=\emptyset}. \tag{18}$$

We adopt a utilitarian perspective to welfare and, accordingly, specify the relevant welfare function $W^{\mu_l}(\lambda; x, k)$ for the group of voters as the sum of their (ex ante) expected utilities, $W^{\mu_l}(\lambda; x, k) = \sum_{j\in N} V_j^{\mu_l}(\lambda; x, k)$. By embedding the derivations in Eq. (16), Eq. (17), and Eq. (18) above into this definition of voters' welfare, we will be able to investigate how the welfare of the voters depends on the distribution of their opinions, for each category of the leader. For expositional reasons, we relegate such derivations that describe the welfare function of the group of voters to Appendix B (Lemma 5-Lemma 7). Using such derivations, we are able to establish that, when the leader is moderate, each voter j prefers that her own critical signal realization $\bar{s}_j$ be as close as possible to the critical signal realization $\bar{s}_k$ of voter k. In this way, it becomes more likely that the preferred alternative of each voter coincides with the outcome of voting. This particular insight is provided by Lemma 5.

As to the question that deals with the majority rules preferred by the group of voters, (i) above, we focus on the case of a moderate leader. Recall that a moderate leader prefers either majority rules $k = n/2$ or $k = (n/2)+1$ (Proposition 7). However, this needs not be always the case for the group of voters. In particular, if we allow k to vary, then our insights (Lemma 5) lead to that some voters may benefit when the critical signal realization of the decisive voter k becomes closer to their own while, at the same time, other voters may be harmed. Assessing the overall impact on the entire groups of voters becomes quite a specific analysis, which depends largely on the particular discrepancies between the players' opinions. In Observation 3 below, we study a particular situation (for which the distributions of the initial opinions and of the critical signal realizations are suitably chosen) such that the group of voters either prefer majority rules $k = n/2$ or $k = (n/2)+1$ (i.e., majority rules close to simple majority).

OBSERVATION 3. Suppose that the leader is a moderate ($\mu_l = 0$) and consider a majority rule $k \leq n/2$. Suppose that the voters' initial opinions are arranged in a way such that $\mu_1 = |\mu_n|$, $\mu_2 = |\mu_{n-1}|, \ldots, \mu_{n/2} = |\mu_{(n/2)+1}|$. In addition, suppose that the induced ordering $\sigma(x)$ of critical signal realizations is such that, for each voter j with $\bar{s}_j < 0$, there exists a voter $m(j)$ with $\bar{s}_{m(j)} > 0$ such that $\bar{s}_{m(j)} \geq |\bar{s}_j|$.

Consider now a one-unit increase, $\Delta k = 1$, in the number of votes k required to approve the new initiative so that we move from the initial majority rule k to the (slightly modified) rule $k+1$. From the proposed distribution of opinions and of critical signal realizations, we obtain the following implication. For each voter j such that $\bar{s}_j < \bar{s}_k < 0$, there exists another voter $m(j)$ such that $\bar{s}_{m(j)} > 0$. Then, note that, as k increases, each such a voter j suffers a loss, whereas each such a corresponding voter $m(j)$ benefits. Moreover, we have $\mathbb{P}_{m(j)}[s < \bar{s}_{m(j)}] \geq \mathbb{P}_j[s \geq \bar{s}_j]$. The latter implication follows simply from the assumption that, from the perspective of a voter i, signals are normally distributed with mean μ_i. Thus, the group of voters prefers $k = n/2$, because aggregate gains overcome aggregate loses, given the considered one-unit increase $\Delta k = 1$.

More in detail, given $\Delta k = 1$, it follows that the voters who have negative critical signal realizations that lie above the one of voter $k+1$ gain. In addition, the voter who passes to position $k+1$ does not lose either because she becomes pivotal now. We can then focus on the change in welfare that stems from those voters with positive critical signal realizations and those voters with negative critical signal realizations that lie below the one of voter $k+1$. This specific change in welfare can be expressed as

$$\begin{aligned} -\lambda \Big\{ & \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_k)]}^{\text{(a)}}\big]\mathbb{P}_j[s \geq \bar{s}_k] \\ & + \overbrace{\big[\mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{k+1})] - \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_k)]\big]}^{\text{(b)}} \sum_{j=1}^{k-1} \mathbb{P}_j[s \geq \bar{s}_j] \\ & + \overbrace{\big[\mathbb{P}_l[s \in [\bar{s}_{k+1}, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)]\big]}^{\text{(c)}} \sum_{j=n/2+1}^{n} \mathbb{P}_j[s < \bar{s}_j] \Big\}. \end{aligned} \tag{19}$$

The expression of the welfare of voters that allow us to derive the change in Eq. (19) is formally established in Lemma 5–(a). A key point here is that all players anticipate how the leader will optimally conceal and disclose signals. Therefore, all voters consider in a common manner how the leader's obfuscation strategy will affect the probability that the outcome of election be either acceptance or rejection. This is why the probabilities that appear in the expression in Eq. (19), according to which some signals are concealed and others are disclosed, are considered from the perspective of the leader.

In the expression in Eq. (19) above, term (a) captures the decrease in the utility of voter k (under the modified majority rule $k+1$), term (b) captures the decrease in the utilities of the voters whose critical signal realizations are below the one of voter k and term (c) captures the increase in the utilities of the voters with positive critical signal realizations. Notice that the magnitudes of the terms in (a), (b) and (c) are the same (in absolute value, (a) and (b) have positive sign, whereas (c) has negative sign). Then, since for each voter j with negative critical signal realization, there exists a voter $m(j)$ with positive critical signal realization such that $\mathbb{P}_{m(j)}[s < \bar{s}_{m(j)}] \geq \mathbb{P}_j[s \geq \bar{s}_j]$, it follows that a unit increase in k raises the welfare of voters.

The analysis is analogous for majority rules $k > n/2$. Therefore, under the particular description of opinions and critical signal realizations suggested here, the group of voters prefer that majority rules be either $k = n/2$ or $k = (n/2) + 1$.

As to the previously posed question regarding the distribution of external sources of information that the voters would prefer, (ii) above, we offer insights for both the cases of a moderate leader (Proposition 9) and of radical leaders (Proposition 10). We should emphasize that our welfare analysis here restricts attention to situations in which, following any rearrangement of the profile x of education levels, voter k remains in the k-th position (within the spectrum of opinions), conditional on $n_e(x)$ voters being educated. In this way, we can ensure that voter k continues to be pivotal after we modify the profile of education levels. To ease the exposition, it will be useful to set

$\alpha \equiv \sum_{j=k+1}^{n} \mathbb{P}_j[s < \bar{s}_j] - \sum_{j=1}^{k-1} \mathbb{P}_j[s \geq \bar{s}_j]$ for $k \leq n/2$ and

$\beta \equiv \sum_{j=1}^{k-1} \mathbb{P}_j[s \geq \bar{s}_j] - \sum_{j=k+1}^{n} \mathbb{P}_j[s < \bar{s}_j]$ for $k > n/2$.

Such terms α and β will be useful to measure whether the group of voters either benefits or are harmed when voter k becomes educated.

PROPOSITION 9. Consider the moderate leader ($\mu_l = 0$). Consider a given majority rule $1 < k \leq n$ and suppose that a number $0 < n_e(x) < k$ of voters are educated under profile x. Then, voters' welfare $W^{\mu_l}(\lambda; x, k)$ is maximized when the set of educated voters takes the form: for $k \leq n/2$, we have $N_e(x) \subseteq N \setminus \{k+1, \dots, n/2\}$ and for $k > n/2$, we have $N_e(x) \subseteq N \setminus \{(n/2)+1, \dots, k-1\}$, provided that $3n/2 - k \geq n_e(x)$. Moreover,

(a) for $k \leq n/2$, there exists a bound $\overline{\alpha}$ on the term α such that if (i) $\alpha < \overline{\alpha} < 0$, then the $n_e(x)$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero must be educated, and (ii) if $\alpha > 0$ is sufficiently high, then the $n_e(x) - 1$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero, together with voter k, must be educated;

(b) for $k > n/2$, there exists a bound $\overline{\beta}$ on the term β such that (i) if $\beta < \overline{\beta} < 0$, then the n_e voters $j \neq k$ whose critical signal realizations in the absence of education are the closest to zero, and (ii) if $\beta > 0$ is sufficiently high, then the $n_e(x) - 1$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero, together with voter k, must be educated.

For $k \leq n/2$, the group of voters prefer that voters with negative critical signal realizations that lie above the one of voter k (i.e., those voters $\{k+1,\ldots,n/2\}$) not be educated. Recall that for more dictatorial majority rules the outcome is acceptance with probability one upon signals above the critical signal realization s_k of voter k. The likelihood that voters in the aforementioned set $\{k+1,\ldots,n/2\}$ prefer rejection increases when they become educated. In particular, voters' welfare decreases when such voters become educated. In addition, voters' welfare increases when other voters $j \notin \{k,k+1,\ldots,n/2\}$ become educated. Note that such voters $j \notin \{k,k+1,\ldots,n/2\}$ are precisely voters $\{1,\ldots,k-1\} \cup \{n/2,\ldots,n\}$. Using totally analogous arguments, it follows that, for $k > n/2$, the group of voters prefer that voters whose positive critical signal realizations lie below the one of voter k (i.e., those voters $\{(n/2)+1,\ldots,k-1\}$) not be educated.

Consider a majority rule $k \leq n/2$. Then, notice that if voter k moves from being uneducated to being educated (so that her critical signal realization increases), then the $k-1$ voters whose critical signal realizations lie below $\bar{s}_k$ suffer a loss, whereas the $n-k$ voters voters whose critical signal realizations lie above $\bar{s}_k$ benefit. As mentioned earlier, the term α measures whether the group of voters gains or loses when voter k becomes educated. Then, a relatively low value of α indicates that an educated voter k inflicts an aggregate utility loss to the group of voters. Then, in order to maximize voters' welfare, we would like that the $n_e(x)$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero be educated. On the other hand, a relatively high value of α indicates that an educated voter k benefits the group of voters in aggregate. Then, voters would prefer that voter k and the $n_e(x)-1$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero be educated. For majority rules $k > n/2$, totally analogous interpretations follow.

We previously derived that, in the equilibrium selection that corresponds to the largest concealment set, the moderate leader prefers that voter k were educated (as suggested in Example 4). Now we observe that this type of distribution of education levels is not necessarily what the group of voters prefer.

We close this section by providing a necessary condition for voters' welfare to be maximized when leaders are radical.

PROPOSITION 10. Suppose that a number $0 < n_e(x) < k$ of voters are educated under profile x. Suppose that voters had the possibility of choosing which voters are educated in order to maximize their welfare $W^{\mu_l}(\lambda;x,k)$. It then follows that

(a) consider a radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$) and consider a majority rule $k > n/2$. Then, in order to maximize voters' welfare, the set of educated voters must have the following form: $N_e(x) \subseteq N \setminus \{(n/2)+1,\ldots,k-1\}$, provided that $3n/2 - k \geq n_e(x)$;

(b) consider a radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$) and consider a majority rule $k \leq n/2$. Then, in order to maximize voters' welfare, the set of educated voters must have the following form: $N_e(x) \subseteq N \setminus \{k+1,\ldots,n/2\}$.

Similarly to our insights in Proposition 9, we obtain that (a) for a radical leader in favor of the new initiative and a more unanimous voting rule, welfare unambiguously decreases when the voters whose positive critical signal realizations lie below the one of voter k (i.e., those voters $\{(n/2)+1,\ldots,k-1\}$) become educated. Recall that in this case the outcome is rejection with probability one when the leader conceals signals below the critical signal realization of voter k. If such voters get educated, it becomes then more likely that they prefer acceptance, which raises their likelihood of suffering a welfare loss. Case (b), relative to the leader in favor of the status quo, offers a totally analogous interpretation.

Interestingly, by comparing our insights from Proposition 8 and from Proposition 10 above, we observe that (under certain majority rules) the (equilibrium) well-beings of the leader and of the voters move in different directions when we are concerned about who gets educated. In particular, radical leaders prefer that voters similar to them in opinions not be educated, whereas (under certain voting rules) the set of voters prefer that voters relatively close in opinions to the voter who turns up decisive for the election outcome not be educated. The forces behind such discrepancies are not obvious as they depend on the fact that individuals consider their expected utilities in the light of their different (prior) opinions. On this point, it is also important to recall that, through education, voters can become better informed about the relevant variable than the leader himself. Based on our model's assumptions and on its underlying mechanisms, we can though give an intuitive description for such implications. In short, voters prefer the group of voters closer in opinions to the voter who would turn decisive for the voting outcome do not have additional sources of information. The reason behind such an implication is subtle and it is critically based on the role of diverse opinions in our model. In particular, the entire set of voters anticipate that, through education, such a group of voters can become better informed than the decisive voter. As a consequence, their preferred alternatives could be different to the alternative approved by the decisive voter, hence decreasing their utilities. On the other hand, a radical leader simply prefers that his own opinion prevails in the voting process. As a consequence, he prefers that voters who would be decisive for the election outcome not be endowed with external means of information. Under the consideration that external means of information are scarce, our model provides a rationale for a fundamental disagreement between radical leaders and voters about which voters should possess such means of information.

7 Comments on Empirical Evidence

We conclude by reviewing some available empirical evidence that may help us illustrate some of the paper's insights.

7.1 *Corporate Governance Voting*

Some empirical evidence that illustrates this paper's insights can be obtained from corporate governance environments. Some recent findings support our model's implication (argued in Observation 2) that moderate leaders are incentivized both to investing more in information and to obfuscating less when voting rules move away from dictatorial and get closer to simple majority. In particular, Mukhopadhyay and Shivakumar (2021) explore the information disclosure implications of regulators requiring firms to approve their proposals through shareholder voting. In 2006, the US Security and Exchange Commission (SEC) introduced direct disclosure regulations to companies that made mandatory the disclosure of compensation-relevant metrics. However, similar in spirit to the mechanism proposed in this paper, in practice, board leaders could still disclose null pieces of evidence to shareholders. Omitting details, or presenting them in "obscure" ways,[24] were commonly reported ways of concealing evidence after the 2006 SEC ruling. Subsequently, in 2011, the SEC introduced a "Say on Pay" voting requirement by shareholders of companies. No further ruling on disclosure was issued by the SEC at that time.

Mukhopadhyay and Shivakumar (2021) take advantage of those two separate regulations to propose an empirical strategy to isolate the role of introducing the simple majority rule for accepting new proposals. Specifically, the authors construct a measure of the key performance indicators disclosures of the companies listed as subject to regulation (between 2007 and 2017). Using such a measure, their analysis shows that the introduction of the simple majority as voting rule accounted for an increase (of roughly 20 percent) in the amount of evidence disclosed by board leaders. This finding can be compared to what our model delivers for the case of the moderate leader. In Observation 2, we derive the implication that, as the voting majority moves away from very dictatorial (which seems a reasonable proxy of situations where, in practice, there is no voting requirement) and approaches simple majority, the leader is incentivized to investing more in information and to obfuscating less (precisely those voters in favor of the proposal). Notably, Mukhopadhyay and Shivakumar (2021) use their empirical strategy to argue that, in such a 2011-2017 period, simple-majority voting incentivized board leaders to disclose more information. Their particular interpretation is that concealing information would enhance the skepticism of the shareholders, which may rise the probability of the raised proposal being rejected. A totally analogous driving force of skepticism (which is fostered by concealed pieces of evidence) is captured by the model investigated in this paper.

7.2 *Governmental Disclosure on the Covid-19 Pandemic*

In July 2020, the US Government changed the rules that applied for hospitals to disclose their Covid-19 hard information to state agencies. In particular, hospitals were required

[24] The corporate governance literature uses *fog indexes* to empirically account for difficulties in interpreting and digesting pieces of reported information.

to stop reporting to the Centers for Disease Control and Prevention (CDC) and to pass on their information instead to the Department of Health and Human Services (HHS). Shortly after the change of rules, media reports on certain Emails involving top health officials at HHS pointed towards an effort of President Trump's administration to silence data that the CDC could potentially gather on the state of the pandemic. At the time, public health experts expressed deep concerns about evidence on the spread and severity of the disease ceasing to be available to researchers, health experts, and to the general public. For example the New York Times reported extensively on these efforts to conceal evidence that research on the evolution of the pandemic could potentially gather.[25] Thus, if we consider that the Trump's administration took action to conceal evidence, then our model would suggest that such an obfuscation strategy would be targeted to bring to the government's side those voters already closer in opinions to the views of the administration. The logic behind our main insights would tell us that changing the opinions of voters far away from the views of the administration would have required to conceal huge amounts of evidence. Our model suggests that concealing larger amounts of evidence would have been detrimental to the administration due to a heightened skepticism on the voters' side.

We now present some empirical evidence that supports this logic. In April 2020, respondents of the *American News Pathways Project* of the Pew Research Center were asked to name the source they relied on most for pandemic news. In August 2021, the Pew Research Center asked Americans adults their vaccination status. Of the 10,348 respondents who took the August 2021 survey, 6,686 had also taken the April 2020 survey. The conclusion was that citizens who relied most on Mr. Trump for Covid-19 news were less likely to be vaccinated. Only 59% who relied most on Trump were vaccinated. The proportions raise for those respondents relying on local (72%), national (83%) or international (78 %)) outlets, public health organizations (82%) and state officials (76%). A sharp distinction is that 92% of those relying most on Trump were either republicans or independents who leaned toward the Republican Party. Conversely, only 7% were Democrats or Democratic leaners. In every other Covid-19 news source category, Democrats accounted for no less than 49% and Republicans accounted for more than 44%.[26] Such empirical data seem to support our model's insights (Proposition 1–Proposition 3) that the concealment strategy of Mr. Trump's administration was largely aimed at persuading those voters that hold aligned opinions aligned with the Republican Party and Mr. Trump's administration.

[25] https://www.nytimes.com/2020/07/14/us/politics/trump-cdc-coronavirus.html; https://www.nytimes.com/2020/09/18/us/politics/trump-cdc-coronavirus.html.

[26] https://www.pewresearch.org/fact-tank/2021/09/23/americans-who-relied-most-on-trump-for-covid-19-news-among-least-likely-to-be-vaccinated/.

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Appendix A

Take a given majority rule $k \in N$ and suppose that the leader has an opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$. Then, upon observing a signal $s \in \mathbb{R}$, let $U_{\mu_l}(s \mid s; x, k)$ be the leader's *interim* expected utility conditional on disclosing such a signal. Analogously, let $U_{\mu_l}(\emptyset \mid s; x, k)$ be the leader's *interim* expected utility when he chooses to conceal such a signal. Recall that $\bar{s}_{i_k} = \bar{s}_{i_k}(x; \varepsilon, 0)$ gives us the signal realization that makes voter i_k's optimal decision to switch between the two alternatives.

Proof of LEMMA 1.

To ease notation in the following arguments, let us simply write $v_i^* = v_i^*(\emptyset \mid C, \lambda)$.

1. Moderate leader ($\mu_l = 0$).

(a) Suppose that $k \leq n/2$. Consider the concealment set $C_k = [\bar{s}_{i_k}, 0)$. Suppose that the leader's investigation efforts allow him to obtain a signal $s < 0$. For such a signal, the leader prefers to remain in the status quo. It follows that the new initiative is approved through the election process if voters $\{i_1, \dots, i_k\}$ prefer acceptance. Note first that only less than k voters (in particular, voters $\{i_1, \dots, i_{k-1}\}$) would prefer to accept when signals $s < \bar{s}_{i_k}$ are disclosed. In addition, using the expression in Eq. (7) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, we observe that voter i_k prefers the alternative A when she observes $s = \emptyset$. Therefore, $\phi_k(C, \lambda) = 1$ for any subset $C \subseteq C_k$.

On the other hand, if the leader obtains a signal $s \geq 0$, then he prefers to disclose such a signal. Conditional on $s \geq 0$, the leader prefers the new initiative and voting would lead to acceptance since at least $n/2$ voters would prefer acceptance (upon observing $s \geq 0$) and we are considering $k \leq n/2$.

(b) Suppose that $k > n/2$. Consider the concealment set $C_k = [0, \bar{s}_{i_k})$. Suppose that the leader investigation efforts allow him to obtain a signal $s \geq 0$. Conditional on such a signal, the leader prefers the new initiative. Notice that the new initiative is not approved through the election process if voters $\{i_{(n/2)+1}, \dots, i_k\}$ do not prefer acceptance. Note first that less than k voters (in particular, voters $\{i_{k+1}, \dots, i_n\}$) would prefer to reject when signals $s \geq \bar{s}_{i_k}$ are disclosed. In addition, using the expression in Eq. (7) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, we observe that voter i_k prefers the alternative R when she observes $s = \emptyset$. Therefore, $\phi_k(C, \lambda) = 0$ for any subset $C \subseteq C_k$.

On the other hand, if the leader obtains a signal $s < 0$, then he prefers to disclose such a signal. Conditional on $s < 0$, the leader prefers the status quo and voting would lead to rejection of the new initiative since less than $n/2$ voters would prefer acceptance (upon observing $s < 0$) and we are considering $k > n/2$.

2. Radical leader in favor of the new initiative ($\mu_l \to +\infty$). Consider the concealment set

$C_k = (-\infty, \bar{s}_{i_k})$. Notice that the new initiative is approved through the election process if voters $\{i_1, \ldots, i_k\}$ prefer acceptance.

Suppose (a) $k \le n/2$. Then, using the expression in Eq. (7) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, note that

$$\begin{aligned}\phi_k(C,\lambda) &\ge \Pr[v^*_{i_1} = A, \ldots, v^*_{i_k} = A] \\ &= \Pr[v^*_{i_1} = A]\Pr[v^*_{i_2} = A \mid v^*_{i_1} = A] \times \cdots \times \Pr[v^*_{i_k} = A \mid v^*_{i_1} = A, \ldots, v^*_{i_{k-1}} = A] \\ &\ge \Pi^k_{j=1}[1 - \pi_{i_j}(C,\lambda)] > 0.\end{aligned}$$

Suppose (b) $k > n/2$. Then, it follows directly that $\phi_k(C,\lambda) = 0$ for any subset $C \subseteq C_k$ since less than k voters prefer alternative A.

3. Radical leader in favor of the status quo ($\mu_l \to -\infty$). We obtain an analogous insight to the one derived in 2. above for the case of the radical leader in favor of the new initiative. In particular, consider now the concealment set $C_k = [\bar{s}_{i_k}, +\infty)$. Notice that the new initiative is rejected through the election process if voters $\{i_k, \ldots, i_n\}$ do not prefer acceptance.

Suppose (a) $k \le n/2$. Then, it follows directly that $\phi_k(C,\lambda) = 1$ for any subset $C \subseteq C_k$ since at least k voters prefer alternative A.

Suppose (b) $k > n/2$. Then, using the expression in Eq. (7) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, note that

$$\begin{aligned}\phi_k(C,\lambda) &\le \Pr[v^*_{i_k} = A, \ldots, v^*_{i_n} = A] \\ &= \Pr[v^*_{i_n} = A]\Pr[v^*_{i_{n-1}} = A \mid v^*_{i_n} = A] \times \cdots \times \Pr[v^*_{i_k} = A \mid v^*_{i_{k+1}} = A, \ldots, v^*_{i_n} = A] \\ &\le \Pi^n_{j=k}[1 - \pi_{i_j}(C,\lambda)] < 1.\end{aligned}$$

∎

Proof of PROPOSITION 1. The required arguments make use of the derivations in Lemma 1. Suppose that the leader is moderate ($\mu_l = 0$).

(a) Consider a majority rule $k \le n/2$. Note then that $\mu_{i_k} > 0$ and, therefore, $\bar{s}_{i_k}(x;\varepsilon,0) < 0$ for voter i_k, given the considered arrangement of the voters' opinions.

Given a signal $s < 0$ observed by the leader, it follows that if any signal $s < \bar{s}_{i_k}$ is publicly observed, then a number less than k voters will vote $v_i = A$. Even though the leader prefers rejection for such signals, those voters are not sufficient to attain the acceptance outcome. By disclosing only those signals $s < \bar{s}_{i_k}$, the interim expected utility of the leader is $U_{\mu_l}(s \mid s;x,k) = 0$. Now, if for a signal realization $s < 0$, we have that $s \ge \bar{s}_{i_k}$, then a number of voters no less than k will vote $v_i = A$ with probability one. For those signals $s \in [\bar{s}_{i_k}, 0)$ the leader prefers rejection and, therefore, his interim utility is either $U_{\mu_l}(s \mid s;x,k) = -1$ or $U_{\mu_l}(\emptyset \mid s;x,k) = -\phi_k(C^*_k,\lambda)L$, for anysubset $C^*_k \subseteq [\bar{s}_{i_k}, 0)$. Now, since $\mu_k > 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} sf(s;\mu_{i_k})ds \geq \bar{s}_{i_k}$ for $C_k^* \subseteq [\bar{s}_{i_k}, 0)$, we observe from Eq. (7) that $\mathscr{J}(k) = 1$ and $\mathscr{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*, \lambda) = 1$. It follows that the leader is indifferent between concealing any subset of signals $C_k^* \subseteq [\bar{s}_{i_k}, 0)$.

Given a signal $s \geq 0$ observed by the leader, if he chooses to disclose it, then a number no less than $n/2$ voters will vote $v_i = A$. Since the majority rule k satisfies $k \leq n/2$, it follows that the outcome of the election will be acceptance with probability one. For such signals $s \geq 0$ the leader strictly prefers acceptance so that, by disclosing them, his interim utility is $U_{\mu_l}(s \mid s; x, k) = 0$. Therefore, the leader optimally chooses any subset $C_k^*(0;x) \subseteq [\bar{s}_{i_k}, 0)$.

(b) Consider a majority rule $k > n/2$. Note then that $\mu_{i_k} < 0$ and $\bar{s}_{i_k} > 0$ for voter i_k, given the considered arrangement of the voters' opinions.

Given a signal $s < 0$ observed by the leader, it follows that no more than $n/2$ voters will now vote $v_i = A$ if the leader decides to disclose such signals. Since the majority rule k satisfies $k > n/2$, we know that the outcome of the election will be rejection with probability one if the leader discloses only such negative signals. For those signals $s < 0$ the leader prefers rejection and, therefore, his interim utility is $U_{\mu_l}(s \mid s; x, k) = 0$. It follows that the leader finds strictly beneficial to disclose all negative of signals.

Given a signal $s \geq 0$ observed by the leader, then at least $n/2$ voters will prefer acceptance upon observing such nonnegative signals (i.e., those voters i with opinions $\mu_i > 0$). For $0 \leq s < \bar{s}_{i_k}$, only $k-1$ voters will prefer acceptance with probability one so that the outcome of the election will be rejection with probability one. In this case, $U_{\mu_l}(s \mid s; x, k) = -1$. On the other hand, if the leader chooses to conceal such signals $s \in C_k^* \subseteq [0, \bar{s}_{i_k})$, his interim utility is $U_{\mu_l}(\emptyset \mid s; x, k) = -[1 - \phi_k(C_k^*, \lambda)]L$. Now, since $\mu_{i_k} < 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} sf(s;\mu_{i_k})ds < \bar{s}_{i_k}$ for $C_k^* \subseteq [0, \bar{s}_{i_k})$, we observe from Eq. (7) that $\mathscr{J}(k) = 0$ and $\mathscr{I}(k, C_k^*) = 0$. Therefore, $\phi_k(C_k^*, \lambda) = 0$. Therefore, the leader is indifferent between concealing any subset of signals $C_k^* \subseteq [0, \bar{s}_{i_k})$.

If the leader observes and discloses a signal $s \geq \bar{s}_{i_k}$, then at least k voters will prefer acceptance with probability one. For those signals $s \in [\bar{s}_{i_k}, +\infty)$ the leader's interim utility when he discloses the signals is $U_{\mu_l}(s \mid s; x, k) = 0$. As a consequence, he will optimally disclose such signals $s \geq \bar{s}_{i_k}$. Therefore, the leader optimally chooses any subset $C_k^*(0;x) \subseteq [0, \bar{s}_{i_k})$. ∎

Proof of PROPOSITION 2. The required arguments are similar to those provided in the proof of Proposition 1. Suppose that the leader is a radical in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$).

(a) Consider a majority rule $k \leq n/2$. Note then that $\mu_{i_k} > 0$ and, therefore, $\bar{s}_{i_k}(x; \varepsilon, 0) < 0$ for voter i_k. Given a signal $s \in \mathbb{R}$ observed by the leader, it follows that if any signal $s < \bar{s}_{i_k}$ is publicly observed, then a number less than k voters will vote $v_i = A$. This radical leader prefers acceptance for any signal that he obtains, and those voters are not sufficient to attain the acceptance outcome. By disclosing those signals $s < \bar{s}_{i_k}$, the interim expected utility of the leader is $U_{\mu_l}(s \mid s; x, k) = -1$. On the other hand, by concealing such signals, his interim utility is $U_{\mu_l}(\emptyset \mid s; x, k) = -[1 - \phi_k(C_k^*, \lambda)]L$, for any-subset $C_k^* \subseteq [\bar{s}_l, \bar{s}_{i_k})$. Now, since $\mu_k > 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds < \bar{s}_{i_k}$ for $C_k^* \subseteq [\bar{s}_l, \bar{s}_{i_k})$, we observe from Eq. (7) that $\mathscr{J}(k) = 1$ and $\mathscr{I}(k, C_k^*) = 0$. Therefore, $\phi_k(C_k^*, \lambda) = 1 - \pi_k(C_k^*, \lambda) \in (0,1)$. It follows that the leader is strictly better off by concealing any subset of signals $C_k^* \subseteq C_k(\overline{\mu}; x) = [\bar{s}_l, \bar{s}_{i_k})$. Notice that $\bar{s}_l \to -\infty$ when $\overline{\mu} \to +\infty$.

(b) Consider a majority rule $k > n/2$. Note then that $\mu_{i_k} < 0$ and $\bar{s}_{i_k} > 0$ for voter i_k. Given a signal $s < \bar{s}_{i_{n/2}}$ observed by the leader, it follows that $s < \bar{s}_{i_k}$. From the arguments given in (a) above, it follows then that the leader has strict incentives to conceal all negative signals $s \in [\bar{s}_l, \bar{s}_{i_k})$ for each $k \leq n/2$. Therefore, the leader is strictly better off by concealing all signals $s \in [\bar{s}_l, \bar{s}_{i_{n/2}})$. On the other hand, given a signal $s \geq \bar{s}_{i_{n/2}}$ observed by the leader, it follows that concealment of signals $s \in B$ for any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k})$ implies $\mathbb{E}_{i_k}[s \mid s \in B] = \int_B s f(s; \mu_{i_k}) ds < \bar{s}_{i_k}$. We then observe from Eq. (7) that $\mathscr{J}(k) = 0$ and $\mathscr{I}(k, B) = 0$. Therefore, $\phi_k(B, \lambda) = 0$. As a consequence, for signals $s \geq \bar{s}_{i_{n/2}}$ observed by the leader, he is indifferent between concealing signals in any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k})$. By putting together the optimal concealment sets that the leader designs for the cases of signals above and bellow the critical realization $\bar{s}_{i_{n/2}}$, it follows that he wishes to conceal all signals that belong to any set $C_k^*(\overline{\mu}; x)$ with the form $C_k^*(\overline{\mu}; x) = [\bar{s}_l, \bar{s}_{i_{n/2}}) \cup B$ for any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k})$. Recall that $\bar{s}_l \to -\infty$ when $\overline{\mu} \to +\infty$. ∎

Proof of PROPOSITION 3. The required arguments are similar to those provided in the proof of Proposition 1. Suppose that the leader is a radical in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). The exposition benefits from presenting the arguments for case (b) first.

(b) Consider a majority rule $k > n/2$. Note then that $\mu_{i_k} < 0$ and, therefore, $\bar{s}_{i_k}(x; \varepsilon, 0) > 0$ for voter i_k. Given a signal $s \in \mathbb{R}$ observed by the leader, it follows that if any signal $s \geq \bar{s}_{i_k}$ is publicly observed, then a number no less than k voters will vote $v_i = A$. This radical leader prefers rejection for any signal that he obtains, and those voters are sufficient to attain the acceptance outcome. By disclosing those signals $s \geq \bar{s}_{i_k}$, the interim expected utility of the leader is $U_{\mu_l}(s \mid s; x, k) = -1$. On the other hand, by concealing such signals, his interim utility is $U_{\mu_l}(\emptyset \mid s; x, k) = -\phi_k(C_k^*, \lambda)L$, for any-subset $C_k^* \subseteq [\bar{s}_{i_k}, \bar{s}_l)$. Now, since $\mu_k < 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds \geq \bar{s}_{i_k}$ for $C_k^* \subseteq [\bar{s}_{i_k}, \bar{s}_l)$, we observe from Eq. (7) that $\mathscr{J}(k) = 0$ and $\mathscr{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*, \lambda) = \pi_k(C_k^*, \lambda) \in (0,1)$. It follows that the leader is strictly better off by concealing any subset of signals $C_k^* \subseteq C_k(\underline{\mu}; x) = [\bar{s}_{i_k}, \bar{s}_l)$. Notice that $\bar{s}_l \to +\infty$ when $\underline{\mu} \to -\infty$.

(a) Consider a majority rule $k \leq n/2$. Note then that $\mu_{i_k} > 0$ and $\bar{s}_{i_k} < 0$ for voter i_k. Given a signal $s \leq \bar{s}_{i_{(n/2)+1}}$ observed by the leader, it follows that $s \leq \bar{s}_{i_k}$. From the arguments given in (b) above, it follows then that the leader has strict incentives to conceal all negative signals $s \in [\bar{s}_{i_k}, \bar{s}_l)$ for each $k \geq (n/2)+1$. Therefore, the leader is strictly better off by concealing all signals $s \in [\bar{s}_{i_{(n/2)+1}}, \bar{s}_l)$. On the other hand, given a signal $s < \bar{s}_{i_{(n/2)+1}}$ observed by the leader, it follows that concealment of signals $s \in B$ for any subset $B \subseteq [\bar{s}_{i_k}, \bar{s}_{i_{n/2}})$ implies $\mathbb{E}_{i_k}[s \mid s \in B] = \int_B s f(s;\mu_{i_k}) ds \geq \bar{s}_{i_k}$. We then observe from Eq. (7) that $\mathscr{J}(k) = 1$ and $\mathscr{I}(k,B) = 1$. Therefore, $\phi_k(B,\lambda) = 1$. As a consequence, for signals $s \leq \bar{s}_{i_{(n/2)+1}}$ observed by the leader, he is indifferent between concealing signals in any subset $B \subseteq [\bar{s}_{i_k}, \bar{s}_{i_{n/2}})$. By putting together the optimal concealment sets that the leader designs for the cases of signals above and bellow the critical realization $\bar{s}_{i_{(n/2)+1}}$, it follows that he wishes to conceal all signals that belong to any set $C_k^*(\underline{\mu};x)$ with the form $C_k^*(\underline{\mu};x) = B \cup [\bar{s}_{i_{(n/2)+1}}, \bar{s}_l)$ for any subset $B \subseteq [\bar{s}_{i_k}, \bar{s}_{i_{n/2}})$. Recall that $\bar{s}_l \to +\infty$ when $\underline{\mu} \to -\infty$. ∎

Proof of LEMMA 2. Suppose that the leader is moderate ($\mu_l = 0$).

(a) Consider a majority rule $k \leq n/2$. Suppose that the leader chooses a research effort $\lambda \in (0, \bar{\lambda}]$. Then, with such a probability λ the leader receives a signal s, and with probability $1-\lambda$ obtains no signal ($s = \emptyset$). First, since $\mu_l = 0$, it follows that, conditional on obtaining a signal, the leader (strictly) prefers the election outcome of rejection with probability $\int_{-\infty}^{0} f(s;0) ds = 1/2$. Proposition 1–(a) showed that, at the interim stage, the leader optimally chooses to conceal a subset of signals $C_k^* = C_k^*(0;x) \subseteq [\bar{s}_{i_k}, 0)$. In this case, the proof of Proposition 1–(a) showed that the leader is able to induce an outcome of acceptance with probability $\phi_k(C_k^*, \lambda)$, so that his expected payoff is $-\phi_k(C_k^*, \lambda) \int_{C_k^*} f(s;0) ds$. As mentioned, this outcome is attained (from an ex ante perspective) with probability $(1/2)\lambda$. Similarly, if the leader obtains no signal, then he has no choice to make with respect to the concealment set. In this case, the leader (strictly) prefers the election outcome of rejection with probability $\int_{-\infty}^{0} f(\omega;0) d\omega = 1/2$. Since voter i_k is voting according to the probability $\phi_k(C_k^*, \lambda)$, the leader will obtain an expected payoff $-\phi_k(C_k^*, \lambda)$, which is attained (from an ex ante perspective) with probability $(1/2)(1-\lambda)$. Furthermore, the leader (strictly) prefers the election outcome of acceptance with probability $\int_0^{+\infty} f(s;0) ds = 1/2$, conditional on obtaining a signal, and, similarly, with probability $\int_0^{+\infty} f(\omega;0) d\omega = 1/2$, conditional on obtaining no signal. However, provided that $k \leq n/2$, the proof of Proposition 1–(a) showed that in such cases the leader chooses optimally to disclose all obtained signals and, at the same time, the election outcome is acceptance with probability one. Thus, the leader obtains a zero payoff in all those cases.

By combining all the arguments above, it follows that the (ex ante) expected utility of the moderate leader takes the form

$$
\begin{aligned}
& U_0(\lambda;x,k) = \\
& (1/2)\lambda \Big[-\phi_k(C_k^*, \lambda) \int_{C_k^*} f(s;0) ds \Big] + (1/2)(1-\lambda) \Big[-\phi_k(C_k^*, \lambda) \Big] - c(\lambda) \\
& \qquad = -(1/2)\phi_k(C_k^*, \lambda) \big[1 - \lambda \mathbb{P}_l[s \notin C_k^*] \big] - c(\lambda).
\end{aligned} \tag{20}
$$

(b) Consider a majority rule $k > n/2$. Suppose that the leader chooses a research effort $\lambda \in (0, \bar{\lambda}]$. Then, with such a probability λ the leader receives a signal s, and with probability $1-\lambda$ obtains no signal ($s = \emptyset$). First, since $\mu_l = 0$, it follows that, conditional on obtaining a signal, the leader (strictly) prefers the election outcome of acceptance with probability $\int_0^{+\infty} f(s;0)ds = 1/2$. Proposition 1–(b) showed that, at the interim stage, the leader optimally chooses to conceal a subset of signals $C_k^* = C_k^*(0;x) \subseteq [0, \bar{s}_{i_k})$. In this case, the proof of Proposition 1–(b) showed that the leader is able to induce rejection with probability $1-\phi_k(C_k^*,\lambda)$, so that his expected payoff is $-\left[1-\phi_k(C_k^*,\lambda)\right]\int_{C_k^*} f(s;0)ds$. As mentioned, this outcome is attained from an ex ante perspective with probability $(1/2)\lambda$. Similarly, if the leader obtains no signal, then he has no choice to make with respect to the concealment set. In this case, the leader (strictly) prefers the election outcome of acceptance with probability $\int_0^{+\infty} f(\omega;0)d\omega = 1/2$. Since voter $i=k$ is voting according to the probability $\phi_k(C_k^*,\lambda)$, the leader will obtain an expected payoff $-\left[1-\phi_k(C_k^*,\lambda)\right]$, which is attained from an ex ante perspective with probability $(1/2)(1-\lambda)$. Secondly, the leader (strictly) prefers the election outcome of rejection with probability $\int_{-\infty}^{0} f(s;0)ds = 1/2$, conditional on obtaining a signal, and, similarly, with probability $\int_{-\infty}^{0} f(\omega;0)d\omega = 1/2$, conditional on obtaining no signal. However, provided that $k > n/2$, the proof of Proposition 1–(b) showed that in such cases the leader chooses optimally to disclose all obtained signals and, at the same time, the election outcome is rejection with probability one. Thus, the leader obtains a zero payoff in all those cases.

By combining all those arguments, it follows that the (ex ante) expected utility of the moderate leader takes the form

$$\begin{aligned} &U_0(\lambda;x,k) = \\ &(1/2)\lambda\Big[-\left[1-\phi_k(C_k^*,\lambda)\right]\int_{C_k^*} f(s;0)ds\Big] + (1/2)(1-\lambda)\Big[-\left[1-\phi_k(C_k^*,\lambda)\right]\Big] - c(\lambda) \\ &\qquad = -(1/2)\phi_k\left[1-\phi_k(C_k^*,\lambda)\right]\left[1-\lambda\mathbb{P}_l[s\notin C_k^*]\right] - c(\lambda). \end{aligned} \tag{21}$$

We can now proceed as follows.

(a) Consider a majority rule $k \le n/2$. Then, since $\mu_k > 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s;\mu_{i_k})ds \ge \bar{s}_{i_k}$ for $C_k^* = [\bar{s}_{i_k}, 0)$, we observe from Eq. (7) that $\mathscr{J}(k) = 1$ and $\mathscr{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*,\lambda) = 1$. From Eq. (20), we obtain then

$$U_0(\lambda;x,k) = -(1/2)\left[1-\lambda\mathbb{P}_l[s\notin C_k^*]\right] - c(\lambda). \tag{22}$$

(b) Consider a majority rule $k > n/2$. Then, since $\mu_{i_k} < 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s;\mu_{i_k})ds < \bar{s}_{i_k}$ for $C_k^* = [0, \bar{s}_{i_k})$, we observe from Eq. (7) that $\mathscr{J}(k) = 0$ and $\mathscr{I}(k, C_k^*) = 0$. Therefore, $\phi_k(C_k^*,\lambda) = 0$. From Eq. (21), we obtain then that $U_0(\lambda;x,k)$ takes the same expression as in Eq. (22) above. ∎

Proof of LEMMA 3. We provided complete arguments for the the proof of Lemma 2, i.e., for the case in which the leader is moderate ($\mu_l = 0$). Most of the arguments required for the case in which the leader is a radical leader in favor of the new initiative ($\mu_l = \overline{\mu}$)

are completely analogous. Therefore, we build upon such arguments for the case $\mu_l = 0$ developed in the proof of Lemma 2. Suppose that the leader is a radical in favor of approving the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Arguments totally analogous to the ones used for the case in which the leader is moderate yield

$$U_{\overline{\mu}}(\lambda; x, k) = -\left[1 - \phi_k(C_k^*, \lambda)\right]\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda), \tag{23}$$

for $C_k^* = C_k^*(\overline{\mu}; x)$ and for any $k \in N$. We can now proceed as follows.

(a) Consider a majority rule $k \leq n/2$. Then, since $\mu_{i_k} > 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} sf(s; \mu_{i_k}) ds < \bar{s}_{i_k}$ for $C_k^* = (-\infty, \bar{s}_{i_k})$, it follows that $\mathscr{J}(k) = 1$ and $\mathscr{I}(k, C_k^*) = 0$. We then observe from Eq. (7) that $\phi_k(C_k^*, \lambda) = 1 - \pi_k(C_k^*, \lambda)$. From Eq. (23), we obtain then that

$$U_{\overline{\mu}}(\lambda; x, k) = -\pi_k(C_k^*, \lambda)\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda). \tag{24}$$

(b) Consider a majority rule $k > n/2$. Then, since $\mu_{i_k} < 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} sf(s; \mu_{i_k}) ds < \bar{s}_{i_k}$ for C_k^*, it follows that $\mathscr{J}(k) = 0$ and $\mathscr{I}(k, C_k^*) = 0$. We then observe from Eq. (7) that $\phi_k(C_k^*, \lambda) = 0$. From Eq. (23), we obtain

$$U_{\overline{\mu}}(\lambda; x, k) = -\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*(\overline{\mu})]\right] - c(\lambda). \tag{25}$$

∎

Proof of Lemma 4. We provided complete arguments for the the proof of Lemma 2, i.e., for the case in which the leader is moderate ($\mu_l = 0$). Most of the arguments required for the case in which the leader is a radical leader in favor of the status quo ($\mu_l = \underline{\mu}$) are completely analogous. Therefore, we build upon such arguments for the case $\mu_l = 0$ developed in the proof of Lemma 2. Suppose that the leader is a radical in favor of approving the status quo ($\mu_l = \underline{\mu} \to -\infty$). Arguments totally analogous to the ones used for the case in which the leader is moderate yield for any $k \in \{1, \ldots, n\}$, we have

$$U_{\underline{\mu}}(\lambda; x, k) = -\phi_k(C_k^*, \lambda)\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda), \tag{26}$$

for $C_k^* = C_k^*(\underline{\mu}; x)$ and for any $k \in N$.

(a) Consider a majority rule $k \leq n/2$. Then, since $\mu_{i_k} > 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} sf(s; \mu_{i_k}) ds \geq \bar{s}_{i_k}$ for $C_k^* = [\bar{s}_{i_k}, +\infty)$, it follows that $\mathscr{J}(k) = 1$ and $\mathscr{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*, \lambda) = 1$. From Eq. (27), we obtain

$$U_{\underline{\mu}}(\lambda; x, k) = -\left[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\right] - c(\lambda). \tag{27}$$

(b) Consider a majority rule $k > n/2$. Then, since $\mu_{i_k} < 0$ and $\mathbb{E}_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} sf(s; \mu_{i_k}) ds \geq \bar{s}_{i_k}$ for C_k^*, we have $\mathscr{J}(k) = 0$ and $\mathscr{I}(k, C_k^*) = 1$. It follows from Eq. (7)

that $\phi_k(C_k^*,\lambda)=\pi_k(C_k^*,\lambda)$.From Eq. (27), we obtain then

$$U_{\underline{\mu}}(\lambda;x,k)=-\pi_k(C_k^*,\lambda)\left[1-\lambda\mathbb{P}_l[s\notin C_k^*]\right]-c(\lambda). \tag{28}$$

∎

Proof of PROPOSITION 4. The proof of the proposition makes use of the expression for the ex ante utility of the leader derived in Lemma 2.

Suppose that the leader is moderate ($\mu_l=0$). Partial derivation of the expression for $U_0(\lambda;x,k)$ in Eq. (22) with respect to λ yields

$$\frac{\partial U_0(\lambda;x,k)}{\partial\lambda}=(1/2)\mathbb{P}_l[s\notin C_k^*]-c'(\lambda). \tag{29}$$

We can resort to explore the required first order conditions in order to maximize the expression for $U_0(\lambda;k)$ in Eq. (22). First, note that $\lambda^*\to 0$ is consistent with a (corner) optimal investigation behavior only if $\lim_{\lambda\to 0}\varphi_0(0;x,k)=\lim_{\lambda\to 0}\partial U_0(0;x,k)/\partial\lambda\leq 0$. Given that $\mathbb{P}_l[s\notin C_k^*]>0$, such a plausible optimal behavior is ruled out by the Inada condition $\lim_{\lambda\to 0}c'(\lambda)=0$ because it directly leads to $\lim_{\lambda\to 0}\varphi_0(\lambda;x,k)=\mathbb{P}_l[s\notin C_k^*]>0$. Secondly, $\lambda^*=\overline{\lambda}$ can be a (corner) solution to the associated problem only if $\varphi_0(\overline{\lambda};k)=\partial U_0(\overline{\lambda};x,k)/\partial\lambda\geq 0$. Given that $\mathbb{P}_l[s\notin C_k^*]\in(0,1)$, this possible solution is ruled out by the Inada condition $\lim_{\lambda\to\overline{\lambda}}c'(\lambda)=+\infty$ because it implies that $\lim_{\lambda\to\overline{\lambda}}\varphi_0(\lambda;x,k)<0$. We are then left only with (well-behaved) interior solutions as possible candidates to maximize the expression for $U_0(\lambda;x,k)$ in Eq. (22).

Then, the first order condition $\varphi_0(\lambda^*;x,k)=\partial U_0(\lambda^*;x,k)/\partial\lambda=0$ yields

$$\mathbb{P}_l[s\notin C_k^*]=2c'(\lambda^*),$$

where $C_k^*=C_k^*(0;x)\subseteq[\bar{s}_{i_k},0)$ for $k\leq n/2$ and $C_k^*=C_k^*(0;\bar{s}_{i_k})\subseteq[0,\bar{s}_{i_k})$ for $k>n/2$.

Inspection of the partial derivative derived in Eq. (29) leads us to conclude that the function $U_0(\lambda;x,k)$ is strictly concave in λ because the cost $c(\lambda)$ is assumed to be strictly convex in λ. Finally, note that our assumptions on the cost function c directly imply that $\varphi_0(\lambda;x,k)$ is a continuous function in the interval $(0,\overline{\lambda})$. Given that the Inada on conditions on c guarantee that $\lim_{\lambda\to 0}\varphi_0(\lambda;k)>0$ and $\lim_{\lambda\to\overline{\lambda}}\varphi_0(\lambda;x,k)<0$, it follows from the intermediate value theorem that we can ensure the existence of a value $\lambda^*\in(0,\overline{\lambda})$ such that $\varphi_0(\lambda^*;x,k)=0$. Furthermore, since the function c is strictly increasing, it follows that such a value $\lambda^*\in(0,\overline{\lambda})$ is unique. ∎

Proof of PROPOSITION 5. The proof of the proposition makes use of the expression for the ex ante utility of the leader derived in Lemma 3. Suppose that the leader is a radical in favor of approving the new initiative ($\mu_l=\overline{\mu}\to+\infty$).

(a) Consider a majority rule $k \le n/2$. Partial derivation of the expression for $U_{\overline{\mu}}(\lambda; x, k)$ given by Eq. (24) with respect to λ yields

$$\frac{\partial U_{\overline{\mu}}(\lambda; x, k)}{\partial \lambda} = \Big\{ \pi_k \mathbb{P}_l[s \notin C_k^*] - (\partial \pi_k / \partial \lambda)\big[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\big] \Big\} - c'(\lambda). \tag{30}$$

From the expression given by Eq. (30), notice first that (in a manner totally analogous as argued earlier in the proof of Proposition 4) the plausible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that can be derived from the problem that faces the leader when he chooses λ are ruled out by the respective Inada conditions $\lim_{\lambda \to 0} c'(\lambda) = 0$ and $\lim_{\lambda \to \overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with (well-behaved) interior solutions (bounded away from $\lambda = 0$) as possible candidates to maximize the expression for $U_{\overline{\mu}}(\lambda; k)$ in Eq. (24). The first order condition $\varphi_{\overline{\mu}}(\lambda^*; x, k) = \partial U_{\overline{\mu}}(\lambda^*; x, k)/\partial \lambda = 0$ yields

$$\pi_k \mathbb{P}_l[s \notin C_k^*] - (\partial \pi_k / \partial \lambda)\big[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\big] = c'(\lambda^*),$$

where $C_k^* = C_k^*(\overline{\mu}; x)$.

To guarantee that the condition above is also sufficient for an interior choice λ^* to maximize the leader's ex ante utility still need to check for the concavity of the expression for the ex ante utility $U_{\overline{\mu}}(\lambda; x, k)$ given by Eq. (24) (with respect to λ). Using the fact that $\partial \pi_i / \partial \lambda = \mathbb{P}_i[s \in C] / \big(1 - \lambda \mathbb{P}_i[s \notin C]\big)^2$ and $\partial^2 \pi_i / \partial \lambda^2 = 2\mathbb{P}_i[s \in C]\mathbb{P}_i[s \notin C] / \big(1 - \lambda \mathbb{P}_i[s \notin C]\big)^3$, (for $C = C_k^*$), further algebra over the expression in Eq. (30) yields

$$\begin{aligned} \frac{\partial^2 U_{\overline{\mu}}(\lambda; x, k)}{\partial \lambda^2} &= \Big\{ 2\frac{\partial \pi_k}{\partial \lambda} \mathbb{P}_l[s \notin C_k^*] - \frac{\partial^2 \pi_k}{\partial \lambda^2}\big[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\big] \Big\} - c''(\lambda) \\ &= \frac{2\mathbb{P}_k[s \in C_k^*]}{\big(1 - \lambda \mathbb{P}_k[s \notin C_k^*]\big)^3} \Big(\mathbb{P}_k[s \in C_k^*] - \mathbb{P}_l[s \in C_k^*] \Big) - c''(\lambda), \end{aligned}$$

for $C_k^* = C_k^*(\overline{\mu}; x) \subseteq (-\infty, \bar{s}_{i_k})$. Then, recall that $\mathbb{P}_k[s \in C_k^*] = \int_{-\infty}^{\bar{s}_{i_k}} f(s; \mu_{i_k}) ds$ and $\mathbb{P}_l[s \in C_k^*] = \int_{-\infty}^{\bar{s}_{i_k}} f(s; \overline{\mu}) ds$. Therefore, for $\bar{s}_{i_k} < 0 < \mu_{i_k} < \overline{\mu} \to +\infty$, we have that $\mathbb{P}_k[s \in C_k^*] < \mathbb{P}_l[s \in C_k^*]$.[27] It follows that $\partial^2 U_{\overline{\mu}}(\lambda; x, k)/\partial \lambda^2 < 0$ and, therefore, that the ex ante utility of $U_{\overline{\mu}}(\lambda; x, k)$ is (strictly) concave for each $\lambda \in (0, \overline{\lambda})$.

Finally, using the expressions of π_k in Eq. (3) and the expression of $\partial \pi_k / \partial \lambda$ above, we observe that $\varphi_{\overline{\mu}}(\lambda; x, k)$ is continuous in $\lambda \in (0, \overline{\lambda})$. In a manner totally analogous as in the proof of Proposition 4, we can invoke the intermediate value theorem to conclude that there exists a unique $\lambda^* \in (0, \overline{\lambda})$ such that $\varphi_{\overline{\mu}}(\lambda^*; x, k) = 0$.

(b) Consider a majority rule $k > n/2$. Partial derivation of the expression for $U_{\overline{\mu}}(\lambda; x, k)$

[27] Leaving aside the technical requirement $\overline{\mu} \to +\infty$, the condition that in fact guarantees the stated argument is that the difference between opinions μ_{i_k} and $\overline{\mu}$ be sufficiently large.

given by Eq. (25) with respect to λ yields

$$\frac{\partial U_{\overline{\mu}}(\lambda;k)}{\partial \lambda} = \mathbb{P}_l[s \notin C_k^*(\overline{\mu})] - c'(\lambda). \tag{31}$$

From the expression given by Eq. (31), notice first that (as argued earlier in the proof of Proposition 4) the possible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that might result from the problem that faces the leader when he chooses λ are ruled out by the respective Inada conditions $\lim_{\lambda\to 0} c'(\lambda) = 0$ and $\lim_{\lambda\to\overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with interior solutions (bounded away from $\lambda = 0$) as possible candidates to maximize the expression for $U_{\overline{\mu}}(\lambda;x,k)$ in Eq. (25). The first order condition $\varphi_{\overline{\mu}}(\lambda^*;x,k) = \partial U_{\overline{\mu}}(\lambda^*;x,k)/\partial\lambda = 0$ yields

$$\mathbb{P}_l[s \notin C_k^*(\overline{\mu})] = c'(\lambda^*),$$

where $C_k^*(\overline{\mu};x) \subseteq (-\infty, \bar{s}_{i_k})$.

Finally, inspection of the derivative obtained in Eq. (31) leads to that, contingent on majority rules $k > n/2$, $U_{\overline{\mu}}(\lambda;x,k)$ is strictly concave in λ because the cost $c(\lambda)$ is assumed to be strictly convex in λ. In addition, under our assumptions on the cost c, we can again apply again the intermediate value theorem to the function $\varphi_{\overline{\mu}}(\lambda;x,k)$ to conclude that there exists a unique $\lambda^* \in (0,\overline{\lambda})$ such that $\varphi_{\overline{\mu}}(\lambda^*;x,k) = 0$. ∎

Proof of Proposition 6. The proof of the proposition makes use of the expression for the ex ante utility of the leader derived in Lemma 4. Suppose that the leader is a radical in favor of remaining in the status quo ($\mu_l = \underline{\mu} \to -\infty$).

(a) Consider a majority rule $k \leq n/2$. Partial derivation of the expression for $U_{\underline{\mu}}(\lambda;x,k)$ given by Eq. (27) with respect to λ yields

$$\frac{\partial U_{\underline{\mu}}(\lambda;x,k)}{\partial \lambda} = \mathbb{P}_l[s \notin C_k^*(\underline{\mu})] - c'(\lambda). \tag{32}$$

From the expression given by Eq. (32), notice first that (as argued earlier in the proof of Proposition 4) the plausible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that might result from the problem that faces the leader when he chooses λ are ruled out by the respective Inada conditions $\lim_{\lambda\to 0} c'(\lambda) = 0$ and $\lim_{\lambda\to\overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with interior solutions as possible candidates to maximize the expression for $U_{\underline{\mu}}(\lambda;x,k)$ in Eq. (27). The first order condition $\varphi_{\underline{\mu}}(\lambda^*;x,k) = \partial U_{\underline{\mu}}(\lambda^*;x,k)/\partial\lambda = 0$ yields

$$\mathbb{P}_l[s \notin C_k^*(\underline{\mu})] = c'(\lambda^*),$$

where $C_k^*(\underline{\mu};x) \subseteq [\bar{s}_{i_k}, +\infty)$.

Finally, inspection of the derivative obtained in Eq. (32) leads to that, contingent on majority rules $k \leq n/2$, $U_{\underline{\mu}}(\lambda;x,k)$ is strictly concave in λ because the cost $c(\lambda)$ is assumed to be strictly convex in λ. In addition, under our assumptions on the cost c, we can again apply again the intermediate value theorem to the function $\varphi_{\underline{\mu}}(\lambda;x,k)$ to conclude that there exists a unique $\lambda^* \in (0,\overline{\lambda})$ such that $\varphi_{\underline{\mu}}(\lambda^*;x,k) = 0$.

(b) Consider a majority rule $k > n/2$. Partial derivation of the expression for $U_{\overline{\mu}}(\lambda;x,k)$ given by Eq. (28) with respect to λ yields

$$\frac{\partial U_{\underline{\mu}}(\lambda;x,k)}{\partial \lambda} = \Big\{\pi_k \mathbb{P}_l[s \notin C_k^*] - (\partial \pi_k/\partial \lambda)\big[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\big]\Big\} - c'(\lambda). \tag{33}$$

From the expression given by Eq. (33), notice first that (as argued earlier in the proof of Proposition 4) the possible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that might result from the problem that faces the leader when he chooses λ are ruled out by the respective Inada conditions $\lim_{\lambda\to 0} c'(\lambda) = 0$ and $\lim_{\lambda\to\overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with interior solutions (bounded away from $\lambda = 0$) as possible candidates to maximize the expression for $U_{\underline{\mu}}(\lambda;x,k)$ in Eq. (28). The first order condition $\partial U_{\underline{\mu}}(\lambda^*;x,k)/\partial\lambda = 0$ yields

$$\pi_k \mathbb{P}_l[s \notin C_k^*] - (\partial \pi_k/\partial \lambda)\big[1 - \lambda \mathbb{P}_l[s \notin C_k^*]\big] = c'(\lambda^*),$$

where $C_k^* = C_k^*(\underline{\mu};x) \subseteq [\bar{s}_{i_k}, +\infty)$.

To guarantee that the condition above is also sufficient for an interior choice λ^* to maximize the leader's ex ante utility still need to check for the concavity of the expression for the ex ante utility $U_{\underline{\mu}}(\lambda;x,k)$ given by Eq. (28) (with respect to λ). By proceeding exactly as in 2.–(a), we derive

$$\frac{\partial^2 U_{\underline{\mu}}(\lambda;x,k)}{\partial \lambda^2} = \frac{2\mathbb{P}_k[s \in C_k^*]}{\big(1 - \lambda \mathbb{P}_k[s \notin C_k^*]\big)^3}\Big(\mathbb{P}_k[s \in C_k^*] - \mathbb{P}_l[s \in C_k^*]\Big) - c''(\lambda),$$

for $C_k^* = C_k^*(\underline{\mu};x) \subseteq [\bar{s}_{i_k}, +\infty)$. Then, recall that $\mathbb{P}_k[s \in C_k^*] = \int_{\bar{s}_{i_k}}^{\infty} f(s;\mu_{i_k})ds$ and $\mathbb{P}_l[s \in C_k^*] = \int_{\bar{s}_{i_k}}^{\infty} f(s;\underline{\mu})ds$. Therefore, for $\underline{\mu} < \mu_{i_k} < 0 < \bar{s}_{i_k}$, with $\underline{\mu} \to -\infty$, we have that $\mathbb{P}_k[s \in C_k^*] < \mathbb{P}_l[s \in C_k^*]$.[28] It follows that $\partial^2 U_{\overline{\mu}}(\lambda;x,k)/\partial\lambda^2 < 0$ and, therefore, that the ex ante utility of $U_{\overline{\mu}}(\lambda;x,k)$ is (strictly) concave for each $\lambda \in (0,\overline{\lambda})$.

Finally, using the expressions of π_k and $\partial\pi_k/\partial\lambda$ given, respectively, in Eq. (3) and **??**, we observe that $\varphi_{\underline{\mu}}(\lambda;x,k)$ is continuous in $\lambda \in (0,\overline{\lambda})$. In a manner totally analogous as in the proof of Proposition 4, we can invoke the intermediate value theorem to conclude that there exists a unique $\lambda^* \in (0,\overline{\lambda})$ such that $\varphi_{\underline{\mu}}(\lambda^*;x,k) = 0$. ∎

Proof of PROPOSITION 7. The proof of the proposition makes use of the expressions for the (ex ante) expected utility of the leader which were derived in Lemma 2–Lemma 4.

(a) Consider the moderate leader ($\mu_l = 0$). Consider any majority rule $k \in N$. Then,

[28] Leaving aside the technical requirement $\underline{\mu} \to -\infty$, the condition that in fact guarantees the stated argument is that the difference between opinions μ_{i_k} and $\underline{\mu}$ be sufficiently large.

note that $\mathbb{P}_l[s \in C_k^*(0)] = \int_{\bar{s}_{i_k}}^0 f(s;0)ds = (1/2) - F(\bar{s}_{i_k};0)$ for $k \leq n/2$, whereas $\mathbb{P}_l[s \in C_k^*(0)] = \int_0^{\bar{s}_{i_k}} f(s;0)ds = F(\bar{s}_{i_k};0) - (1/2)$ for $k > n/2$. Consider a given optimal investigation effort $\lambda^* \in (0, \overline{\lambda})$ for this leader. Let us use the short-hand notation $U_0 = U_0(\lambda^*;x,k)$ for simplicity. Then, by the envelope theorem, it follows from the expression derived in Lemma 2 that $\partial U_0/\partial \bar{s}_{i_k} = (1/2)\lambda^* f(\bar{s}_{i_k};0) > 0$ for $k \leq n/2$, whereas $\partial U_0/\partial \bar{s}_{i_k} = -(1/2)\lambda^* f(\bar{s}_{i_k};0) < 0$ for $k > n/2$. Therefore, if the moderate leader had the ability to choose $\bar{s}_{i_k}$, he would prefer $\bar{s}_{i_k} \to 0$.

(b) Consider a radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Consider any majority rule $k \in N$. Then, note that $\mathbb{P}_i[s \in C_k^*(\overline{\mu};x)] = F(\bar{s}_{i_k};\mu_i)$ so that

$$\pi_k = \pi_k(C_k^*(\overline{\mu};\lambda);\mu_{i_k}) = \frac{\lambda F(\bar{s}_{i_k};\mu_{i_k})}{\lambda F(\bar{s}_{i_k};\mu_{i_k}) + (1-\lambda)}.$$

Therefore,

$$\frac{\partial \pi_k}{\partial \bar{s}_{i_k}} = \frac{\lambda(1-\lambda) f(\bar{s}_{i_k};\mu_{i_k})}{\left[\lambda F(\bar{s}_{i_k};\mu_{i_k}) + (1-\lambda)\right]^2} > 0.$$

Consider a given optimal investigation effort $\lambda^* \in (0, \overline{\lambda})$ for this leader. Let us use the short-hand notation $U_{\overline{\mu}} = U_{\overline{\mu}}(\lambda^*;x,k)$ for simplicity. Then, by the envelope theorem, it follows from the expression derived in (a) of Lemma 3 for $k \leq n/2$, that

$$\frac{\partial U_{\overline{\mu}}}{\partial \bar{s}_{i_k}} = -\left[\frac{\partial \pi_k}{\partial \bar{s}_{i_k}}\left[(1-\lambda^*) + \lambda^* F(\bar{s}_{i_k};\overline{\mu})\right] + \pi_k \lambda^* f(\bar{s}_{i_k};\overline{\mu})\right] < 0.$$

In addition, it follows from the expression derived in (b) of Lemma 3 for $k > n/2$, that

$$\frac{\partial U_{\overline{\mu}}}{\partial \bar{s}_{i_k}} = -\lambda^* f(\bar{s}_{i_k};\overline{\mu}) < 0.$$

Therefore, if the radical leader had the ability to choose $\bar{s}_{i_k}$, he would prefer $\bar{s}_{i_k} \to -\infty$.

(c) Consider a radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). Consider any majority rule $k \in N$. Then, note that $\mathbb{P}_i[s \in C_k^*(\underline{\mu};x)] = 1 - F(\bar{s}_{i_k};\mu_{i_k})$ so that

$$\pi_k = \pi_k(C_k^*(\underline{\mu};x);\mu_{i_k}) = \frac{\lambda - \lambda F(\bar{s}_{i_k};\mu_{i_k})}{1 - \lambda F(\bar{s}_{i_k};\mu_{i_k})}.$$

Therefore,

$$\frac{\partial \pi_k}{\partial \bar{s}_{i_k}} = \frac{-\lambda(1-\lambda) f(\bar{s}_{i_k};\mu_{i_k})}{\left[1 - \lambda F(\bar{s}_{i_k};\mu_{i_k})\right]^2} < 0.$$

Consider a given optimal investigation effort $\lambda^* \in (0, \overline{\lambda})$ for this leader. Let us use the short-hand notation $U_{\underline{\mu}} = U_{\underline{\mu}}(\lambda^*;x,k)$ for simplicity. Then, by the envelope theorem, it follows from the expression derived in (a) of Lemma 4 for $k \leq n/2$, that

$$\frac{\partial U_{\underline{\mu}}}{\partial \bar{s}_{i_k}} = L\lambda^* f(\bar{s}_{i_k};\underline{\mu}) > 0.$$

In addition, it follows from the expression derived in (b) of Lemma 3 for $k > n/2$, that

$$\frac{\partial U_{\underline{\mu}}}{\partial \bar{s}_{i_k}} = -\left[\frac{\partial \pi_k}{\partial \bar{s}_{i_k}}\left[1-\lambda^* F(\bar{s}_{i_k};\underline{\mu})\right] - \pi_k \lambda^* f(\bar{s}_{i_k};\underline{\mu})\right] > 0.$$

Therefore, if the radical leader had the ability to choose $\bar{s}_{i_k}$, he would prefer $\bar{s}_{i_k} \to +\infty$. ∎

Proof of PROPOSITION 8. Consider a majority rule $1 < k \leq n$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that a number $0 < n_e(x) < k$ of voters are educated under profile x and that the leader has the possibility of choosing the composition of the set $N_e(x)$.

(a) Consider a radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). From the results of Proposition 2, we know that the optimal concealment set has the form $C_k^*(\overline{\mu};x) = (-\infty, \bar{s}_{i_k}(x;\varepsilon,\overline{\mu}))$ for any majority rule $k \in N$. Then, notice that the size of the optimal concealment set $C_k^*(\overline{\mu};x)$ is minimized if the profile of education levels x induces an ordering $\sigma(x)$ such that the corresponding signal realization $\bar{s}_{i_k}(x;\varepsilon,\overline{\mu})$ is as low as possible. Since $\bar{s}_i(x;\varepsilon,\overline{\mu}) \to -\infty$ for each $i \in N_e(x)$ the resulting $\bar{s}_{i_k}(x;\varepsilon,\overline{\mu})$ results as low as possible if we choose that voters $i = 1,2,\dots,k-n_e$ do not receive education and that n_e voters from the remaining set $N \setminus \{1,\dots,k-n_e\}$ receive education. This strategy makes voter $i_k = k-n_e$ to be the pivotal voter. The corresponding optimal concealment is therefore given by $C_k^*(\overline{\mu};x) = (-\infty, -\mu_{k-n_e})$. This gives the set of minimal size that can be attained for the optimal concealment set for any ordering of critical signal realizations $\sigma(x)$, under the restriction that $n_e = n_e(x)$. In turn, from the results of Proposition 7, we observe that such an induced minimal size optimal concealment set maximizes the leader's (ex ante) utility in equilibrium when $\mu_l = \overline{\mu} \to +\infty$.

(b) Consider a radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). From the results of Proposition 3, we know that the optimal concealment set has the form $C_k^*(\overline{\mu};x) = (\bar{s}_{i_k}(x;\varepsilon,\underline{\mu}), +\infty)$ for any majority rule $k \in N$. Then, notice that the size of the optimal concealment set $C_k^*(\underline{\mu};x)$ is minimized if the profile of education levels x induces an ordering $\sigma(x)$ such that the corresponding signal realization $\bar{s}_{i_k}(x;\varepsilon,\underline{\mu})$ is as high as possible. Since $\bar{s}_i(x;\varepsilon,\underline{\mu}) \to +\infty$ for each $i \in N_e(x)$ the resulting $\bar{s}_{i_k}(x;\varepsilon,\underline{\mu})$ results as high as possible if we choose that voters $i = k-n_e, k-n_e+1,\dots,n$ do not receive education and that n_e voters from the remaining set $N \setminus \{k-n_e,\dots,n\}$ receive education. This strategy makes voter $i_k = k-n_e$ to be the pivotal voter. The corresponding optimal concealment is therefore given by $C_k^*(\underline{\mu};x) = (-\mu_{k-n_e}, +\infty)$. This gives the set of minimal size that can be attained for the optimal concealment set for any ordering of critical signal realizations $\sigma(x)$, under the restriction that $n_e = n_e(x)$. In turn, from the results of Proposition 7, we observe that such an induced minimal size optimal concealment set maximizes the leader's (ex ante) utility in equilibrium when $\mu_l = \underline{\mu} \to -\infty$. ∎

Proof of PROPOSITION 9. Let us use Δ_W^j to capture the change in the welfare of voters when an arbitrary voter j moves from being uneducated to being educated. Then, let $\Delta_W^j >$ $(\leq)$ 0 indicate an increase (respectively, no change, and decrease) in voters' welfare.

Consider a moderate leader, $\mu_l = 0$, and suppose that $k \leq n/2$. Then, voters' welfare is given by Lemma 5-(a). Let us begin from a situation where no voter is educated. To propose distributions of education levels across voters in order to maximize their welfare, notice first that those $(n/2) - k$ voters whose negative critical signal realizations lie above the one of voter k (e.g., those voters $\{k+1, \ldots, n/2\}$) must not be educated. The reason for this is that we are interested in minimizing the distance between the critical signal realizations of such voters and the one of voter k. In this way, we are able to decrease the the probability according to which, conditional on the leader not obtaining any signal, such voters prefer rejection. Recall, in this case the outcome is acceptance with probability one, that is, $\phi(C_k^*; \lambda) = 1$. Therefore, voters' welfare unambiguously decreases when such voters are educated, whereas voters' welfare unambiguously increases when a voter $j \neq k$ outside of this set, is educated. In consequence, $N_e(x) \subseteq N \setminus \{k+1, \ldots, n/2\}$.

Consider now the set of voters $\{1, \ldots, k-1\} \cup \{n/2, \ldots, n\}$. Then, in order to maximize voters' welfare, we wish that the $n_e(x) - 1$ voters whose critical signal realizations (in the absence of education) are the closest one to zero be educated. The reason for this lies in that, from the perspective of any voter j, the signals received by the leader are normally distributed with mean μ_j. Thus, the voters j whose critical signal realizations in the absence of education are closest to zero, experience the highest possible reduction in the probability that the signal obtained by the leader lis below their own critical signal realizations. In other words, $\mathbb{P}_j[s < \bar{s}_j]$, when such voters j become educated. Also, when such critical signal realizations of such voters tend to zero, we obtain the highest possible reduction in probability $\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)]$. The reason for this lies in that, from the point of view of the moderate leader, signals are normally distributed with zero mean. More specifically, consider two voters, j and m, such that $0 < \bar{s}_j < \bar{s}_m$. If voter j becomes educated so that $0 < \bar{s}'_j < \bar{s}_j$, it then follows that

$$\Delta_W^j \equiv \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)]}^{a} \overbrace{\mathbb{P}_j[s < \bar{s}_j]}^{b} - \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}'_j)]}^{a'} \overbrace{\mathbb{P}_j[s < \bar{s}'_j]}^{b'} > 0.$$

If voter m becomes educated so that $0 < \bar{s}'_m < \bar{s}_m$, we have that:

$$\Delta_W^m \equiv \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_m)]}^{c} \overbrace{\mathbb{P}_m[s < \bar{s}_m]}^{d} - \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}'_m)]}^{c'} \overbrace{\mathbb{P}_m[s < \bar{s}'_m]}^{d'} > 0.$$

The claim is that $\Delta_W^j > \Delta_W^m$. Given that form the point of view of voters j and m

signals are normally distributed with means $0 > \mu_j > \mu_m$, we have that:

$$\overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)]}^{a} - \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}'_j)]}^{a'} > \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_m)]}^{c} - \overbrace{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}'_m)]}^{c'}$$

and

$$\overbrace{\mathbb{P}_j[s < \bar{s}_j]}^{b} - \overbrace{\mathbb{P}_j[s < \bar{s}'_j]}^{b'} > \overbrace{\mathbb{P}_j[s < \bar{s}_m]}^{d} - \overbrace{\mathbb{P}_j[s < \bar{s}'_m]}^{d'},$$

that is: $a - a' > c - c' > 0$ and $b - b' > d - d' > 0$.

The inequality $a - a' > c - c' > 0$ and the fact that $b - b' > 0$, imply that $(a - c)b > (a' - c')b' > 0$, or equivalently $ab - a'b' > cb - c'b'$. The inequality $b - b' > d - d' > 0$ and the fact that $c - c' > 0$, imply that $(b - d)c > (b' - d')c'$, or equivalently $cb - c'b' > cd - c'd'$. Hence, $ab - a'b' > cd - c'd'$, that is, $\Delta_W^j > \Delta_W^m$. The case in which a pair of individuals have critical signal realizations of negative sign or of different sign, is analogous.

At this point, notice that if the number of voters j such that $\bar{s}_j < |\bar{s}_k|$ is smaller than $n_e(x)$, then all these voters must be educated in order to maximize the welfare of the voters. Then, there remain those voters j such that $|\bar{s}_j| > |\bar{s}_k|$, as well as voter k, that might still become educated.

The education of voter k benefits the $n - k$ voters whose critical signal realizations are above the one of this voter because and hurts the $k - 1$ voters whose critical signal realizations are below. Specifically,[29]

$$\Delta_W^k \equiv \left\{\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)]\right\}\alpha + \gamma.$$

In the expression above, we have $\bar{s}'_k > \bar{s}_k$,

$\alpha \equiv \sum_{j=k+1}^{n} \mathbb{P}_j[s < \bar{s}_j] - \sum_{j=1}^{k-1} \mathbb{P}_j[s \geq \bar{s}_j]$, and

$\gamma = \sum_{k+1}^{n/2} \pi_j \mathbf{1}_{\mathbb{E}_{i_j}[s|s \in C_k^*] < \bar{s}_j} - \sum_{k+1}^{n/2} \pi'_j \mathbf{1}_{\mathbb{E}_{i_j}[s|s \in C_k^{*'}] < \bar{s}_j} \geq 0$, with $\pi_j > \pi'_j$ and $C_k^{*'} \subseteq C_k^*$.

Suppose that the voter j who has the smallest critical signal realization such that $\bar{s}_j > |\bar{s}_k|$, is educated. Then, it follows that

$$\Delta_W^j \equiv \mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j] - \mathbb{P}_l[s \in [\bar{s}_k, \bar{s}'_j)]\mathbb{P}_j[s < \bar{s}'_j] > 0,$$

where $\bar{s}'_j < \bar{s}_j$. The reasoning here is analogous to the situation in which the voter m who has the smallest signal realization $\bar{s}_m < 0$ such that $|\bar{s}_m| > |\bar{s}_k|$, is educated. In this case, it

[29] Notice that $\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j]] > 0$ takes the same value for each voter j whose critical signal realization is above the one of voter k. That is also the case for $\mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_k] - \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}'_k]] < 0$ and each voter each voter j whose critical signal realization is below the one of voter k. Moreover, both expressions have the same value and opposite sign.

follows $\Delta_W^m > 0$ as well. Therefore, if

$$\alpha \leq \overline{\alpha} \equiv -\gamma/\{[\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)]\} < 0$$

so that $\Delta_W^k \leq 0$, then voters' welfare is maximized when the n_e voters $j \neq k$ whose critical signal realizations are the closest ones to zero be educated.

Consider now that $\alpha > \overline{\alpha}$. Given that signals are normally distributed with zero mean from the point of view of the leader, for the aforementioned voter j such that $\bar{s}_j > |\bar{s}_k|$ we have that $\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)] > \mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}_k, \bar{s}'_j)]$. Thus, $\overline{\Delta}_W^j \equiv \mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}'_j]$ is an upper bound for Δ_W^j. Further, $\Delta_W^k \geq \overline{\Delta}_W^j$ if and only if:

$$\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)][\alpha - \mathbb{P}_j[s < \bar{s}_j]] + \beta \geq \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)][\alpha - \mathbb{P}_j[s < \bar{s}'_j]]. \tag{34}$$

Given that signals are normally distributed with means zero and $\mu_j < 0$, respectively, from the point of view of the leader and the aforementioned voter j, it follows that

$$\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)] > \mathbb{P}_j[s < \bar{s}_j] - \mathbb{P}_j[s < \bar{s}'_j],$$

or equivalently,

$$\mathbb{P}_j[s < \bar{s}'_j] > \mathbb{P}_j[s < \bar{s}_j] - \left[\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)]\right].$$

Thus, since $\mathbb{P}_j[s < \bar{s}'_j] = \mathbb{P}_j[s < \bar{s}_j] - [\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j)] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)]$ ensures that the difference in right hand side of Eq. (34) attains its highest possible value, we are able to obtain the following sufficient condition in order to maximize voters' welfare: $\alpha \geq \alpha_j \equiv \mathbb{P}_j[s < \bar{s}_j] + \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)]$. As a consequence, in order to maximize the welfare of the voters, voter k must be educated. Finally, for a voter m with $\bar{s}_m < 0$ such that $|\bar{s}_m| > |\bar{s}_k|$, the analysis is completely analogous and, therefore, $\alpha \geq \alpha_m \equiv \mathbb{P}_m[s \geq \bar{s}_m] + \mathbb{P}_l[s \in [\bar{s}_l, \bar{s}'_k)]$ is a sufficient condition. Again, in order to maximize the welfare of the voters, voter k must be educated.

In summary, if $\alpha \leq \overline{\alpha}$, then voters' welfare is maximized when the n_e voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero be educated. On the other hand, if $\alpha > 0$ is sufficiently high, then voters' welfare is maximized when voter k and the $n_e - 1$ voters $j \neq k$ whose critical signal realizations in the absence of education are the closest ones to zero be educated.

The case in which the number of voters j such that $\bar{s}_j < |\bar{s}_k|$ is at least n_e, is analogous. Once the $n_e - 1$ voters $j \neq k$ whose critical signal realizations are the closest ones to zero become educated, there still remains one voter that might be educated. This voter would be either voter k or a certain voter $h \neq j,k$ who satisfies that $0 < \bar{s}_h < |\bar{s}_k|$ and, at the same time, that her critical signal realization $\bar{s}_h$ is the closest one to zero within the opinion spectrum. The change in voters' welfare when voters k and h become educated, are defined in case (a). If $\alpha \leq \overline{\alpha}$, voters' welfare is maximized when the n_e, voters $j \neq k$ whose critical signal realizations are the closest one to zero be educated. Otherwise, $\alpha \geq \alpha_h = \mathbb{P}_h[s < \bar{s}_h] + \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_h)]$ is a sufficient condition for voters' welfare to be maximized when voter k and the $n_e - 1$ voters $j \neq k$ whose critical signal realizations are the closest ones to zero be educated.

The reasoning for $k > n/2$ is analogous and hence omitted. The expression for voters' welfare is provided in Lemma 5-(b). In this case voters in the set $\{(n/2)+1,\ldots,k-1\}$ must not be educated in order to maximize voters' welfare. That can be achieved if $3n/2 - k \geq n_e$, that is, if the number of voters $j \neq k$ outside of the set $\{(n/2)+1,\ldots,k-1\}$ is at least as high as the required number of educated voters, n_e. With regard to the notation in Proposition 9, let

$$\overline{\beta} \equiv -\gamma' / \{[\mathbb{P}_l[s \in [\bar{s}_k, \bar{s}_j] - \mathbb{P}_l[s \in [\bar{s}'_k, \bar{s}_j)]\} < 0.$$

Analogously to case (a), $\gamma' \geq 0$ captures the change in voters' welfare when voter k becomes educated. This change is non-negative because: (i) the concealment set may shrink and hence the expected value of the concealed signals may decrease and (ii) for any voter j whose positive critical signal realization is below the one of voter k, π_j decreases if the concealment set shrinks. ∎

Proof of PROPOSITION 10. Suppose that leader is a radical in favor of the new initiative, $\mu_l = \overline{\mu} \to +\infty$, and that the majority rule satisfies $k > n/2$. Then, the expression for the welfare of the voters is given by Lemma 6. Let us begin from a situation in which no voter is educated. By analogous reasons to the ones provided in the proof of Proposition 9, we know that, in order to maximize voters' welfare, the $k-1-n/2$ voters whose positive critical signal realizations lie below the one of voter k, must not be educated. These voters are the ones in the set $\{(n/2)+1,\ldots,k-1\}$. This would allow to minimize the distance between the critical signal realizations of these voters and the one of voter k. In turn, this would minimize the probability that these voters prefer acceptance, as recall, in this case the outcome is rejection with probability one. The feature that such voters become educated unambiguously reduces voters' welfare. Additionally, if a voter from the set $N \setminus \{(n/2)+1,\ldots,k-1\}$ becomes educated, then voters' welfare unambiguously increases. Therefore, provided that $3n/2 - k \geq n_e(x)$, the set of educated voters must satisfy $N_e(x) \subseteq N \setminus \{(n/2)+1,\ldots,k-1\}$.

The analysis is fully analogous when we consider a radical leader in favor of the status quo, $\mu_l = \underline{\mu} \to -\infty$, and consider that the majority rule satisfies $k \leq n/2$. The expression for the welfare of the voters is given by Lemma 7. Thus, in order to maximize voters' welfare, it must be the case that the set of educated voters satisfies $N_e(x) \subseteq N \setminus \{k+1,\ldots,n/2\}$. This is always possible because there are at least $n/2$ voters such that, if they become educated, then voters' welfare increases whenever $n_e < k \leq n/2$. ∎

Online Appendix B—Additional Derivations

The following Lemma 5- Lemma 7 provide the expressions for voters' welfare depending on the leader's type. For simplicity, as in Section 5, we focus on situations in which voter i_k continues to be pivotal when the leader moves from a hypothetical situation of not concealing signals to doing so (so that $C_k \neq \emptyset$), and voters receive $s = \emptyset$.

In order to understand the expression for voters' welfare, it is important to emphasize that all players anticipate how the leader will optimally conceal and disclose signals. Therefore, all voters consider in a common manner how the leader's obfuscation strategy will affect the probability that the outcome of election be either acceptance or rejection. This is why the probabilities that appear in the subsequent expressions in Lemma 5 - Lemma 7, according to which some signals are concealed and others are disclosed, are considered from the perspective of the leader.

LEMMA 5. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a (centrist) moderate opinion $\mu_l = 0$. Then, conditional on the optimal selection of the concealment set $C_k^*(0;x)$, and for an investigation effort λ, voters' welfare is expressed as follows:

(a) For $k \leq n/2$

$$W^0(\lambda;x,k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})] \mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] \mathbb{P}_j[s < \bar{s}_j]$$

$$+ (1-\lambda)[n/2 + \sum_{j=k+1}^{n/2} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s \in C_k] < \bar{s}_j}] \Big\}.$$

(b) For $k > n/2$

$$W^0(\lambda;x,k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})] \mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] \mathbb{P}_j[s < \bar{s}_j]$$

$$+ (1-\lambda)[n/2 + \sum_{j=n/2+1}^{k-1} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s \in C_k] \geq \bar{s}_j}] \Big\}.$$

Proof of LEMMA 5. The expression for voters' welfare when $k \leq n/2$ directly comes from observing that given the induced ordering $\sigma(x)$ of the voters' critical signal realizations, it follows that the leader's optimal concealment set has the form $C_k^* \subseteq [\bar{s}_{i_k}, 0)$. It is important to advance that although the expressions for voters' welfare are stated by asuming that for $k \leq n/2$, $C_k^* = [\bar{s}_{i_k}, 0)$, they are basically the same, and we arrive to the same conclusions, when $C_k^* \subset [\bar{s}_{i_k}, 0)$ and non-empty as $\phi_k(C_k^*;\lambda) = 1$.[30] The same observation holds for $k > n/2$ where $C_k^* \subseteq [0, \bar{s}_{i_k})$ and we focus on $C_k^* = [0, \bar{s}_{i_k})$, as $\phi_k(C_k^*;\lambda) = 0$. See Lemma 1.

Consider a given voter $j \in N$. When the leader receives a signal (an event which happens with probability λ), we have that:

$\Pr[o(v) = A \mid s < \bar{s}_j] = 0 \quad \text{if } \bar{s}_j < \bar{s}_{i_k}$,

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda)\mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] \quad \text{if } \bar{s}_{i_k} \leq \bar{s}_j < 0$ and

$\Pr[o(v) = A \mid s < \bar{s}_j] = \mathbb{P}_l[s \in [0, \bar{s}_j)] + \phi_k(C_k^*, \lambda)\mathbb{P}_l[s \in C_k^*] \quad \text{if } \bar{s}_j \geq 0$.

Similarly:

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})] + [1 - \phi_k(C_k^*, \lambda)]\mathbb{P}_l[s \in C_k^*] \quad \text{if } \bar{s}_j < \bar{s}_{i_k}$,

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = [1 - \phi_k(C_k^*, \lambda)]\mathbb{P}_l[s \in [\bar{s}_j, 0)] \quad \text{if } \bar{s}_{i_k} \leq \bar{s}_j < 0$, and

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = 0 \quad \text{if } \bar{s}_j \geq 0$.

In this case, by the proof of Lemma 1, $\phi_k(C_k^*, \lambda) = 1$.

When the leader does not receive a signal (an event which happens with probability $1-\lambda$), voters in the set $\{i_1, \ldots, i_k\}$ prefer acceptance. Thus, as stated, $\phi_k(C_k^*, \lambda) = 1$. Then, the voters who lose are those with positive critical signal realizations, as they prefer rejection. Also, voters j such that $\bar{s}_{i_k} \leq \bar{s}_j < 0$ whenever they prefer rejection. That happens for each of these latter voters with probability π_j if $\mathbb{E}_j[s|s \in C_k^*] < \bar{s}_j$. See Eq. (7). We therefore express voters' welfare as:

$$W^0(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})]\mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[n/2 + \sum_{j=k+1}^{n/2} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s \in C_k^*] < \bar{s}_j}]\Big\}.$$

Consider now that $k > n/2$. Consider a given voter $j \in N$ and that. When the leader receives a signal (an event which happens with probability λ), we have that:

$\Pr[o(v) = A \mid s < \bar{s}_j] = \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] + \phi_k(C_k^*, \lambda)\mathbb{P}_l[s \in C_k^*] \quad \text{if } \bar{s}_{i_k} \leq \bar{s}_j$,

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda)\mathbb{P}_l[s \in [0, \bar{s}_j)] \quad \text{if } 0 < \bar{s}_j < \bar{s}_{i_k}$ and

$\Pr[o(v) = A \mid s < \bar{s}_j] = 0 \quad \text{if } \bar{s}_j \leq 0$.

Similarly:

[30] The only minor difference when $C_k^* = \emptyset$ is that when the leader does not receive a signal, the voters whose negative (respectively positive) critical signal realizations are above (respectively below) the one of voter i_k for $k \leq n/2$ (respectively $k > n/2$) accept (respectively reject) with probability one. Recall that the outcome is indeed acceptance (respectively rejection). All the stated results go through in this case.

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = 0 \quad \text{if } \bar{s}_{i_k} \leq \bar{s}_j,$

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = [1 - \phi_k(C_k^*, \lambda)]\mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})] \quad \text{if } 0 < \bar{s}_j < \bar{s}_{i_k}, \text{ and}$

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = \mathbb{P}_l[s \in [\bar{s}_j, 0)] + (1 - \phi_k(C_k^*, \lambda))\mathbb{P}_l[s \in C_k^*] \quad \text{if } \bar{s}_j \leq 0.$

In this case, by the proof of Lemma 1, $\phi_k(C_k^*, \lambda) = 0$.

When the leader does not receive a signal (an event which happens with probability $1-\lambda$), voters in the set $\{i_k, \ldots, i_n\}$ prefer rejection. Thus, as stated, $\phi_k(C_k^*, \lambda) = 0$ as less than k voters prefer acceptance. The voters who lose are those with negative critical signal realizations, as they prefer acceptance. Also, voters j such that $0 < \bar{s}_j < \bar{s}_{i_k}$ lose whenever they prefer acceptance. For each of these latter voters that happens with probability π_j if $\mathbb{E}_j[s|s \in C_k^*] \geq \bar{s}_j$. See Eq. (7). We therefore express voters' welfare as:

$$W^0(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})]\mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[n/2 + \sum_{j=n/2+1}^{k-1} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s \in C_k^*] \geq \bar{s}_j}] \Big\}.$$

∎

LEMMA 6. Consider a majority rule $k > n/2$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical in favor of the new initiative, $\mu_l = \overline{\mu} \to +\infty$. Then, conditional on the optimal selection of the concealment set $C_k^*(\overline{\mu}; x)$ and for an investigation effort λ voters' welfare is expressed as follows:

$$W^{\overline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})]\mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[\sum_{j=1}^{n/2} \mathbf{1}_{\mathbb{E}_j[s|s \in C_k^*] \geq \bar{s}_j} + (1 - \mathbf{1}_{\mathbb{E}_{i_j}[s|s \in C_k^*] \geq \bar{s}_j})(1-\pi_j) + \sum_{j=n/2+1}^{k-1} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s \in C_k^*] \geq \bar{s}_j} \Big\}.$$

Proof of LEMMA 6. The expression for voters' welfare when $k > n/2$ directly comes from observing that given the induced ordering $\sigma(x)$ of the voters' critical signal realizations it follows that $C_k^* = [-\infty, \bar{s}_{i_{n/2}}) \cup B$, for $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k}(x; \varepsilon, \overline{\mu}))$ and $\overline{\mu} \to +\infty$.[31] Consider a given voter $j \in N$. When the the leader receives a signal (an event that happens with probability λ), we have that:

[31] As in the previous Lemma 5, the expressions for voters' welfare are stated by assuming that for $k > n/2$, C_k^* is the largest concealment set. The analysis holds when the optimal concealment set shrinks, in the set inclusion order, as in this case by the proof of Lemma 1, $\phi(C_k^*; \lambda) = 0$. The only minor difference when C_k^* shrinks is that, when the leader does not receive a signal, some voters whose positive critical signal realizations are below the one of voter i_k may reject with probability one. All the stated results go through in this case.

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda)\mathbb{P}_l[s \in (-\infty, \bar{s}_j)]$ if $\bar{s}_j < \bar{s}_{i_k}$, and

$\Pr[o(v) = A \mid s < \bar{s}_j] = \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] + \phi_k(C_k^*, \lambda)\mathbb{P}_l[s \in C_k^*]$ if $\bar{s}_j > \bar{s}_{i_k}$.

Similarly:

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = (1 - \phi_k(C_k^*, \lambda))\mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})]$, if $\bar{s}_j < \bar{s}_{i_k}$ and

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = 0$ if $\bar{s}_j > \bar{s}_{i_k}$.

By the proof of Lemma 3, in this case $\phi_k(C_k^*, \lambda) = 0$.

When the leader does not receive a signal (an event which happens with probability $1 - \lambda$), voters in the set $\{i_k, \ldots, i_n\}$ prefer rejection. Thus, as stated, $\phi_k(C_k^*, \lambda) = 0$, as less than k voters prefer acceptance. Then, voters j with $\bar{s}_j < 0$ lose when they voter for acceptance. That happens for each of them with probability one if $\mathbb{E}_j[s|s \in C_k^*] \geq \bar{s}_j$ or with probability $1 - \pi_j$ otherwise. Also, the voters j such that $0 < \bar{s}_j < \bar{s}_{i_k}$ lose when they prefer acceptance. That happens for each of tem with probability π_j if $\mathbb{E}_j[s|s \in C_k^*] \geq \bar{s}_j$. See Eq. (7). We therefore express voters' welfare as:

$$W^{\overline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})]\mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[\sum_{j=1}^{n/2} \mathbf{1}_{\mathbb{E}_j[s|s\in C_k^*] \geq \bar{s}_j} + (1 - \mathbf{1}_{\mathbb{E}_j[s|s\in C_k^*] \geq \bar{s}_j})(1 - \pi_j) + \sum_{j=n/2+1}^{k-1} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s\in C_k^*] \geq \bar{s}_j}] \Big\}.$$

∎

Lemma 7. Consider a majority rule $k \leq n/2$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical in favor of the status quo initiative, $\mu_l = \underline{\mu} \to -\infty$. Then, conditional on the optimal selection of the concealment set $C_k^*(\underline{\mu}; x)$ and for an investigation effort λ voters' welfare is expressed as follows:

$$W^{\underline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})]\mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[\sum_{j=k+1}^{n/2} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s\in C_k^*] < \bar{s}_j} + \sum_{j=n/2+1}^{n} \mathbf{1}_{\mathbb{E}_j[s|s\in C_k^*] < \bar{s}_j} + (1 - \mathbf{1}_{\mathbb{E}_j[s|s\in C_k^*] < \bar{s}_j})(1 - \pi_j)] \Big\}.$$

Proof of Lemma 7. The expression for voters' welfare when $k \leq n/2$ directly comes from observing that given the induced ordering $\sigma(x)$ of the voters' critical signal realizations it follows that that $C_k^* = (\bar{s}_{i_{(n/2)+1}}, +\infty] \cup B$, for $B \subseteq (\bar{s}_{i_k}(x; \varepsilon, \underline{\mu}), \bar{s}_{i_{(n/2)+1}}]$ and $\underline{\mu} \to -\infty$.[32] Consider a given voter $j \in N$. When the leader receives a signal (an event which happens with probability λ), we have that:

[32] As in the previous Lemma 5- Lemma 6, the expressions for voters' welfare are stated by assuming

$\Pr[o(v) = A \mid s < \bar{s}_j] = 0 \quad \text{if } \bar{s}_j < \bar{s}_{i_k} \text{ and}$

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda)\mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] \quad \text{if } \bar{s}_{i_k} \leq \bar{s}_j.$

Similarly:

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})] + (1 - \phi_k(C_k^*, \lambda))\mathbb{P}_l[s \in C_k] \text{ if } \bar{s}_j < \bar{s}_{i_k} \text{ and}$

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = (1 - \phi_k(C_k^*, \lambda))\mathbb{P}_l[s \in (s_j, +\infty)] \quad \text{if } \bar{s}_{i_k} \leq \bar{s}_j.$

By the proof of Lemma 1, in this case $\phi_k(C_k^*, \lambda) = 1$.

When the leader does not receive a signal (an event which happens with probability $1 - \lambda$), voters in the set $\{i_1, \ldots, i_k\}$ prefer acceptance. Thus, as stated, $\phi_k(C_k^*, \lambda) = 1$. Then, voters j such that $\bar{s}_{i_k} < \bar{s}_j < 0$ incur a loss whenever they prefer rejection. That happens for each of them with probability π_j if $\mathbb{E}_j[s|s \in C_k^*] < \bar{s}_j$. Voters j such that $\bar{s}_{i_k} \geq 0$ also incur a loss whenever they prefer rejection. That happens for each of them with probability one if $\mathbb{E}_j[s|s \in C_k^*] < \bar{s}_j$ or with probability $1 - \pi_j$ otherwise. See Eq. (7). We therefore express voters' welfare as:

$$W^{\underline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda \mathbb{P}_l[s \in [\bar{s}_j, \bar{s}_{i_k})]\mathbb{P}_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda \mathbb{P}_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]\mathbb{P}_j[s < \bar{s}_j]$$

$$+ (1 - \lambda)[\sum_{j=k+1}^{n/2} \pi_j \mathbf{1}_{\mathbb{E}_j[s|s \in C_k^*] < \bar{s}_j} + \sum_{j=n/2+1}^{n} \mathbf{1}_{\mathbb{E}_j[s|s \in C_k^*] < \bar{s}_j} + (1 - \mathbf{1}_{\mathbb{E}_j[s|s \in C_k^*] < \bar{s}_j})(1 - \pi_j)] \Big\}.$$

∎

that for $k \leq n/2$, C_k^* is the largest concealment set. The analysis holds when the optimal concealment set shrinks, in the set inclusion order, as in this case by the proof of Lemma 1, $\phi(C_k^*; \lambda) = 1$. The only minor difference when C_k^* shrinks is that when the leader does not receive a signal, some voters whose negative critical signal realizations are above the one of voter i_k, may accept with probability one. All the stated results go through in this case.